THE EFFECT OF CHANGED DATA UPON REASONING¹

BY EDWARD L. THORNDIKE

Institute of Educational Research, Teachers College, Columbia University

The older psychology, perpetuated in current educational doctrines and practices, regarded reasoning as a force largely independent of associative habits, which worked back to correct or oppose them. Our present psychology finds that the mind is ruled by habit throughout, the correction or opposition being of certain more simple, thoughtless and coarse habits, by others which are more elaborate, selective, and abstract. It defines reasoning as the organization and coöperation of habits rather than as a special activity above their level; and expects to find 'reasoning' and habit or association working together in almost every act of thought.

One interesting and rather important consequence of this view is the theorem that "Any disturbance whatsoever in the concrete particulars reasoned about will interfere somewhat with the reasoning, making it less correct or slower or both." It is the purpose of this article to give illustrations of and evidence for this theorem drawn from a simple experiment which can be easily given to any class, and is perhaps deserving of inclusion in a list of group experiments in psychology.

Consider the two sets of tasks in algebra, printed below. Each pair of tasks demands the application of the same principle, but the concrete situation in the one case is that with which our ordinary associative habits have been made, whereas in the other case the concrete particulars are somewhat altered. The alterations vary from slight ones, such as using p instead of x , or $b_1 + b_2$ instead of $x + y$, to a

¹ The experiments reported here were part of an investigation made possible by a grant from the Commonwealth Fund.

change from a very customary statement of a set of relations to a very rare statement, as in No. 9.

HA.

Habitual

1. What is the square of $x + y$?
2. What is the square of a^2x^3 ?
3. Simplify $4ac + \left(\frac{b^2}{c^2} \times \frac{c}{d^2} \times \frac{d^3}{b}\right)$.
4. What are the factors of $x^2 - y^2$?
5. Multiply x^a by x^b .
6. Simplify $ac - [a(b + c)]$.
7. Solve
$$\begin{aligned} x + y + z &= 15, \\ 2x + y + 3z &= 22, \\ x + 2y + z &= 25. \end{aligned}$$
8. $e^2 + ef = \frac{g}{x}$.

What does x equal?

9. There are two numbers.
The first number plus 3 times the second number equals 7.
The first number plus 5 times the second number equals 11.

What are the numbers?

The probability of error and delay in the necessary reasoning is clearly increased when the task is of the habitual sort (Ha.) rather than the changed sort (Ch.).

The subjects which I used were ninety-seven graduate students, divided at random into three groups of 34, 32 and

CH.

First Changed Form

1. What is the square of $b_1 + b_2$?
2. What is the square of $r_1^3r_{11}^2$?
3. Simplify $5a^2b + (b^2 \div c^2)(c^2 \div d^2)(d^3 \div b)$.
4. What are the factors of $1/x^2 - 1/y^2$?
5. Multiply 4^a by 4^b .
6. Simplify $p_1p_3 - [p_1(p_2 - p_3)]$.
7. Solve
$$\begin{aligned} c_1 + c_2 + c_3 &= 15, \\ 2c_1 + c_2 - 3c_3 &= 22, \\ c_1 + 2c_2 + c_3 &= 25. \end{aligned}$$
8. $e^2 + ef = \frac{g}{p}$.

What does p equal?

9. $y = ax + b$.
When $x = 3$, $y = 7$.
When $x = 5$, $y = 11$.
What does a equal?
What does b equal?

31. Group *A* did tasks 1 and 7 of Ha., tasks 4, 5, 6, 8 and 9 of Ch. and two other tasks as follows: "What is the square of k^2p^3 ?" and "Simplify $3ab + [b^2cd^3/bc^2d^2]$." Group *B* did tasks 3, 6 and 9 of Ha., tasks 1 and 2 of Ch. and four other tasks as follows: "What are the factors of $x - y$?" "Multiply x^4 by x^3 ." "Solve $x + y = 13$, $y + z = 12$, $x + z = 5$." " $e^2 + ex = g/a$. What does a equal?" Group *C* did tasks 2, 4, 5 and 8 of Ha., tasks 3 and 7 of Ch. and three other tasks as follows: "Multiply $(b_1 + b_2)$ by $(b_2 - b_1)$." "Simplify $mn - [2n(m - p)]$." "The cost of a season ticket is b dollars plus k times the cost of a month ticket. When the cost of a month ticket is \$6, the cost of a season ticket is \$15. When the cost of a month ticket is \$15, the cost of a season ticket is \$33. What is the value of b ? What is the value of k ?"¹

The relative abilities of the groups may be estimated by comparing the scores in tasks estimated to be of equal difficulty, such as:

$(x + y)^2 = ?$ and Solve $e^2 + ef = g/p$ for p , for *A*,
 $x^4 \times x^3 = ?$ and Solve $e^2 + ex = g/a$ for a , for *B*,
 and Factor $(x^2 - y^2)$ and Solve $e^2 + ef = g/x$ for x , for *C*.
 Using these combinations, the percent of right answers was 71 for Group *A*, 84 for Group *B*, and 63 for Group *C*. There is thus an approximate equalization of ability of subjects between the *Habitual* and the *Changed* series.²

All three groups worked by the same time-schedule, which was: "0, Begin; 1 min. 30 sec., even if you have not finished 1, begin on 2; 2.30, even if you have not finished 2, begin on 3; 4.30, even if you have not finished 3, begin on 4;" and so on with 6.30 for proceeding to 5; 8.30 for 6; 9.30 for 7; 12.30 for 8; 13.30 for 9; 16.30 to stop. The subject could thus use time saved on one task for another, absolute uni-

¹ The fractions in the above and throughout were all presented in the usual 'horizontal' form, e.g. $\frac{g}{a}$.

² The *B* group is probably really somewhat more superior than these figures show, and the differences shown in Table I. below are somewhat greater than would be the case with perfect equalization. However, there can be no doubt that a substantial difference would remain.

formity being had only in respect to the 16 min. 30 sec. spent on the entire set of nine tasks.

The percents wrong or incomplete for the nine tasks where customary associations were favored and for the nine where some change was made were, in order:

TABLE I

	Customary	Changed	Nature of Change
1.....	6	28	$(x + y)$ to $(b_1 + b_2)$
2.....	34	47	a^2x^3 to $r_1^2r_{11}^2$
3.....	$37\frac{1}{2}$	$64\frac{1}{2}$	— to $\div$ form
4.....	22	41	$x^2 - y^2$ to $\frac{1}{x^2} - \frac{1}{y^2}$
5.....	55	$70\frac{1}{2}$	x^2 to 4^2
6.....	25	53	ac , etc., to p_1p_2 , etc.
7.....	62	61	xyz to $c_{12}c_{23}$
8.....	52	53	x to p
9.....	16	70	form of problem

In all but two cases (7 and 8) there is a substantial interference with thought by the change. In No. 9 the amount of novelty introduced is much greater than in the other cases, and is perhaps more truly called a change in the principles and operations used than a change in the concrete particulars reasoned about. In No. 6 the use of a minus sign in place of the plus may have added a little difficulty over and above the change from literal to subscript distinctions.

We may now examine certain facts from the accessory tasks which were used partly to conceal the special point of the experiment from the participants and partly to secure additional data.

"Multiply $(b_1 + b_2)$ by $(b_2 - b_1)$ " is not strictly comparable with "What is the square of $x + y$?" but the rise in errors from 6 percent for the latter (Group *A*) to 61 percent for the former (Group *C*) would probably have been much less if "Multiply $(x + y)$ by $(y - x)$ " had been used.

The change from "What is the square of a^2x^3 ?" (Group *C*) to "What is the square of k^2p^3 ?" (Group *A*) raises the errors and omissions from 34 percent to 47 percent.

Simplify $3ab + [b^2cd^3/bc^2d^2]$ produces more errors than the same task when arranged in $+$ ($\times \times$) or in $+$ ($\div \div$) form (71 percent in Group A). This may seem to be in contradiction to our general theorem, but an inspection of the errors makes it probable that the parentheses were a protection against two habits, one of canceling upper and lower numbers seen in proximity, the other of clearing of fractions any expression that contains one. The former leads to canceling the b of $3ab$ with the b of bc^2d^2 , which is done by one of the thirty-four subjects. The latter adds an operation in which errors are made by four of the subjects, and makes the needed cancellations more difficult. It also encourages the error of dropping out the common denominator, which is done by four more, and permits the worker to feel that he has done his duty by the task, though he has not cancelled. Finally, there are further erroneous manipulations of the denominator (as in $(3ab + b^2cd^3)/bc^2d^2$) from which the parentheses and signs of multiplication and division were perhaps a protection in the other forms of the task.

The question "What are the factors of $x - y$?" evokes responses which show that when a novel situation is met, one's associative habits do not retire while reason attacks. On the contrary, the forces of habit are nowhere more evident than in the treatment of novel situations. Which habits will act depends on which elements or features of the situation are given weight and upon the amount of weight given to each. With our subjects, the features that this is the difference not of two particular numbers but of any number from any number and that 'any number' is as truly the square of ' $\sqrt{\text{any number}}$ ' as '(any number)²' is of 'any number,' are usually neglected or underweighted, there being only four answers of

$$(\sqrt{x} + \sqrt{y})(\sqrt{x} - \sqrt{y}) \text{ or } (x^{1/2} + y^{1/2})(x^{1/2} - y^{1/2})$$

The underweighting of these features permits the habits of not factoring prime numbers in arithmetic, and of not factoring expressions like $x - y$ in algebra, to act acceptably, answers of $1(x - y)$ being the commonest given (11 out of 32).

"Multiply x^4 by x^3 " is, consistently with our general theorem, easier than x^a by x^b .

On the whole, it seems certain that even such slight changes as from the customary a , b , x and y , to k and p or to p_1 , p_2 and p_3 or to p_1 , p_{11} and p_{111} , impede thought, and that the general theorem does hold that "Any disturbance whatsoever in the concrete particulars reasoned about will interfere with reasoning."