outline seems wholly feasible. Its scope is within the capacities of the instructors and the time limit allowed for mathematical work. And the proposed unification means economy in presentation of vocabulary, a knitting together of dependent ideas, a constant review of the old material, and a perspective of additional knowledge, and pleasure in the study of mathematics.

DISCUSSION OF THE UNIFICATION OF SECONDARY MATHEMATICS.¹

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The Central Association is getting its bearings. For several years now it has been trying to improve mathematical teaching. Many experiments have been tried; many schemes have failed. But we now see light. The report on unification of mathematics read by Mr. Cobb shows wherein, to a great degree, the real trouble with mathematics lies.

In science work, in all applied mathematics one does not use arithmetic, algebra, geometry, trigonometry. One uses *mathematics*.

Until now we have built up a systematic course in arithmetic; followed this with a somewhat vigorous course in algebra, then came a course in geometry, carefully compiled so that few rules of logic were violated and every rule of pedagogy forgotten.

Throughout each course were glaring sign posts:

"We must not do *this* because *that* has not been proven."

"We must not do *this* because *that* has not been defined."

"We cannot admit this problem in algebra because it is essentially arithmetical."

One of the greatest difficulties arising from the teaching of mathematics in compartments, is that arithmetic and algebra are taught largely as memory and rule of thumb processes, then comes a geometry course that is all thought process, and all three subjects suffer.

I have been asked to discuss our committee's report on unification. Perhaps as satisfactory a discussion as I can offer is a report of what my teachers have accomplished in the past year.

This report offered by the committee is necessarily general in its nature. It is my purpose to give the report my support, show

¹Read before the Mathematics Section of the Central Association of Science and Mathematics Teachers at the University of Chicago, November 27, 1909.

how some of the details may be worked out, and to urge that the teachers, in so far as they have a free hand in their respective schools, combine quite largely the subjects above mentioned.

It is my good fortune to be in a school where such unification is necessary, essential, desired by those in control, and opportunity offered to try out such courses on pupils from all nationalities and all classes of homes. Such constituency gives the test an exceptional value.

Last year we began this course:

First year, arithmetic and algebra combined.

Second year, arithmetic, algebra, geometry, trigonometry combined.

These combinations were effected by using the regularly adopted texts, rearranging some portions, omitting others and supplementing with mimeograph sheets.

Beginning with our summer quarter, last July, we started a few sections on a combined course of the four subjects, covering a period of two years. We have some 300 boys doing this work.

At the same time we are continuing last year's arrangement in other classes. This affords us a basis for comparison of the two kinds of unification.

We find that it is a success to start with the study of number. We cannot call it arithmetic. The pupil knows all about that subject and is tired of it. We study therefore a new subject—*the composition of number*. Beginning with the digit—the decimal arrangement—and following with the factor phase.

E. g. $17 \cdot 6 = (10 + 7)6 = 60 + 42 = 102$.
which is analogous to $(a + b)c$.

When a boy gets $16 \cdot 12$ mentally, $116 \cdot 12$ does not bother him.

Combine $24 = 20 + 4$ with digit problems. If x = ten's digit, y = unit's digit, $10x + y$ = the number.

Then the factor phase—of how great value is it?

$$(1) 6 = 2 \cdot 3 \quad (2) 6^2 = 2^2 \cdot 3^2$$

$$(3) 12 = 2^2 \cdot 3 \quad (4) 144 = 2^4 \cdot 3^2$$

$$(5) (15)^2 \cdot (12) = 3^2 \cdot 5^2 \cdot 2^2 \cdot 3 = 2^2 \cdot 3^3 \cdot 5^2$$

$$(6) 2^2 \cdot 3^4 \cdot 5 \quad \text{Compare with} \quad a^2 b^4 c$$

$$2 \cdot 3^2 \quad a b^2$$

$$2^3 \cdot 3^3 \cdot 5 \quad a^3 b^3 c$$

I have used no exponent law, but have appealed to the common sense of the pupil.

Oral drill in number makes much algebraic work simple.

During the first twelve weeks we teach oral control of numbers in arithmetic, the four fundamentals of algebra, geometric construction, four theorems, about a dozen originals and simultaneous equations of two and three unknowns.

During the first year our course covers arithmetic, algebra through proportion, all of straight line geometry found in Books I and III.

Second year, the straight line part of Book IV, trigonometry of the right triangle, evolution, quadratics, the curvilinear parts of Books II, III, IV, V.

We make some use of the lever, do something of the proportion of chemistry, and simple problems involving gravity. Much is done toward combining the processes of arithmetic, algebra and geometry.

Compare:

$$45 = 3^2 \cdot 5$$

$$48 = 2^3 \cdot 3$$

$$54 = 2^1 \cdot 3^3 \cdot 5$$

$$\text{l. c. m.} = 2^3 \cdot 3^3 \cdot 5$$

and

$$a^2 - 8a + 15 = (a-3)(a-5)$$

$$a^2 - 81 = (a+9)(a-9)$$

$$a^4 - 18a^2 + 81 = (a+3)^2(a-3)^2$$

$$\text{l. c. m.} = (a-3)^2(a-5)(a+3)^2(a^2+9)$$

Such examples as this are helpful:

(1) $25x^2 + 30x + 9$ is the area of a square. What is one side?

(2) $x^2 + 8x + 15$ is a rectangle. One side is $x + 3$. What is the other side? Find the dimensions when $x = 1, 2, 3, -1, -2, -3, -5$. Draw each rectangle.

(3) The area of a square is $4x^{\frac{1}{2}} - 12x^{\frac{1}{4}} + 9$. Find the diagonal correct to three decimal places when $x = 16$.

Such work is of course all oral.

Multiplication by factors are useful and helpful when such cases as these arise:

$$81.25, 216.3333, 10.3333, 10.301, 10.4771, 10.8451,$$

$$(10^2)(10.4771), \text{ etc.}$$

Thus far the work is most satisfactory. The pupils are interested. They know what they are doing because they think. We do not work by rule. Our entire product is based on principle, axiom, reason.