with its necessary (automatic) appearance in $(n\rho_1, n\rho_1')$; it is one main connecting link between the original vectors and those obtained by inversion (crossed pairing). The perpendicularity of $(\mathbf{M}, \mathbf{s})$, requiring equal orthogonal projections upon the line of $(\mathbf{s})$ for the vectors $(\mathbf{Q}_2, \mathbf{Q}_2'')$, considered by itself demands no more than equality of scale-factors applied to the same segment of that line. The consideration that here precedes makes the selection (ns) natural.

The material of this particular discussion traverses familiar ground, so far as the final facts about conjugate vectors are concerned. But the line of attack there formulated is capable of deducing novel results elsewhere.

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LXIV. *Contact Difference of Potential and Thermionic Emission.* By O. W. RICHARDSON, *F.R.S., Wheatstone Professor of Physics*, and F. S. ROBERTSON, *M.I.E.E., Lecturer in Electrical Engineering, University of London, King's College* *.

IT has been pointed out by one of us † that the following relation should subsist between the contact potential difference V between two surfaces at the absolute temperature T and their thermionic electron saturation currents i_1, i_2 per unit area at the same temperature:—

$$V = \frac{kT}{e} \log \frac{i_2}{i_1}. \quad . \quad . \quad . \quad . \quad . \quad (1)$$

In this equation, k is Boltzmann's constant and e the electronic charge.

Observations which confirm the validity of this equation, approximately at any rate, have been made by us in the course of some experiments with a thoriated tungsten filament. The filament was of tungsten containing 1 per cent. of thorium and was kindly supplied to us by the Tungsten Manufacturing Co. Ltd., 231 Strand, London. It was 3.0 cm. long and 0.100 mm. in diameter, and was mounted axially in a cylindrical glass tube and surrounded by a coaxial copper anode. The general arrangements for evacuation and for controlling the temperature of the filament, etc., were similar to those described in our paper on the effect of gases on the contact difference of potential between metals ‡. After baking out

* Communicated by the Authors.

† O. W. Richardson, *Phil. Mag.* vol. xxiii. p. 265 (1912); and 'Emission of Electricity from Hot Bodies,' p. 41. Second Edition (1916).

‡ *Phil. Mag.* vol. xliii. p. 162 (1922).

the tube, exhausting with the mercury-vapour pump, etc., and heating the filament for some hours, it was found to have a fairly stable characteristic curve in the neighbourhood of that to the left-hand side of fig. 1. Later on, the emission was found to have risen to an enormously higher value, and was

Fig. 1.

right outside the scale of the available measuring instruments with the same temperature as indicated by the resistance of the filament. The emission was now lowered by reducing the filament temperature to a value comparable with that ruling before the change occurred, and the characteristic was now found to be stable in the neighbourhood of the right-hand curve in fig. 1. This displacement of the characteristic

corresponds to a change of 0.71 volt in the potential difference between cathode and anode, and is in such a direction as to indicate that the filament had suddenly become relatively more electropositive to that extent.

There seems no reason to doubt that most of this change in the contact potential occurred at the heated filament. Assuming that the whole change is at the filament, these observations furnish the material for an approximate test of equation (1). The test is only approximate in any event because it was not possible to ascertain the temperature of the filament with accuracy. When its inverse resistance was 0.521 ohm^{-1} , corresponding to the left-hand curve in fig. 1, its temperature was estimated with a Cambridge & Paul Optical Pyrometer at 1583°K . Another estimate can be obtained if we assume that it was giving the pure tungsten emission at this stage. Using K. K. Smith's tungsten data we calculate $T = 1450^\circ \text{K}$; and using Langmuir's $T = 1507^\circ \text{K}$ for this limit, the average of these three estimates is 1480°K .

The emission for a resistance of $(.521)^{-1}$ after the rise was too large for us to measure with our instruments, but was obtained by extrapolation. It was found that the currents at various values of the inverse resistance were as follows :—

$i_2 (I = 2.06 \times 10^{-10} \text{ amp.})$	10.2	21.0	45	91.5	165	318
$\log_{10} i_2$	1.01	1.323	1.653	1.962	2.218	2.503
$R^{-1} (\text{ohm}^{-1})$	.774	.764	.740	.726	.714	.698

The relation between the logarithm of this current and R^{-1} is found to be linear, and from the plot it appears that the value of $\log_{10} i_2$ for $1/R = .521$ is 5.96. The value of $\log_{10} i_1$ for $1/R = .521$ was 2.62. Substituting the known values of k and e and the experimental value of T , viz. 1480°K , and inserting the factors for volts and natural logarithms, it appears that these data give $V = 0.96$ volt as compared with the measured value 0.71 volt.

This is probably as substantial an agreement as could be expected, considering the uncertainty of some of the data. We are engaged in further experiments which it is believed will eliminate these elements of uncertainty. The difficulties, however, are not inconsiderable, and it is clear that some time must elapse before further results of value are obtained.

A reference to these experiments was made by one of us in his Presidential Address to Section A of the British Association at the Edinburgh meeting, 9th September 1921.