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The Society has suffered a heavy loss in the death of the Treasurer, Col. E. H. Grove-Hills, which occurred on October 2.

On the Determination of the Colour-equivalent of a Star, with special reference to the Effective Wave-length and its Relation to Spectral Class: A Review of the Different Methods. By Knut Lundmark, Ph.D., and W. J. Luyten, Ph.D.

As colour-equivalent of a star may be taken every well-measurable quantity which in a simple way depends on the colour itself. From its nature the latter can, strictly speaking, be determined only by direct estimates with the naked eye, and is as such not very well suited to measuring purposes.

Colours of the stars were mentioned by the ancients (1), were known to Ptolemy, and have in recent years been determined by visual estimates, sometimes with the aid of small telescopes, by Santini, Schmidt, Krueger, Osthoff, Lau, Müller, and Kempf, and others. In this way the colour of a star can be found only as a relative quantity, and it has to be referred to some standard system in somewhat the same way that the apparent magnitude of a star is referred to the Harvard system (2).

As a theoretical scale of colours there might be used a sequence of colour photographs of bright stars obtained by the Lippmann method.

Although these visual estimates can be very accurate (Osthoff), they are restricted to bright stars; for this reason, and also because modern astronomy concerns itself with measurable quantities, various other methods for determining colour-equivalents have been developed. Among the principal ones may be mentioned:—

I. The colour-index; *i.e.* the difference in intensity, expressed in

magnitude, between two different parts of the spectrum; or, in mathematical symbols,

$$\text{C.I.} = 2{\cdot}5 \log \left[\int_{\lambda_3}^{\lambda_4} \Phi(\lambda) d\lambda - \int_{\lambda_1}^{\lambda_2} \Phi(\lambda) d\lambda \right] \quad . \quad . \quad (1)$$

where $\Phi(\lambda)$ is the observed spectral intensity at wave-length λ.

The different colour-indices enumerated below are then found by varying both the function Φ and the limits of the integrals. From this point of view it should be possible, from spectral-photometric work, to *compute* colour-indices.

In practice there have been used:—

A. The colour-index proper, the difference photographic – visual magnitude;

B. The difference radiative – visual magnitude; and

C. The difference radiative – photographic magnitude, as developed by Coblentz (3) and Burns (4);

D. Ultraviolet-photographic magnitude introduced by Seares (5);

E. The difference between photoelectric magnitudes obtained without and with colour filter (6);

F. The colour-index obtained by the use of Nordmann's photomètre hétérochrome (7).

II. Seares's method of exposure ratios (8).

III. King's method of taking simultaneous photographs of a star through a yellow and a blue filter, each covering one half of the objective (9).

IV. Tamm's extrafocal photography through UV lenses (10).

V. Effective wave-lengths.

Undoubtedly other equivalents will be used in future. Care should be taken, however, when developing any new equivalent, not to introduce quantities too sensitive to such instrumental errors as achromatism and photographic effects, or quantities too hard to measure, such as λ_{min}, to be explained later.

We shall now try to give a more detailed description of the methods mentioned here, and compare the pros and cons of each method.

I. Determination of colour-index rests on the fundamental scales for all of the different magnitudes used. For the photographic magnitudes there is the well-established photographic North Polar Sequence, as determined and confirmed by the co-operation of Harvard, Mount Wilson, and Greenwich. For the brighter visual magnitudes we have the Harvard system, which seems also fairly well established, although there is still the systematic scale difference Harvard – Parkhurst that needs explanation. A North Polar Sequence of photovisual magnitudes for fainter stars has been determined by Seares (11), but this can only be regarded as an extension of the visual series if no systematic difference between the two exists. The facts are, however, that the two scales coincide only at 6th and 12th magnitudes, and that the difference $H_{\text{vis}} - S_{\text{photovis}}$ depends on the colour. What this finally comes to, is that in the colour-index are involved systematic errors depending on the colour; or, in other words, what we actually determine is a function of the colour and some unknown parameters. Fortunately, these unknown

quantities do not play an important part, and do not sensibly diminish the value of a colour-index determined in this way. Von Zeipel and Lindgren (12) have determined photovisual magnitudes on Seares's scale and visual magnitudes extrapolated from the Potsdam system. From a comparison of these values for 218 stars they find a mean systematic correction in the sense

$$m_{\text{vis}} - m_{\text{p.v.}} = -0^{\text{m}}\cdot01 \pm 0^{\text{m}}\cdot005,^{*}$$

whereas Seares (13) finds for the same difference

$$+0^{\text{m}}\cdot19 \pm 0^{\text{m}}\cdot02 \text{ from } 12 \text{ stars.}$$

Von Zeipel concludes from this that his visual magnitudes are in error. This is probably the case, but still the connection between the visual and photovisual scales needs further attention.

Regarding the colour-indices mentioned under B and C, these, although they are extremely important for individual stars, cannot as yet be used in general considerations, as they are at present, unfortunately, restricted to bright stars. Method D is only in the development stage, and not much can be said of it.

The same holds for E, which has been applied by Guthnick and Hügeler to Nova Aquilæ, and which seems to give results of extremely high accuracy. A comparison with other colour equivalents shows a linear relation in most cases. It is very remarkable, on the other hand, that this colour-index, F_1, decreases so considerably from M*a* to M*d*, but increases towards N.

Nordmann and Le Morvan have determined colour-indices by measuring visually through red and blue screens. The range in colour-index obtained in this way is very close to that determined by the difference photographic – visual.

In deriving the actual magnitudes from the images on photographic plates we have the advantage that either the diameter or the blackness can be used. Both quantities are suited to refined measurement, especially in the last case, when a "Schraffier Kassette" or intrafocal images in conjunction with a microphotometer are used. The personal element in measuring can be entirely eliminated by using the self-recording microphotometer of Koch, or its recent improvement by Moll (14).

Among the disadvantages of the colour-index determinations may be mentioned:

(*a*) Dyeing of plates may introduce systematic errors, or possibly a non-uniform sensitivity of the plates (15).

(*b*) Corrections have to be applied corresponding to distance from centre and these may contain uncertainties.

(*c*) If an auxiliary scale is used, and magnitudes are determined by estimates, as in the Argelander method, trouble will arise from the definition of the images on the plates. This can be greatly overcome by using different scales corresponding to varying definitions. There also seems to be a change in personal equation with the time (15).

Among the more extensive determinations of colour-index, apart from the standard-sequence work, may be mentioned those of Park-

* By a typographical error given as $0^{\text{m}}\cdot05$ by von Zeipel.

hurst (16), Schwarzschild (17), King (18), Shapley (19), von Zeipel and Lindgren (12).

II. Seares's method.

Advantages:—1. Every part of the field can be used, as the shape of the images will not greatly influence the accuracy; and

2. Measuring is not necessary; estimates will do, especially as a large number of images are secured on the same plate.

Disadvantages:—1. The connection between exposure ratios and colour is not known *a priori*, and has to be determined in the course of the work (from known colour-indices);

2. It is tacitly assumed that the filters and colour-sensitiveness of the plates are invariable (this applies, of course, to all methods using dyed plates and colour filters);

3. The variation in the transparency of the air will undoubtedly influence the results. (This will, however, not be of great importance for the observatories on the Pacific Coast); and

4. The colour derived by this method is dependent on the intensity of the image (photographic Purkinje effect).

As a knowledge of standard sequences is not necessary, this method of determining the colour-index looks very promising for the future, and it can at the same time afford a valuable check on the colour-indices obtained by method I. (20).

III. and IV. Not much can as yet be said of these methods, as we have only a preliminary knowledge of them. In method IV. the differently-coloured stars give a varying intensity of the outside ring as compared with the central dot. From the appearance of records for the different spectral types (judging from prints and information kindly communicated by Mr. Tamm) satisfactory results may be predicted. The method has the advantage that an achromatic lens is not required, a simple lens being, in fact, preferable.

V. Effective wave-lengths.

The term effective wave-length was first applied by Comstock (21) to his visual determinations of the wave-length for the maximum intensity in short objective-grating spectra. For the different spectral classes the differences in effective wave-length are very small in the visual region, and determinations of this quantity are therefore made mostly by photography.

A grating of equal bars and spaces is placed in front of the objective, as was first done by the Brothers Henry (22) and by Schwarzschild (23), and later applied to the determination of colour equivalents by Hertzsprung (24) and Bergstrand (25). More recently measures of effective wave-length have been made by Lindblad (26), Lundmark (27), Wolf (28), Davidson and Martin (29), Balanovsky (30), and others. There is no need here to go into the theory of the objective grating, as this can be found in the earlier of the papers referred to.

If $\Phi(\lambda)$ is the spectral intensity for the wave-length λ, then the effective wave-length can be defined by the expression

$$\lambda_{\text{eff}} = \frac{\int \Phi(\lambda)\lambda d\lambda}{\int \Phi(\lambda) d\lambda} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

both integrals extending over all wave-lengths acting on the photographic plate. As this quantity is intimately related to the effective temperature of a star and thus also to the colour, the λ_{eff} as measured, even by using small instruments, gives a valuable check on the colour-index determined by other methods. Used in connection with powerful instruments λ_{eff} will afford material for an extended knowledge of the colours of faint stars.

The construction of a coarse grating does not introduce such delicate technical problems as are met in the manufacture of colour screens. Besides, an objective grating may be more easily kept in order and controlled than a colour screen.

If always used in the same position relative to the objective, no trouble will arise, even if the grating is slightly tilted relatively to the optical axis, as this introduces only a small systematic error. As is the case with Seares's, King's, Tamm's, and Nordmann's methods, work can be done for λ_{eff} independently of a photometric scale. Neither is any dyeing of the plates needed, with accompanying uncertainties, and almost any kind of high-speed commercial plates can be used to advantage. Leaving out of consideration the correction of λ_{eff} for the relative strength of the image, the reduction of the measures is simple and does not involve much labour.

One of the principal advantages with the λ_{eff} method is perhaps its usefulness for objects of measurable extension on the plate, such as nebulæ, planets, and comets. The use of other methods, except Seares's, and perhaps Guthnick's, seems to present difficulties in these cases.

On the other hand, it must be admitted that the method is of rather idiosyncratic behaviour, and, if not carefully used, it certainly contains several disadvantages. A number of these have come within the experience of the authors in preliminary work undertaken with the grating, and on account of the growing interest in this subject a brief discussion of some of them is given here.

Every observer of λ_{eff} has met with the so-called photographic Purkinje effect, *i.e.* the dependence of λ_{eff} on the intensity of the images on the plate. (This follows immediately from formula (2).) To overcome this, the material has generally been reduced to an arbitrarily selected value of the diameter of the central image. Now the effect depends also on the spectral class, and the correction can thus be obtained only by successive approximations, as λ_{eff} is unknown. The sometimes complicated forms of the empirical reduction curves used in correcting for this effect show how much care should be taken in order to get an accurate correction for this important source of error. For some instruments the effect has been found to be so large that the measures cannot properly be used for obtaining knowledge of λ_{eff} (31). On the other hand, this effect appears to be very small, if sensible at all, in the determinations of λ_{eff} for surface objects with the Upsala 6-inch twin telescope; at least so long as large intensities are not used.

The effective wave-lengths as measured are in a high degree dependent on the optical properties of the instruments used. Regarding refractors, nothing can be said before a thorough test has been made. The system of λ_{eff} depends entirely on achromatism. Thus it was

found in Upsala that changes in the optical system (the middle lenses of the triplets were slightly readjusted to obtain good focus over as large a field as possible) changed the λ_{eff} so much that another scale had to be established. Reflectors should be well adapted for work of this kind. Their behaviour has, however, not quite come up to theoretical expectations. A comparison between the results of Bergstrand, Wolf, and Greenwich is, in this respect, very instructive. Rather obscure is the apparent relation of the system of λ_{eff} to a resilvering of the mirror, as found by Wolf, and described in his paper.

A connection between λ_{eff} and changes in focal length, arising from temperature, has been suspected.

Rosenberg (32) has investigated this problem and found, by using a grating of high precision, that changes of the focal length as small as $\frac{1}{1680}$ could be detected in the measured effective wave-lengths. Of still greater importance is his result that small focal changes influence very considerably the shape of the reduction curves for the photographic Purkinje effect.

Other optical effects are the distortion of the images by distance from centre and by atmospheric dispersion. These can be made less serious by reducing the aperture of the objective, by avoiding too long exposures, or by working at small zenith distances. The Greenwich observers have measured their λ_{eff} for stars with distorted images, as they found those no less fit for accurate measurement. They do not state, however, whether there exists any systematic difference between the λ_{eff} for those images and for images near the centre of the plate.

In an instrument used by Lindblad and one of the authors a rotational error was found, *i.e.* λ_{eff} changes when the grating is rotated in front of the objective.

Sometimes a chromatic irregularity of the lens is shown, which acts in much the same way as the atmospheric dispersion. The most natural explanation is that the lens acts as a prism, and in the resulting spectra there is dispersion in two directions.

The state of the atmosphere is of some importance for work of this kind. Messrs. Vallin and Lundmark thus found a fogging effect for λ_{eff} at Upsala. When comparing λ_{eff} obtained through a clear sky and low humidity with those obtained through a foggy sky and high humidity, a mean difference of 30 Å was found in the sense that a fog increases λ_{eff}. Davidson and Martin (29) also mention an influence of the same sort. They do not state, however, whether this effect is systematic and in the same direction.

The influence of chemical fog or other photographic effects has not yet been investigated; neither has any comparison been made between different kinds of plates. Preliminary work of Vallin, as given in Lindblad's paper, shows the importance of a proper selection of plates. In the measurement of the plates a personal element is introduced, as the short spectra generally are not symmetrical and their appearance varies with the spectral class. When measuring λ_{eff} of spirals and clusters Lindblad and Lundmark found very good agreement, taking into account the difficulty of measuring objects of this kind. From a great number of measures there resulted a mean difference between

the two observers of 1 Å, which is quite negligible. Comparison of measures of stars has shown very little systematic difference between these two observers for all spectral classes except B, where Lindblad measures λ_{eff} decidedly smaller. Another comparison between two observers is given in figure 1 where, among others, are shown the results of Bergstrand and Lindblad, both using the same instrument and the same kind of plates.

No clear definition exists in the literature of the subject as to whether the λ_{eff} is the wave-length found by bisecting the blackened area or by setting on the "centre of gravity," then giving λ_{eff} as defined by (2). It is evident that in practice the last way had been followed, and although a measure of this kind may seem only vaguely defined, accuracy in measuring can certainly be obtained after some experience. Among the disadvantages of the λ_{eff} method may also be mentioned that in dense regions the position of the short spectra can be influenced by photographic effects, sometimes called photographic repulsion (investigated by F. E. Ross) (33). This applies also to the other colour-equivalent methods, but in less degree, because, when dealing with effective wave-lengths, we have at least three times as many images on the plate. Compared with the colour-index method, λ_{eff} is relatively somewhat inefficient; for it has been found that when using the grating, exposures must be increased at least ten to fifteen times, whereas the colour screen increases the exposures only six times. Moreover, in the latter case, underexposed images are still measurable, which is not true for the short spectra.

It is very doubtful whether λ_{eff} has any real significance for classifying purposes when applied to objects with bright emission lines. Thus the extraordinarily low value of 3940 Å for the Ring Nebula, as found by Lundmark and Lindblad, must be ascribed to the presence of the bright doublet at 3727 Å. The spirals gave wave-lengths corresponding to the spectral classes G–K, in close accordance with the results from actual spectral classification. For the planetaries a mean effective wave-length of 4130 Å was found, corresponding to spectral class B–F. It seems thus possible with this instrument to separate the spirals from the planetaries, which is sometimes impossible to do from considerations of structure alone. In a recent paper Balanovsky finds for 16 objects, probably spirals, a mean effective wave-length of 4420 Å (in his system corresponding to G), in good agreement with Lindblad and Lundmark. On the other hand, 4 of his planetaries give in the mean 4760 Å, which abnormal value seems to be hard to explain for the present, even if we may assume that N_1 and N_2 have contributed to the recorded spectrum. Although he thus gets λ_{eff} higher for planetaries than for spirals, which is contrary to the Upsala results, still his system can be used for distinguishing between these two kinds of nebulæ.

The close dependence of λ_{eff} on the optical system and the several other effects mentioned are the reason why no two results of different observers are directly comparable. It is thus to be regretted that no standard sequence has been established. It may be suggested to perform this by laboratory experiments with monochromatic light.

However, until this has been accomplished, astronomers should be careful in their interpretation of observations of effective wave-lengths, and should not use them without reduction, as has in some cases been done for the derivation of colour-equivalents. Likewise, we are of the

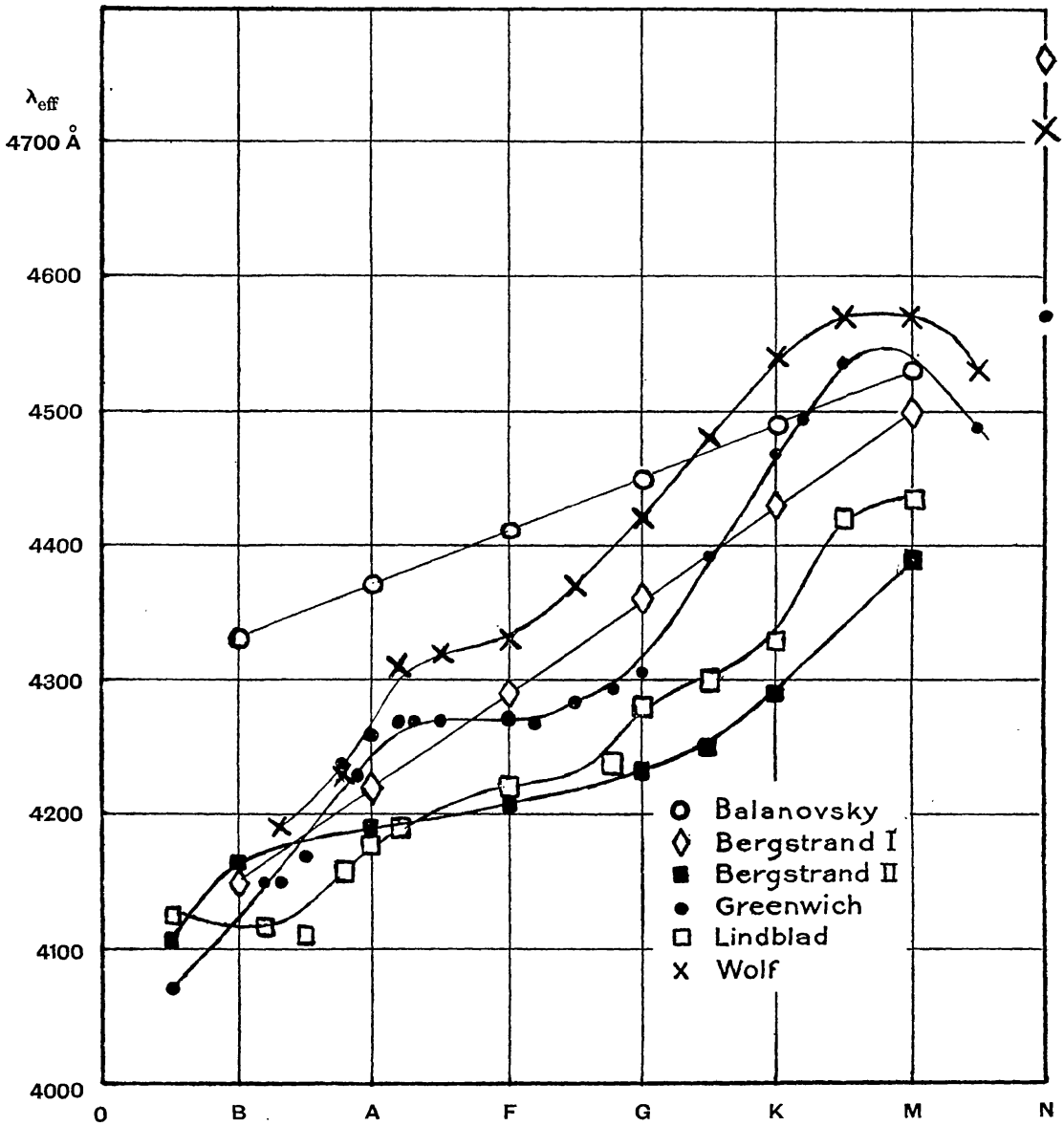

FIG. 1.

opinion that it is not justifiable to use the value of the effective wave-lengths for a certain spectral class determined by *another* observer, with *another* instrument, on *other* plates, as a point of reference for one's own measures, which procedure has been followed by two investigators (34).

In spite, however, of all the systematic errors to which a determination of λ_{eff} is subject, there is, unquestionably, a relation between λ_{eff} and spectral class. On the main features of this relation there is a

general agreement among observers, as may be seen from fig. 1, where the effective wave-length measures of Balanovsky, Bergstrand I., Bergstrand II., Greenwich, Lindblad, and Wolf are plotted against spectral class. It appears from this that the increase of λ_{eff} is rather rapid between B and A, is slow (for some observers almost zero) from A to F,* and large again from these onward, till at M it goes down toward Mb, c, and up toward N.

In their paper Davidson and Martin publish an extensive diagram giving the relation between their individual λ_{eff} and spectral class; from this it will be seen that at G5, *e.g.* the measured effective wave-lengths cover the entire range of the mean λ_{eff} from A to K5. When it comes to discussing the possible causes of this unexpected effect, we are, at present, able to arrive at definite conclusions respecting the error in spectral classification. For, apart from the Harvard spectra used by Davidson and Martin, we have for the stars in this part of the sky another determination of the spectral classes by ten Bruggen Cate (35). As his paper deals with a comparatively small number of stars, and great care was taken in the classification, we can give this determination a weight at least equal to that of the New Henry Draper Catalogue. A comparison between the two different classifications will then give us an idea of the error to be expected in a single determination of spectral class. If A0 is denoted by 0 and K0 by 30, then the relation between Harvard and ten Bruggen Cate can be written

$$B = 0{\cdot}89\ H + 0{\cdot}4.$$

The deviations from this straight line lead to the following mean errors in the classification:

For A0	± 0·7	
F0	2·2	
F5	3·1	unit: one spectral subdivision
G0	3·4	
G5	2·8	
K0	2·2.	

It is clear from this that errors in spectral classification alone cannot be responsible for the large discordance in λ_{eff} for G5 – K, as the mean errors decrease where the discordances increase. In fig. 2 are plotted the Greenwich λ_{eff} against ten Bruggen Cate's spectra; each star is represented by an ellipse with axes equal to the mean errors in the two co-ordinates.

Another estimate of the allowable error in spectral classification is found from a comparison between Harvard and Mount Wilson spectra for 489 stars (between 0 and 8 hours of R.A., and between F0 and M, taken from *Mount Wilson Contributions*, No. 199).

The spectral classes were again denoted by numbers and a rigorous solution by least squares was made in the following way: Let each star be represented by a point whose x equals the Harvard spectrum, and

* Due to the well-known ultraviolet absorption in Class A stars.

whose y equals the Mount Wilson spectrum; then the problem is to lay a straight line through these points in such a way that the sum of

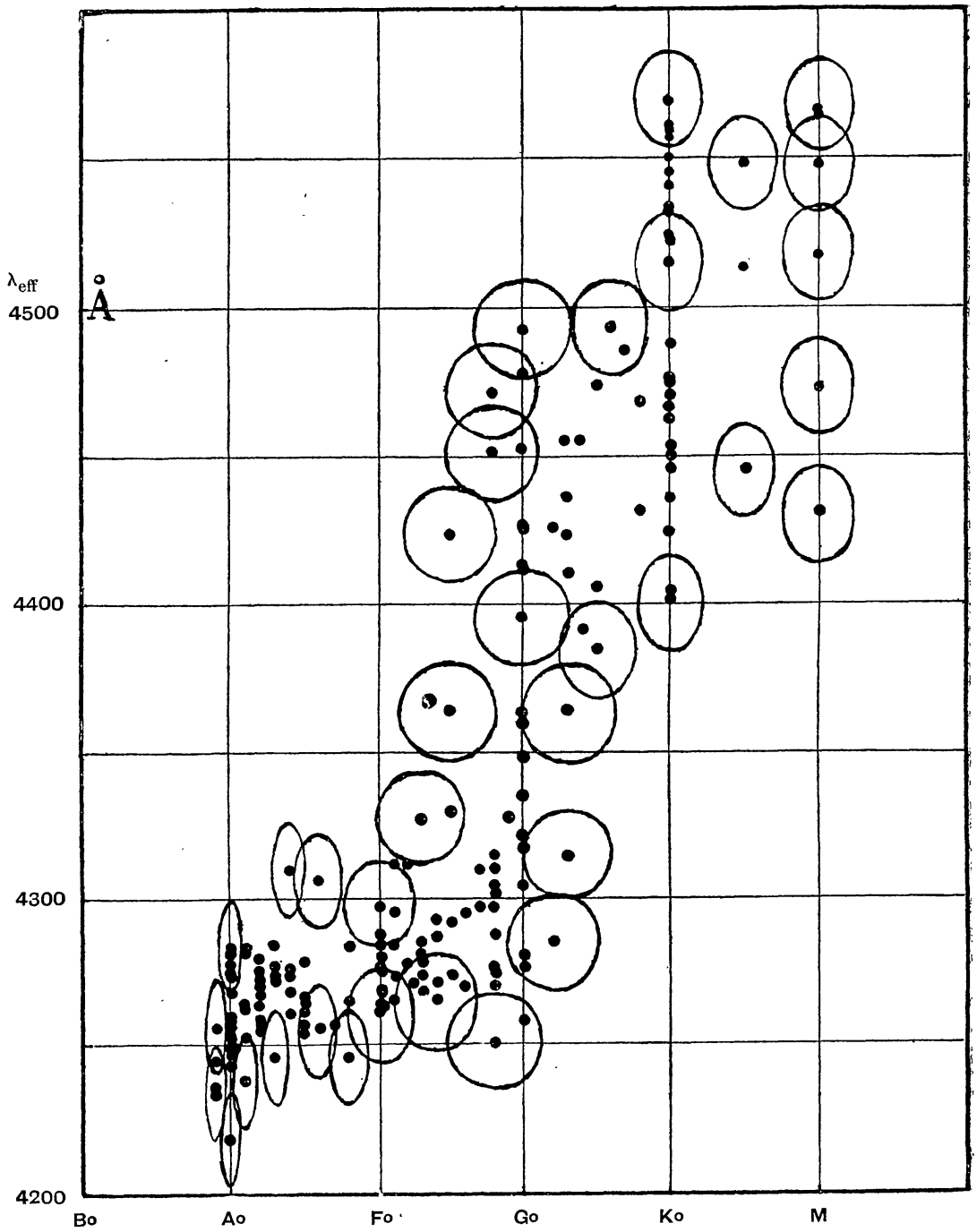

FIG. 2.

the squares of the perpendicular distances of these points from the line is a minimum. Write the equation of the line as follows: $L \equiv -x \sin \alpha + y \cos \alpha - l = 0$ (where α is the angle between the line and the axis of

x); then $D(x_1y_1) = L(x_1y_1)$; accordingly, $\Sigma L^2(x_1y_1) = \text{Min.}$ After differentiation and substitution we find for a and l

$$\tan 2a = \frac{2\Sigma xy - \frac{2}{n}\Sigma x \Sigma y}{\Sigma x^2 - \Sigma y^2 + \frac{1}{n}\Sigma y \Sigma y - \frac{1}{n}\Sigma x \Sigma x} \text{ and } l = \frac{\cos a \,.\, \Sigma y - \sin a \Sigma x}{n}$$

where n is the number of stars. This derivation, however, only holds when the mean errors in both co-ordinates are equal; to obtain this the ordinates were increased corresponding to giving double weight to Mount Wilson.

Solution of the equations then yields

$$W = 1{\cdot}15(\pm{\cdot}17)H - 1{\cdot}6(\pm 2{\cdot}0)$$

on a scale where Fo = 0 and Ma = 30.

The mean errors in spectral classification then turn out:

$$\pm 2{\cdot}0 \text{ for Harvard, } \pm 1{\cdot}4 \text{ for Mount Wilson,}$$

these being of the same order of magnitude as previously found.

Another useful parameter to indicate errors in spectral classification is the correlation constant. This constant r is in the case Harvard-Mount Wilson

$$r = 0{\cdot}932.$$

This is relatively low; for Charlier (36) found as high a value as 0·958 for the correlation constant between Harvard spectra and Parkhurst's colour-indices.

The difference Mount Wilson-Harvard was previously investigated by Adams and Joy (37), who found $-1{\cdot}6$ spectral subdivisions. On the basis of a more comprehensive inquiry they derived later on (38) a difference of opposite sign, varying irregularly with the spectral class itself. It may also be mentioned that they found a pronounced difference in the values H – W for giants and dwarfs, in the sense that Mount Wilson classifies the dwarfs comparatively redder than it does the giants (by approximately two spectral subdivisions). We shall refer to this fact later on.

If we now consider another possible cause of the discrepancy in the λ_{eff} for late-type stars, bearing on the irregularities in the spectra (and accordingly in the λ_{eff}) of the stars, depending on the giant or dwarf character (pointed out by Seares and by Lindblad), we come to the conclusions that this cannot produce such a marked effect on the range. In agreement with present data we assume that among the Ko stars the dwarfs outnumber the giants *in space* by 100 to 1; further, that their respective mean absolute magnitudes are $+6^{\text{m}}{\cdot}0$ and $+0^{\text{m}}{\cdot}8$, both with a Gaussian distribution. After making use of Kapteyn and van Rhijn's density law (39) we find that at apparent magnitude $8^{\text{m}}{\cdot}5$ (lower than the mean of the Greenwich stars) the dwarfs form only 30 per cent. of the total number. This would accordingly cause a spread on the *blue* side of the maximum, whereas Davidson and Martin's figure shows most of the dispersion on the *red* side.

As the evidence we have regarding the errors in the measures of effective wave-length (for one instrument only) does not account for discrepancies of the size of those indicated in the plot, we come to the conclusion that at present individual effective wave-lengths cannot serve to predict spectral class with certainty.

Whether this is due to the several causes mentioned above or to real differences in λ_{eff} even among giants of the same spectral class, we cannot yet say, but the facts seem to point largely towards the latter possibility.

An inspection of fig. 1 will strengthen our conclusion that individual λ_{eff} cannot serve for the prediction of spectral class. It will be seen that between A and G λ_{eff} is nearly independent of the spectral class, whereas between G and M the uncertainty is too large, or the curve reverses (Wolf), and the only safe predictions that can be made are for the classes O and N (*cf.* Wolf's list in *A.N.*, 5092, where his estimates differ materially from the spectral classification, *e.g.* as the star called B8 by him is classed F8 by Harvard). We can therefore say that the relation between spectral class and effective wave-length is not in the nature of a one to one correspondence. The most that can be said is that upon the spectral class a rough estimate of λ_{eff} may be based, but that the direction of inference may not be reversed.

It is clear from the foregoing considerations that we must receive with much scepticism any method of deriving the absolute magnitude from a combination of effective wave-length and another colour equivalent. This is especially so, when for this second colour equivalent the minimum effective wave-length is used, as this asymmetrical quantity is subject to a large personal element in measuring.

Such a method was developed by Lindblad (40), and although from a theoretical point of view a difference in λ_{min} should exist between giants and dwarfs, it is very doubtful whether this can be detected even in the refined measures of Lindblad. For with his arrangement of the grating (wires north and south, and therefore length of spectra in Right Ascension) any error in guiding will affect λ_{min} much more than it will λ_{eff}. A more serious objection that can be raised against the principle of using λ_{min} and λ_{eff} together for prediction of luminosity is the high correlation that exists between them. From the 64 stars measured by Lindblad (41) the correlation constant r is computed to be

$$r = 0{\cdot}969,$$

from which we draw the conclusion that the correlation is absolute and that his measures are extremely accurate.

But, as thus λ_{min} can be computed with high accuracy from λ_{eff}, a combination of the two cannot give any more information about the absolute magnitude than can λ_{eff} alone. A similar case is presented by the following:—The systematic difference Harvard – Mount Wilson spectral class differs largely for giants and dwarfs (38), and as the correlation constant between H and W is only 0·932, it would be possible to conclude whether a star is a giant or a dwarf if its spectral class in both the Harvard and Mount Wilson systems is known. It is evident that this cannot be so.

That there is, nevertheless, a change in λ_{eff} and λ_{min} with absolute magnitude is clearly shown in figs. 3 and 4, which have been arranged in such a way as to correspond approximately to the usual spectral-class-luminosity diagram. However, it appears from these that λ_{eff} is more suitable for the indication of M_{abs} than is λ_{min}, which fact is quite natural and results from the more accurate measuring of λ_{eff}. It may also be remarked that as shown in fig. 3, λ_{min} apparently does not change appreciably with the absolute magnitude.

From the data represented in fig. 4 Lindblad has suggested that stars

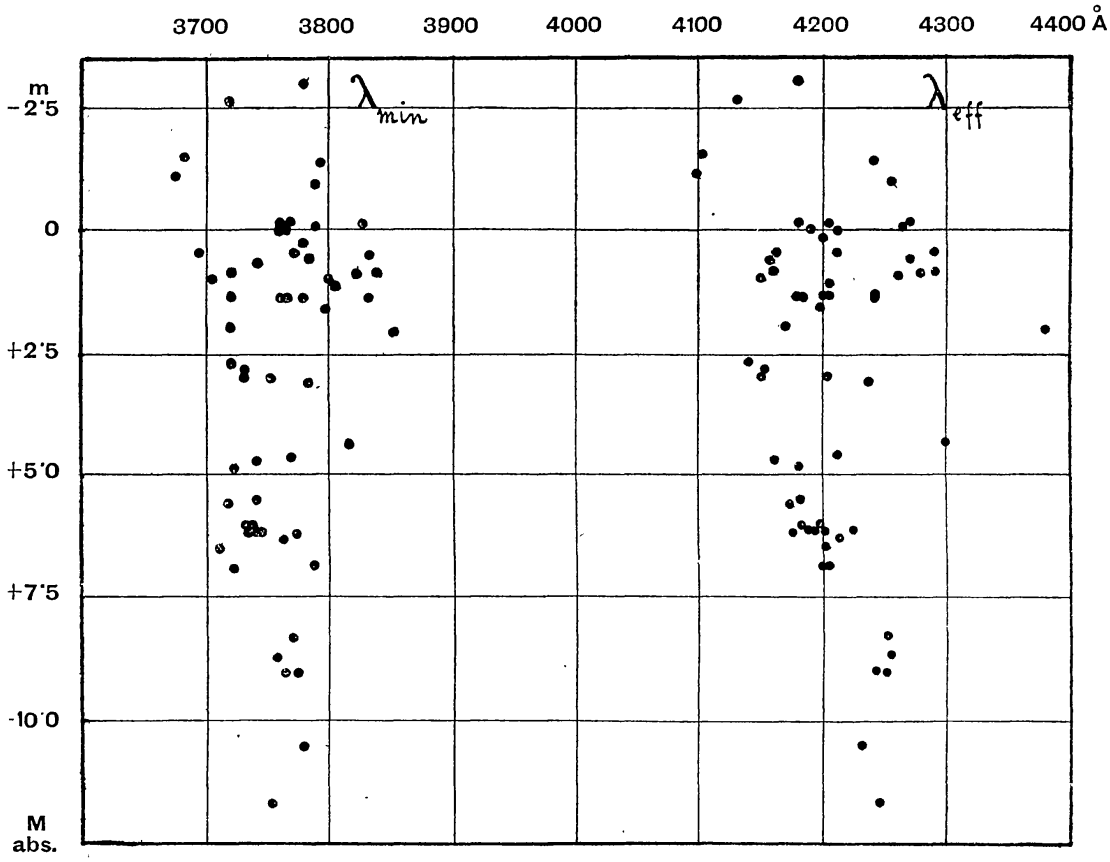

FIG. 3. FIG. 4.

with λ_{eff} larger than 4260 Å on his system are red giants. With the fact in view, however, that for many observers the effective wave-length decreases again from K5 to Mc (giants) we find that the number of stars thus separated from the rest as giants is unfortunately very small and practically restricted to K2–K5. If we also take account of the fact that the personal difference in λ_{eff} for late-type stars can amount to as much as 86 Å (between Bergstrand and Lindblad, same instrument, same kind of plates) we must regard the evidence mentioned here, although extremely interesting, as only preliminary and in need of confirmation.

It is interesting, therefore, to compare with this a statement of Seares (42), who arrives at the conclusion that "the problem of finding the luminosity of a star from observations of colour alone seems to

be indeterminate, owing to the fact that the effect of M_{abs} on colour enters in such a manner that it cannot be separated from that due to spectral class."

On the other hand, it seems not entirely impossible to obtain knowledge of the luminosity of a star if both the spectral class and λ_{eff} are known. It is in this case sufficient to know whether the star is a giant or a dwarf, for Hertzsprung and Russell (43) have shown that the luminosity of a dwarf is so closely related to the spectral class that from a knowledge of the latter quantity we can make a fair estimate of the former. The correlation constant for these two quantities comes out as high as 0·730, thus fully justifying the above-cited conclusion. (For this computation it was necessary to increase the scale in spectral class considerably between K and M, in order to make the relation between M_{abs} and Sp as closely linear as possible.) As giants are extremely rare among the F and early G stars (first pointed out by Hertzsprung), the fainter stars of these classes can generally be regarded as dwarfs. The M dwarfs must, on account of their low luminosity, betray their presence by a large value of the reduced proper motion (the quantity $m+5+5\log\mu$); the brighter stars of this spectral class can therefore be treated as giants. (The New Henry Draper Catalogue contains probably not more than 20–30 dwarfs in a total number of about 3800 M stars.)

It is therefore a desideratum to develop criteria of luminosity for the stars between spectral classes G5 and K5, and it would be extremely valuable if future development of effective wave-length methods could accomplish something in this direction. Lindblad's criterion, if confirmed, looks very promising, as this seems to work exactly for those spectral classes where it is needed most. But at present it seems to be out of the question to derive any such general criteria without first establishing a standard scale of effective wave-lengths to which all observers could reduce their measures. Such work has already been taken up by Professor Bergstrand, according to private information obtained from him.

Summary.

1. So long as no standard scale of effective wave-lengths has been established, and their determination is subject to large personal equations and systematic differences which have been but meagrely investigated, they cannot be regarded as equivalent to spectral class in individual cases.

2. A thorough and extensive investigation as to all sources of systematic differences affecting measures of effective wave-length is highly desirable.

3. When it becomes possible to master these difficulties, effective wave-lengths may prove valuable as substitutes for spectral classes in dealing with faint stars, and in indicating luminosities when used in connection with known spectral classes.

4. Instead of Lindblad's λ_{min}, a λ_{min} obtained through ultraviolet screens is suggested (for application with reflectors only).

5. Investigation of a possible relation between λ_{min}, λ_{eff}, and the

intensity measures in short spectra (44) will be of importance. In this connection the wave-length of maximum intensity, as determined from shortly exposed low-dispersion slit-spectrograms, may be used as a substitute for λ_{eff}.

Lick Observatory:
1922 *May* 8.

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Note on the Actual Determination of Photographic and Photovisual Magnitudes. By W. J. Luyten.

As is stated in the preceding paper, photographic magnitudes can be determined only with photographic instruments and with reference to photographic standards; photovisual magnitudes only by means of isochromatic plates together with yellow screens and with reference to visual or photovisual standard stars. There seems need to emphasise the necessity of this, as it has not always been clearly understood.

Thus *photographic* magnitudes of Nova Cygni * have been determined with the aid of the *visual* magnitudes of comparison stars of spectral class F8. It is clear that the magnitudes obtained in this way cannot be called photographic, as they differ from those by the difference in colour-index between the Nova and the comparison stars, and are also dependent on the colour correction of the telescope used. This explains the large discordance between the magnitudes derived in that way and those obtained by the Greenwich observers † by comparison with photographic standards.

One investigator ‡ has determined *photovisual* magnitudes of Nova Cygni by using isochromatic plates *without* yellow filter, and by comparing these with the photographic magnitudes reaches the following statement: "This second curve, which we call photovisual, follows the photographic curve rather closely; however, not much importance should be attached to this." Magnitudes determined in this way do not in the least approach photovisual magnitudes, and the statement mentioned above cannot therefore be considered as remarkable. For it is a well-known fact that the use of an isochromatic plate without yellow filter instead of an ordinary photographic plate will alter the magnitude, *ceteris paribus*, by only a small fraction of the colour-index (probably near one-fiftieth). In fact, this is the basis of Seares's method of exposure ratios.

The Planes of the Spiral Nebulæ in Relation to the Line of Sight. By J. H. Reynolds.

In a paper on "The Galactic Distribution of the Large Spiral Nebulæ," § attention was drawn to the planes of the spiral nebulæ of 10′ in diameter and upwards in relation to the line of sight, and it was shown that there was a considerable excess of these nebulæ inclined at angles under 30°, which angle would be expected to divide the total number equally.

The random distribution of the planes of spirals inclined at all angles over the whole sky is given by the formula

$$n = \mathrm{N}(\sin\theta_1 - \sin\theta_2),$$

where n is the number between any two angles θ_1 and θ_2, and N is the total number considered.

* Smart and Green, *M.N.*, **81**, 179, 1921. † *M.N.*, **81**, 37, 61, 1920.
‡ F. Henroteau, *Journ. R.A.S. Canada*, 1921, p. 155.
§ *M.N.*, **81**, 129.