Mathematics of Elasticity.

A Treatise on the Mathematical Theory of Elasticity. By Prof. A. E. H. Love. Third edition. Pp. xviii+624. (Cambridge: At the University Press, 1920.) Price 37*s*. 6*d*. net.

THIS is the third edition of the classical treatise in the English language upon the theory of elasticity, and all students of the subject will be grateful to Prof. Love for having brought his masterly exposition of this difficult but fascinating branch of mathematical science up to date. The book is now thoroughly well established as part of the education of such mathematicians as have to deal with the engineering or physical applications of elasticity; indeed, it needs some hardihood, even for a specialist, to criticise it; every fresh perusal convinces the careful reader of the inadequacy of his own knowledge.

The changes made in this edition have been slight in appearance, but attention to details will reveal many improvements in both form and matter. The numbering of the sections has not been changed, which is of great help to those students who have learnt to use the second edition. New sections have been interpolated here and there, and these have been distinguished by a letter—*e.g.* 79A.

As previously, a great many references are given to the experimental side of the subject, and very rightly, for in many ways the mathematical theory of elasticity is more closely verified by experience (where verification has been seriously attempted) than the cognate theory of hydrodynamics. Where discrepancies have occurred they can usually be traced either to the inherent difficulty of obtaining an exact mathematical solution of the problem, or to unintelligent experimenting. Far too much experimental work, for example, has been done with systematic disregard of the elastic limits, or without due precautions, in anticipation of 5 per cent. accuracy. For various reasons, the engineer does not find it convenient to *isolate* effects, and he rarely carries out experiments for the express purpose of testing a mathematical theory. Thus what may be called the *physics* of elasticity has advanced comparatively little. The methods of photo-elasticity, first used by Clerk Maxwell, who applied the effect of stress on polarised light (discovered by Brewster) to the investigation of stress-distributions, and recently developed as a working engineering method by Prof. Coker, promise to do much to remove this reproach and to get rid of the difficulty mentioned by Prof. Love that "the components of stress or of strain within a solid body can never, from the nature of the case, be measured directly" (p. 94).

New sections have been added in chap. iv. on the results of Hopkinson and Sears concerning stresses maintained for a very short time, and also on elastic hysteresis. The term "perfect elasticity" to denote that condition in which the stress-strain diagram is closed, although the loading and unloading graphs do not coincide, seems unfortunate, as elasticity can scarcely be called perfect when elastic energy is being dissipated. "Perfect recovery" might denote this case, "perfect elasticity" being reserved for the condition in which loading and unloading graphs coincide. "Linear elasticity" explains itself, but surely the statement on p. 113, given on the authority of Bauschinger, that the limits of linear elasticity are higher than those of perfect recovery, can scarcely be right, since the former condition should imply the latter. An important appendix has been added at the end of chap. ix. on Volterra's theory of dislocations in the case of multiply connected bodies. A simpler proof of Weingarten's theorem that the discontinuities in the displacements on crossing a "barrier" correspond to a rigid body displacement can, however, be given. For if u_0, v_0, w_0 be one value of the displacement at a point P, and u_1, v_1, w_1 the displacement at the same point P after describing an irreducible circuit, u_1-u_0, v_1-v_0, w_1-w_0 are solutions of the equations of elasticity which necessarily (since the strains are supposed one-valued) correspond to zero strain everywhere, and such displacements must be rigid-body displacements. In this connection it would make things clearer for the beginner if in the proof of the uniqueness theorem given in § 118 the limitations as to the nature of the functions and the simply connected quality of the space were stated. Todhunter and Pearson have pointed out that the existence of more than one solution for a multiply connected body is immediately evident to anyone who turns a short piece of indiarubber tubing inside out. The realisation of this fact is apt to shake the student's faith if warning has not been given.

In the chapter on the sphere a very valuable new section gives the alternative method developed by the author in his essay on "Some Problems in Geodynamics," and another section gives a number of new and important references to work on geophysical problems, a branch of elasticity which is assuming nowadays an increasing importance. The work of Lamb and of G. W. Walker in connection with seismology is noticed on p. 314.

§§ 226A and 226B deal with the torsion of a

bar of varying cross-section and with end-effects in torsion.

In the chapter dealing with the elastica, the section (265) which gives the computation of the strain energy of the strut has been practically rewritten and much improved. It might be useful, in dealing with buckling, to dispose of a fallacy common among engineers that Euler's limit implies *failure* of the strut, whereas all that occurs is passage from one type of stable equilibrium to another.

Southwell's method of dealing with problems of elastic stability comes in, naturally, for considerable notice. The buckling of a strut (§ 267A), of a rectangular plate (§ 332), and of a tube (§ 341) are discussed as examples of this theory.

An entirely new chapter (xxiv. A) has been added, dealing very exhaustively with the equilibrium of thin shells in the shape of surfaces of revolution, including in particular a discussion of Meissner's work on the spherical and conical shells.

A feature of this edition (as of the previous ones) is the extraordinarily complete and careful set of references to all the original papers and memoirs dealing with the subject. Needless to say, these references, which have been most thoroughly brought up to date, are invaluable to the reader who takes up the book as a guide to research. The example set by such a master as Prof. Love might well be commended to the younger generation of scientific writers. Too often nowadays, especially in papers dealing with applied science, one comes across a statement of references which betrays the author's ignorance of the literature of his subject, both by the omission of work (sometimes of fundamental importance) done by his predecessors, and by the undue prominence accorded to the minor efforts of contemporaries in his own circle. L. N. G. F.

Behaviourism.

Psychology from the Standpoint of a Behaviorist. By Prof. John B. Watson. (Lippincott's College Texts.) Pp. xiii+429. (Philadelphia and London: J. B. Lippincott Co., 1919.) Price 10*s*. 6*d*. net.

THERE has been a great deal of controversy, especially in the philosophical journals of America, concerning the theory of behaviourism. Prof. Watson is, we believe, the originator of the term and the recognised leader in its application as a method in psychology. The book before us is not an exposition of the theory; it takes it as accepted, and puts forward an elementary, but nevertheless complete, schematic outline of the science of psychology, its scope and its method, regarded from this point of view. It therefore, better than any detailed exposition, sets before us the advantages and the disadvantages, the limitations and inclusions and exclusions, of psychology as the behaviourist conceives it.

Behaviourism is a theory of the science of psychology based on two postulates. The first is that the only thing the psychologist can study scientifically is behaviour. The second is that there is nothing else in psychology to study but behaviour. When the description of an individual's behaviour is exhausted there is no remainder, no psyche, left out of the account. The first postulate is explicit, the second implicit. It is clear at once, however, that the second is fundamental. Analyse the response of an organised material being to the stimulus of a situation, and you have exhausted psychology. Not only have you gone as far as you can go, but there is also no farther to go.

When you have simplified your science to this extent, the difficulty is to justify it at all. What is the subject-matter of psychology which demands a special method? This is Prof. Watson's difficulty. Physiology is already in the field; it has accomplished a vast amount of this very behaviour study. What is there left over for psychology? What sort of responses are there to which the physiologist can be, and is, completely indifferent, and which fall under the class-heading, psychological? The further we read in this book, the more intensely does this inquiry present itself as the crucial question. Three chapters of the book (no inconsiderable portion of the whole) are acknowledged to be pure physiology, and not psychology, and the reader is told in the preface that he may skip these if he likes, and that if he does so he need be at no disadvantage from his point of view as psychologist. But the physiology is not all so easily excised. When Prof. Watson defines an emotion he has to apologise for the impossibility of avoiding physiological terms. How much, one wonders, would be left of the book if all the physiology were taken out and only pure psychology left? The present writer, at least, as he reads the book finds himself in continual expectation that now he is coming to the end of the physiology and the beginning of the psychology, but is continually disappointed, and the reason is clear enough when Prof. Watson gives at last his definition of the distinction of the two sciences. Whenever, he tells us, we are studying the response of a part of the organism to a