571. The Bee and the Pentagon

Author(s): W. Hope-Jones

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The question would be accurate if it had read "any two points of AC ," or else "a general point of AC ." UBV is always straight for one position of the point, and then UV is $\perp$ to AC .

For, let X be the point on AC , and O the orthocentre of ABC ; let AO, CO meet CV, AU in Q, P respectively.

Then, by parallels, $PU : UA = CX : XA = CV : VQ$. Hence COP, AOQ, UV are concurrent, and UV always passes through O . If it also passes through B , then either B coincides with O and ABC is a right angle, or else UV coincides with BO and is at right angles to AC . H. P. H.

Let P denote "any point" in AC . For different positions of P , UV passes through the orthocentre O of the triangle ABC . Hence, when O and B do not coincide, there is still *one* position of P in AC for which UV passes also through B . W. J. DOBBS.

AB, CD are given parallel lines.

From any point P in AC , draw PM, PN in given fixed directions to cut AB, CD in M, N respectively.

MN must pass through a fixed point.

If MP cut CD in U , and $CLE \parallel UPM$ cut MN in E and PN in L ,

$$\frac{CL}{LE} = \frac{UP}{PM} = \frac{CP}{PA}; \quad \therefore AE \parallel PN;$$

$\therefore E$ is fixed; $\because CE, AE$ are parallel to given fixed directions.

(AE and CE are special positions of MN .)

If $AM = x, CN = \xi$, the line is known when x, ξ are known.

You might call x, ξ tangential coordinates of M, N , and a linear relation between x and ξ signifies " MN passes through fixed point."

FIG. 1.

FIG. 2.

In the B.Sc. problem, when it is recognised that UV passes through a fixed point, AX, CY being special positions, UV always passes through the orthocentre. If the orthocentre is $B, ABC = 90^\circ$.

If $x\xi = K^2$, envelope of MN is an ellipse, AB, CD being parallel tangents at ends of diameter AC .

The point of contact of MN and curve cuts MN as $-dx/d\xi$, i.e. as x to ξ .

R. F. DAVIS.

571. [I. 2. δ .] *The Bee and the Pentagon.*

In matters hexagonal the bee is our oldest teacher: but her connection with the regular pentagon is unsuspected by many naturalists, and very likely by herself.

The female bee has two parents, like man and other respectable animals; but the male is a parthenogenetic product, and owns a mother only. From this it follows that the number of ancestors of n generations back possessed by the ordinary worker, or her sister the queen, is

$$\frac{2^{n+2}}{\sqrt{5}} (\cos^{n+2} 36^\circ - \cos^{n+2} 108^\circ).$$

W. HOPE-JONES.