GENERAL ABILITY, CLEVERNESS AND PURPOSE.

BY J. C. MAXWELL GARNETT.

1. *Introduction.*
2. *A 'single general factor' in dissimilar mental tests.*
3. *The cosine expression for correlation; and the consequent condition that three correlated variables should be completely expressible in terms of only two independent factors.*
4. *'Cleverness,' a group factor in tests of certain intellectual qualities; and its relation to 'General Ability.'*
5. *Group factors in tests of similar qualities.*
6. *'Purpose,' a group factor in tests of certain character qualities.*
7. *Summary of Conclusions.*

1. *Introduction.*

THE object of this paper is to show that, in addition to the 'single general factor' which along with specific factors tends (as Professor Spearman has shown) wholly to account for the correlations between any set of sufficiently diverse mental tests, there are other independent factors which also enter to a significant extent into tests of certain groups of similar qualities. Thus there are several intellectual qualities which may be regarded as compounded of 'g'—the 'general ability' which tends to occur without significant group factors in every set of sufficiently dissimilar tests—and of one other independent (group) factor which we shall find it convenient to describe as 'cleverness.' There is also, as Dr Webb has shown¹, a number of character qualities into which there enters, along with 'g,' another (group) factor which has much in common with 'purpose' and which is independent both of general ability and of cleverness. When the degrees in which any particular person possesses general ability, cleverness and this third (group) factor are known, his character is, in large measure, determined; although of course a very great number of other factors, independent of these three, may also enter into many of his mental qualities.

¹ This *Journal*, Monogr. Suppl. 1. 3.

2. A 'single general factor' in dissimilar mental tests.

It has been shown that if $q_1, q_2, \dots, q_n$ be a number of correlated variables, as for example the measures of the mental qualities tested in such investigations as those of Mr Burt¹ or Dr Webb², each of which is distributed according to the normal law with the same standard deviation, then all these n inter-dependent q 's may be regarded as made up of the sum of fractions of n or more independent variables $x_1, x_2, \dots, x_N$ (where $N \geq n$) each of which is distributed according to the normal law with the same standard deviation³. In other words, any one of the q 's, say q_s , may be expressed as a linear function of the x 's thus

$$q_s = {}_s l_1 \cdot x_1 + {}_s l_2 \cdot x_2 + \dots + {}_s l_N \cdot x_N \dots \dots \dots (1),$$

where the l 's are numerical coefficients such that

$${}_s l_1^2 + {}_s l_2^2 + \dots + {}_s l_N^2 = 1 \dots \dots \dots (2).$$

(In order to avoid wearisome repetition, we state here that all the variables discussed in this paper are assumed to be distributed according to the normal law with the same standard deviation unless, in any particular case, the context explicitly provides to the contrary.)

It has further been shown that, if the correlations between the q 's satisfy Professor Spearman's 'correlation between columns' conditions the differences, $q_1', q_2', \dots, q_n'$, between the n q 's and a constant multiple k (which becomes zero when Mr Burt's conditions for a hierarchy are fulfilled) of an $n+1$ th variable, y , that is independent of the q 's may be expressed in terms of $n+1$ independent x 's of which one, say $x_{n+1} \equiv x_g$, is a single general factor, while all the other x 's are specific factors⁴. In other words, if r_{st} be the correlation between q_s and q_t , and if when the r 's be arranged in a correlation table thus

—	r_{12}	r_{13}	...	r_{1n}
r_{12}	—	r_{23}	...	r_{2n}
r_{13}	r_{23}	—	...	r_{3n}
$\vdots$	$\vdots$	$\vdots$		$\vdots$
r_{1n}	r_{2n}	r_{3n}	...	—

the correlation between every pair of columns is ± 1 , we may write

$$q_s' \equiv (q_s - y \cdot k) (1 + k^2)^{-\frac{1}{2}} = r'_{sg} \cdot x_g + (1 - r'^2_{sg})^{\frac{1}{2}} \cdot x_s \quad (s = 1, 2, \dots, n) \dots \dots (3),$$

¹ This *Journal*, 1909-10, III. 94-177.

² *Loc. cit.*

³ Garnett, *Proc. R. S.*, 1919. Cf. also Bravais, *Mém. de l'Inst. de France*, 1846, IX. 260 *et seq.*

⁴ Garnett, *loc. cit.*

where r'_{sg} is the coefficient of correlation between q'_s and x_g ; so that x_g is a single general factor of the q 's and all the other x 's are specific factors¹. When Mr Burt's conditions for a hierarchy, namely

$$\frac{r_{su}}{r_{sv}} = \frac{r_{tu}}{r_{tv}}$$

for all values of s, t, u, v from 1 to n inclusive, are satisfied, $k=0$ and equations (3) reduce to

$$q_s = r_{sg} \cdot x_g + (1 - r_{sg}^2)^{\frac{1}{2}} \cdot x_s \quad (s = 1, 2, \dots n) \quad \dots\dots\dots(4),$$

where r_{sg} is the coefficient of correlation between q_s and x_g . Under certain conditions, which are not fulfilled in the case of published results of mental tests examined by the present writer, r_{sg} in equations (4) or r'_{sg} in equations (3) would become imaginary.

It is easy to show that Mr Burt's conditions are equivalent to $\frac{1}{2}n(n-3)$ independent conditions. We note that when $n=3$ no conditions need be satisfied by the correlations between the q 's in order that the q 's may be expressed in terms of four independent factors, one of which is a single general factor and the remainder are specific factors.

In what follows we shall use the term 'single general factor' to denote only that one of $n+1$ independent factors which, when n correlated q 's are expressed in terms of them alone, enters into each of the q 's while the remaining n independent factors are specific, each entering into a different q . This definition of a single general factor is equivalent to saying that, if n correlated q 's can be expressed by means of equations (4) in terms of $n+1$ x 's, so that all the r 's are finite, then x_g is a single general factor.

That such a definition of a single general factor is necessary in order to avoid confusion is readily seen by observing that it is always possible, by choosing a sufficient number of independent factors (x 's) in terms of which to express n given q 's, so to express these q 's that any desired number of x 's will have finite coefficients in the right-hand sides of all the n equations (1) obtained by giving s all values from 1 to n in that equation. For example in equation (4) we might replace x_g by

$$\frac{1}{\sqrt{R}}(m_1y_1 + m_2y_2 + \dots + m_ky_k), \text{ where } m_1^2 + m_2^2 + \dots + m_k^2 = 1,$$

where all the y 's are independent of each other and of $x_1, x_2, \dots x_n$. There would then be k independent factors (y 's) instead of only one (x_g) entering into each of the q 's. Moreover, by another similar trans-

¹ Cf. Garnett, *loc. cit.*

formation it is possible to replace the right-hand sides of equations (4) by expressions into which there enter any desired number of group factors: namely, independent factors whose coefficients are finite in the expressions for two or more but not all of the q 's. If, for example, we were to write

$$y_1 = r_{1g} \cdot x_g + \sqrt{1 - r_{1g}^2} \cdot x_1,$$

$$y_2 = \sqrt{1 - r_{1g}^2} \cdot x_g - r_{1g} \cdot x_1,$$

so that

$$x_g = r_{1g} \cdot y_1 + \sqrt{1 - r_{1g}^2} \cdot y_2,$$

and to substitute for x_g in the n equations (4), they would be replaced by n equations into the right-hand sides of which there entered $n + 1$ mutually independent variables $y_1, y_2, x_2, x_3, \dots, x_n$, but so that y_2 appeared to be a group factor entering into all but one of the q 's.

3. *The cosine expression for correlation; and the consequent condition that three correlated variables should be completely expressible in terms of only two independent factors.*

It has been shown¹ that the correlation r_{st} between any two of the q 's, q_s and q_t , expressed by means of equations (1), is given by

$$r_{st} = l_1 \cdot l_1 + l_2 \cdot l_2 + \dots + l_N \cdot l_N \dots \dots \dots (5).$$

However great may be the number, N , of independent variables we can, by a linear transformation, choose new independent variables $y_1, y_2, y_3, \dots$ such that two variables, q_s and q_t , depend on two and two only among them, say y_1 and y_2 . Equations (1) may then be replaced by

$$\left. \begin{aligned} q_s &= y_1 \cos \theta_s + y_2 \sin \theta_s \\ q_t &= y_1 \cos \theta_t + y_2 \sin \theta_t \end{aligned} \right\} \dots \dots \dots (6),$$

so that

$$r_{st} = \cos (\theta_s \sim \theta_t) \dots \dots \dots (7).$$

This equation may be interpreted by saying that, if we measure the y_1 and y_2 of any subject along two axes Oy_1 and Oy_2 at right angles to one another, and plot the point $P (y_1, y_2)$ corresponding to that subject so that y_1 and y_2 are the projections of OP on Oy_1 and on Oy_2 respectively, and if we then draw a line Oq_s , which we may call the axis of q_s , making with Oy_1 an angle θ_s (measured in the direction from Oy_1 to Oy_2), the degree in which the subject possesses the quality measured by q_s will be equal to the projection of OP on Oq_s . Moreover, the

¹ Garnett, *Proc. R. S.*, 1919. Cf. also Bravais, *Mém. de l'Inst. de France*, 1846, ix, 271.

correlation between q_s and q_t is equal to the cosine of the angle $q_s O q_t$; so that this correlation represents the average deviation in q_s (or q_t) corresponding to unit deviation in q_t (or q_s), affording us a very simple geometrical conception of the measure of correlation.

The condition that three qualities q_1 , q_2 and q_3 should be expressible in terms of two and only two independent factors follows at once. It is the condition that Oq_1 , Oq_2 and Oq_3 should lie in a plane; or that the sum of the angles $q_2 \hat{O} q_3$, $q_3 \hat{O} q_1$ and $q_1 \hat{O} q_2$ should be zero; or that

$$\cos^{-1} r_{23} + \cos^{-1} r_{31} + \cos^{-1} r_{12} = 0 \dots \dots \dots (8);$$

or that

$$r_{23}^2 + r_{31}^2 + r_{12}^2 - 2r_{23}r_{31}r_{12} = 1 \dots \dots \dots (9).$$

4. '*Cleverness*,' a group factor in tests of certain intellectual qualities; and its relation to '*general ability*.'

Dr Webb, in his paper on "Character and Intelligence¹," describes an investigation of forty-eight mental qualities. The subjects of his enquiry were ninety-eight men students (average age twenty-one) at a Training College during the last six months of their second year of training (January—July, 1912), and a similar group of ninety-six students during the corresponding period of the following year (January—July, 1913). Dr Webb also investigated a similar number of mental qualities of four groups of London schoolboys, with whom we shall be only once again concerned. Of the students' forty-eight mental qualities investigated, forty-three were estimated by pairs of prefects who acted as judges; to each pair a group of twenty (or nineteen) students was assigned. The measurements of the qualities were so chosen as to give the same constant (standard deviation) to the frequency distribution of each. The forty-three mental qualities estimated by the prefects included degree of sense of Humour (No. 8) and two described by Dr Webb as intellectual qualities, namely, Quickness of apprehension (No. 35) and Originality of ideas (No. 38).

Among the five qualities not estimated by the prefects, two were objectively measured, namely Examinational Ability and Professor Spearman's single general factor whose measure is g^* . The manner in which ' g ' was measured is fully described in Dr Webb's paper. Following

¹ *Loc. cit.*

* It will be remembered that g , like the measures of the other qualities with which we are concerned, is distributed according to the normal law and may have any value, $-\infty$ to $+\infty$, its mean value being zero. For the future we shall briefly write ' g ' in place of 'the quality of which the measure is g .'

Dr Hart and Professor Spearman, Dr Webb described '*g*' as a 'General Factor of Intellective Energy'¹; but Dr Webb produced no additional evidence for regarding *g* as a measure of intellective energy rather than as, for example, a measure of power of voluntarily concentrating nervous energy, or, what amounts to the same thing, voluntarily concentrating attention.

Now it has been shown² that the correlations between the three qualities, '*g*,' Humour and Originality satisfy equation (8) within the limits of probable error, while the correlations of Quickness with any pair of the other three qualities very nearly do so; so that '*g*,' as measured experimentally, together with Humour and Originality, as estimated in a very large number of cases by Dr Webb's collaborators, is compounded of two, and only two, independent factors, while Quickness consists almost, if not quite, of some combination of the same independent factors. We may therefore construct a diagram (Figure 1) by drawing lines *Og*, *Oh*, *Oo* in one plane, making the following angles with each other:

$$gOo = \cos^{-1} r_{go} = 100^\circ,$$

$$hOo = \cos^{-1} r_{ho} = 38^\circ,$$

$$oOg = \cos^{-1} r_{og} = 62^\circ.$$

Fig. 1.

And we may add to the diagram another line *Oq*, very nearly coincident³ with *Oo*.

If the training college students who formed the subjects of Dr Webb's investigation constitute a fair sample of adult Englishmen, the '*g*,' sense of Humour, Originality, and (probably) Quickness of apprehension of any Englishman can be represented by a single point (*P*, say) on the plane of the diagram—the 'intellectual plane,' as it has been

¹ *Loc. cit.* 37.

² Garnett, *loc. cit.*

³ The correction coefficient between Humour and Originality given in Dr Webb's table is $r_{go} = 1.04$.

called; for if P be determined so that the projection of OP on any two of the axes, Og , Oq , Oo or Oh , measures the corresponding two qualities of the subject in question, it follows (as we saw in § 3 above) that the degrees in which the same subject possesses the remaining two qualities will be measured by the projections of OP on the remaining two axes; and the proportion of Englishmen whose intellectual qualities (if for the moment we may confine this term to the four qualities just named) are represented by points lying within any small area δA , will be equal to the volume of a cylinder having its axis perpendicular to δA , its base on δA , and bounded at its other extremity by the surface

$$z = \frac{1}{2\pi\sigma^2} e^{-\frac{1}{2\sigma^2}(x^2+y^2)} = \frac{1}{2\pi\sigma^2} e^{-\frac{\epsilon^2}{2\sigma^2}} \dots\dots\dots(10),$$

where x and y are the measures of the two independent variables of which each of the four qualities is compounded, and where

$$\epsilon^2 = x^2 + y^2;$$

so that ϵ measures the degree in which the subjects represented by points in δA are *exceptional*. We observe that since equation (10) is independent of the particular axes chosen, the number of individuals represented by points in δA is independent of the orientation of the radius joining O to δA .

The solid bounded on one side by the surface whose equation is given in (10) above, and on the other by the plane $z = 0$, is a useful conception when we have to consider qualities dependent upon only two independent variables measured along Ox and Oy . For example, the number of men whose Originality lies between o and $o + \delta o$ is equal to the volume of this solid included between the two planes which are at right angles to Oo (and therefore to the plane gOh), and which cut Oo at distances o and $o + \delta o$ from the origin.

Now imagine the small area δA to move, in the plane of the diagram, at right angles to Og (or to Oh , or to Oo). It is clear that the projections of the radius ($O . \delta A$) on the lines Oh and Oo (or on Og and Oo , or on Oh and Og) vary in strict proportion to one another. In other words, if any one of the three qualities ' g ,' Humour, or Originality be kept constant, the other two qualities will be strictly proportional to each other. That is to say, the partial correlation between Humour and Originality, with ' g ' constant, should be ± 1 ; between Originality and ' g ,' with Humour constant, should be ± 1 ; and between ' g ' and Humour, with Originality constant, should be ± 1 . Actually, the partial correlations in question, obtained by Yule's formula from the total correlations given

above, are 1.00, 1.0*, - 0.98, thus verifying all our anticipations within the probable error.

The nature of the two independent qualities or factors which, combined in different proportions according to equation (5), make up each of the four intellectual qualities 'g,' Humour, Originality and Quickness, has been thus discussed¹:

Since the equation (10) is independent of the particular pair of perpendicular axes chosen in the 'intellectual' plane, we may choose *Og* as one axis. What, then, is the other axis, represented by a line *Oc* at right angles to *Og*? Evidently, since the Humour axis makes with it an angle of only some 10°, it is very nearly identical with Humour.

Now Mr McDougall has suggested that the process of 'reproduction by similars,' or, as William James² called it 'association by similarity,' is due to "a partial identity of the complex neural systems involved in the perception of two objects. Each system consists of many sub-systems, and one or more of these sub-systems is common to the two. When the one system is excited, its excitement spreads, not, as is most commonly the case, through some association-path previously established by some temporal contiguity, but from the sub-system, which forms also part of another system, radiates itself through that other system. In the commonplace type of mind this process comparatively rarely occurs. It would seem that in the brains of such persons neural systems tend to become circumscribed and individualized, whereas in a higher type of brain the neural systems are more complexly interwoven, sub-systems becoming freely associated with many principal systems. In a brain so constituted, reproduction of similars will frequently occur, causing the dull chain of simple reintegration, the serial reproduction of impressions associated by temporal contiguity, to be broken across. The possessor of a brain so constituted will never be a commonplace person; he may be a crank or an original thinker, or merely a wit³."

There is no evident reason why such a constitution of brain should have anything to do with capacity voluntarily to concentrate attention, the capacity which Binet and others have identified with the quality whose measure is *g*. Now the quality independent of 'g' for which we are seeking is very closely connected with Wit or Humour, and also closely connected with Originality. Its correlation with Humour is, as we see from our diagram (Fig. 1) $\cos 10^\circ$ or 0.98 and its correlation with Originality is $\cos 28^\circ$ or 0.88. Let us call it Cleverness. Then Cleverness is defined as a quality which is independent of 'g' but which, combined with 'g' in different proportions, wholly constitutes Humour or Originality, and wholly or mainly constitutes Quickness of apprehension. Being closely connected with Humour and with Originality, Cleverness as thus defined is also closely connected with the form of brain constitution described by Mr McDougall, in the passage we have

* The actual figure is 1.65, but the correlation cannot, of course, exceed +1. If, for example, $\cos^{-1} r_{gh}$ were .70 instead of .79 a difference approximately equal to the probable error, the partial coefficient $r_{og.h}$ would become 1.19.

¹ Garnett, *loc. cit.*

² *Principles of Psychology*, 1. 578.

³ *Physiological Psychology*, 139.

quoted. Moreover, so far as we can see, this form of brain constitution, like Cleverness as just defined, is independent of 'g.' We have therefore grounds for identifying it with Cleverness.

Let us now consider our diagram (Fig. 1) in the light of this suggestion.

Still having regard only to the five intellectual qualities named in the diagram, we observe that people may be intellectually exceptional in any number of different ways: the radius from O to the small area δA , while remaining of length ϵ , may make any angle θ with any one of the fixed axes, say Og . If ϵ remain large and θ increase, we start with men having great 'g'—great 'Ability,' perhaps great power of concentrating attention—but only average Cleverness. Such men will, according to the diagram, possess much more than average Originality and Quickness, but less than the average sense of Humour. As θ increases, ϵ remaining constant, δA will come to Oq and soon after to Oo , when θ is about 60° ; for $gOq = \cos^{-1} r_{gq} \equiv \cos^{-1} 0.53$, and $gOo = \cos^{-1} r_{go} = \cos^{-1} 0.47$, according to Dr Webb's table. When, therefore, for a given degree of intellectual exceptionality, Quickness or Originality is greatest, Cleverness is about $\sqrt{3}$ times as great as 'g,' while Sense of Humour is well above the average. As θ continues to increase, the rotating radius brings δA to Oc . Then Cleverness is at a maximum (ϵ being given); Sense of Humour is nearly at a maximum, for the angle cOh is only about 10° ; Originality and Quickness are much above the average; but Ability ('g') is only equal to the average. Finally, when δA comes to lie on Oh (so that Sense of Humour¹, or Wit, is the most exceptional quality of the exceptional men represented by points in δA), Cleverness, Originality and Quickness are all much above the average, but Ability is slightly below the average. This does not, of course, mean that very able men (men with very high 'g') may not have a great Sense of Humour; but only that, the greater their Ability ('g'), the greater must be their Cleverness to produce a given degree of Sense of Humour.

The distinction between Ability ('g') and Cleverness has been emphasised by Dr Mercier in an essay on "Cleverness and Capability²." He maintains that these two qualities "are quite different from one another, and may be developed to very different degrees in the same person," as the point representing that person in our diagram moves

¹ We here assume the correlation $r_{gh} = -.17$, which is about double the probable error, to be a significant figure; so that the correlation between Ability and Humour is low and negative.

² *Human Temperament*, 1917.

from the neighbourhood of *Og* to the neighbourhood of *Oc*. Dr Mercier is further of opinion that "capability [*g*]... may be inculcated by a proper training, but no training will make a stupid person clever¹." Dr Mercier goes on: "Capability can be acquired, and it should be one of the main objects of education to see that it is acquired." The clever man of science "is fertile in hypotheses." "The clever shopman amuses his customers"; he is, in fact, a wit. "Capable people concentrate their attention on the matter in hand, think it out in all its bearings..." "Clever [but incapable] people are apt to make mistakes and go wrong because their attention is discursive." "From this lack of concentration it results that they do not think matters out."

William James, after pointing out that there are two stages in reasoning, the first of which—association by similarity—merely operates to call up cognate thoughts (Cleverness), and in the second of which attention is concentrated (*g*) upon the bond of identity between these cognate thoughts, adds: "So men of genius may be divided into two main sorts, those who notice the bond and those who merely obey it. The first are the abstract reasoners properly so called, the men of science and philosophers—the analysts, in a word; the latter are the poets, the critics—the artists, in a word, the men of intuitions²." According to this classification, great men of science and philosophers would be represented by points lying far from the origin but near to *Og*, while equally great poets and artists would be represented by points lying equally far from the origin but near to *Oc*. The importance to the poet of having a large ratio of *c* to *g* was emphasized, although in other words (!), by Schiller in a letter written in 1788, and quoted by Professor Freud³. Schiller wrote (to a friend who complained of his own lack of creativeness): "The reason for your complaint lies, it seems to me, in the constraint which your intelligence imposes upon your imagination."

We may note in this connexion that the word 'genius' is more commonly used to denote exceptional Cleverness than exceptional '*g*.' Thus William James writes:

Geniuses are, by common consent, considered to differ from ordinary minds by an unusual development of association by similarity... And as the genius is to the vulgarian, so the vulgar human mind is to the intelligence of a brute⁴.

¹ The view that '*g*' is educable is inconsistent with that expressed by Mr Burt (*This Journal*, III. 176); but Mr Burt's argument that '*g*' is innate seems to me inconclusive. On the contrary, there seems to me to be a considerable balance of evidence that '*g*' is educable, although its educability may be innate.

² *Loc. cit.* II. 361.

³ *The Interpretation of Dreams*, English edition, 85, 86.

⁴ *Loc. cit.* II. 348.

Again, in another place William James says that

Geniuses are commonly believed to excel other men in their power of sustained attention. In most of them, it is to be feared, the so-called 'power' is of the passive sort. Their ideas coruscate, every subject branches infinitely before their fertile minds, and so for hours they may be rapt. But it is their genius making them attentive, not their attention making geniuses of them. . . . It is probable that genius tends actually to prevent a man from acquiring habits of voluntary attention, and that moderate intellectual endowments [moderate c] are the soil in which we may best expect, here as elsewhere, the virtues of the will [$'g'$]¹, strictly so called, to thrive. But, whether the attention come by grace of genius or by dint of will, the longer one does attend to a topic the more mastery of it one has. And the faculty of voluntarily bringing back a wandering attention, over and over again, is the very root of judgment, character and will².

At the same time, it is probable that genius, as the word is commonly used, is more directly measured by $\epsilon = \sqrt{g^2 + c^2}$ than by g (measuring general Ability or capacity to concentrate attention) alone, or even by c (measuring Cleverness, as we have defined it, or tendency to associate by similarity) alone.

If the independence of ' g ' and ' c ' is confirmed by further investigation, and if of these two ' g ' alone be educable (although its educability may be innate), the distinction between ' g ' and ' c ' should have important consequences for education.

The correlation of c (the measure of Cleverness, as we have defined it) with the forty-eight qualities investigated by Dr Webb may be calculated as follows. Let us take the general factor measured by g , and Cleverness measured by c , as two of the independent variables in terms of which any of Dr Webb's qualities may be expressed. Then the measure of any of his qualities is given by equation (1), which now becomes:

$$q = r_{gq} \cdot g + r_{cq} \cdot c + l_3 x_3 + \dots + l_N x_N \dots \dots \dots (11),$$

where $x_3, x_4, \dots x_N$ are the remaining independent variables. Any of the qualities represented in our diagram, say Humour, is given by

$$h = r_{gh} \cdot g + r_{ch} \cdot c = r_{gh} \cdot g + (\sqrt{1 - r_{gh}^2}) \cdot c,$$

so that, according to equation (5),

$$r_{qh} = r_{gq} \cdot r_{gh} + r_{cq} \cdot \sqrt{1 - r_{gh}^2},$$

¹ In identifying ' g ' with 'will strictly so called,' it is necessary clearly to distinguish the momentary effort of *will* that we identify with ' g ' from the persistence of *purpose* that in English is often called by the same name. For example, when in the Lord's Prayer we say 'Thy will be done,' we mean 'Thy purposes be fulfilled.'

² *Loc. cit.* i. 423-4.

356 *General Ability, Cleverness and Purpose*

from which we obtain

$$r_{ca} = \frac{r_{gh} - r_{ga} \cdot r_{gh}}{\sqrt{1 - r_{gh}^2}} \dots\dots\dots(12).$$

All the r 's on the right-hand side of this equation are given in Dr Webb's table of corrected coefficients for the students. If we substitute Dr Webb's values and calculate r_{ca} for all values of g , except $g = g$ and $g = h$, we obtain the following series of coefficients of correlation:

TABLE I.

No. in Dr Webb's schedule	Name of Quality	Correlation with 'Cleverness'
1	General tendency to be cheerful (as opposed to being depressed and low-spirited)	.97
2	Tendency to <i>quick</i> oscillation between cheerfulness and depression (as opposed to permanence of mood)	-.05
3	Occasional liability to extreme depression	-.57
4	Readiness to become angry	.15
5	Readiness to recover from anger	.33
6	Occasional liability to extreme anger	-.18
7	Degree of aesthetic feeling (love of the beautiful for its own sake)	.39
8	Degree of sense of humour	.985*
9	Desire to excel at performances (whether at work, play or otherwise) in which the person has his chief interest	.46
10	Desire to impose his own will on other people (as opposed to tolerance)	.58
11	Eagerness for admiration	.18
12	Belief in his own powers	.32
13	Esteem of himself as a whole	.30
14	Offensive manifestation of this self-esteem (superciliousness) ...	.12
15	Fondness for large social gatherings	.85
16	Fondness for small circle of intimate friends	.01
17	Impulsive kindness (to be distinguished from No. 18)	.50
18	Tendency to do kindnesses on principle	.40
19	Degree of corporate spirit (in whatever body interest is taken) ...	.68
20	Trustworthiness (keeping his word or engagement, performing his believed duty)	.07
21	Conscientiousness (keenness of interest in the goodness and wickedness of actions)	-.05
22	Interest in religious beliefs and ceremonies (regardless of denomination)	-.39
23	Readiness to accept the sentiments of his associates	-.29
24	Desire to be liked by his associates	.56
25	Wideness of his influence	.67

* This figure cannot be calculated from equation (12). The equivalent formula

$$r_{ch} = \frac{r_{gh} - r_{gh} \cdot r_{go}}{\sqrt{1 - r_{go}^2}} \text{ has therefore been used.}$$

TABLE I (*continued*).

No. in Dr Webb's schedule	Name of Quality	Correlation with 'Cleverness'
26	Intensity of his influence on his special intimates	·84
27	Degree of 'tact' in getting on with people	·60
28	Extent of mental work bestowed upon usual studies	·01
29	Extent of mental work bestowed upon pleasures (games, etc.) ...	·31
30	Degree of bodily activity during business hours	·64
31	Degree of bodily activity in pursuit of pleasures (games, etc.) ...	·37
32	Degree with which he works with distant objects in view (as opposed to living 'from hand to mouth')	·07
33	Tendency <i>not</i> to abandon tasks in the face of obstacles	·39
34	Tendency <i>not</i> to abandon tasks from mere changeability	·06
35	Quickness of apprehension	·95
36	Profoundness of apprehension	·59
37	Soundness of common-sense	·51
38	Originality of ideas	·88
39	Pure-mindedness (extent to which he shuns telling or hearing stories of immoral meaning)	·45
40	Power of getting through mental work <i>rapidly</i>	·59
41	<i>Physique</i> , estimated by visiting doctor and lecturer in physical exercises	·16
42	General excellence of character, estimated by lecturers	·02
43	Estimate of general excellence of character, supplied by each prefect	·33
44	Examination ability	·30
45	Athletics, estimated by captains and by a member of the college staff	·27
46	Experimental tests of intelligence, furnishing the correlations of 'g' with the forty-seven other qualities, as explained on pp. 35 to 38 of Dr Webb's paper	·00*
47	Degree of strength of will	·41
48	Degree of excitability (as opposed to being phlegmatic)	·22

Inspection of these figures indicates that Cleverness may be recognised in practice—as, for example, when interviewing a candidate for an appointment, to whose general Ability ('g') testimonials or examination results bear witness—by noting his sense of humour, general tendency to cheerfulness (which is perhaps difficult to judge on the occasion of such an interview!), or quickness of apprehension.

* The correlation between the independent variables *c* and *g* cannot be calculated from equation (12). That its value is zero follows from our definition of *c*.

5. *Group factors in tests of similar qualities.*

It is important to observe that, if Mr Burt's conditions for a hierarchy were completely satisfied by every set of mental tests, g would be a single general factor within the meaning of our definition; and that, if Professor Spearman's correlation between columns conditions were completely satisfied, g would be a single general factor of the q 's in equations (3). But the fact is that, while Professor Spearman's conditions are approximately satisfied by every set of sufficiently diverse tests, they are not satisfied by a set of tests of very similar qualities, such for example as ability to translate Latin and knowledge of Latin grammar. The absence of a single general factor within the meaning of our definition—or, in other words, the inevitable presence of group factors,—when the tests relate to a set of such similar qualities, has been repeatedly mentioned throughout the literature of this subject beginning with Professor Spearman's paper of 1904¹. Thus when a set of tests relates to sufficiently dissimilar qualities the results of the tests may be approximately expressed by means of equations (3) or, if Mr Burt's conditions are satisfied, equations (4) containing on the right-hand sides a single general factor $\bar{g} \equiv x_g$ (and therefore no group factors) together with specific factors, x_s ($s = 1, 2, \dots n$); and, in so far as these approximate expressions give correlations that differ from those experimentally determined by more than the probable error, the necessary small correction may be applied by adding to the right-hand sides of equations (3) or (4) group factors of which the coefficients are small compared with those of g or of the specific factors.

Next suppose that a set of tests is concerned with similar qualities. Each one of these qualities (or, if Mr Burt's conditions are not satisfied, its difference from a real multiple of an independent $n + 1$ th quality), if considered in conjunction with dissimilar qualities, could be approximately expressed by means of the right-hand side of equation (3) or (4) into which only g and a specific factor enter; and it could be completely expressed by adding group factors of which the coefficients were small compared with those of g and of the specific factor. When, therefore, a set of similar tests is in question it is still convenient to express the measures (q 's) of the qualities tested in terms of independent variables that include g among their number. But now the independent variables

¹ *Loc. cit.* 273. Cf. also Dr Webb, *loc. cit.* 53 (first line).

other than g that enter into the various q 's (or q 's of equations (3)) will no longer all be specific factors: some will be group factors with significant coefficients.

Our new factor c is such a group factor. Along with g it enters into a number of intellectual qualities several of which as we have seen are expressible in terms of c and g alone without specific factors by means of equations of the form

$$q_i = g \cos \theta_i + c \sin \theta_i \dots\dots\dots(13),$$

where q_i is the measure of one of these intellectual qualities and θ_i is the angle which its axis makes with the axis of g measured in the direction from that axis to the axis of c . But, so far as this group of intellectual qualities is alone concerned, neither g nor c is a 'single general factor' within our definition.

6. '*Purpose*,' a group factor in tests of certain character qualities.

Another group factor which, along with g , enters into a number of purpose qualities—notably those numbered 34, 33, 18, 20, 21, 32, and 28 in the above Table I—was described by Dr Webb¹. So far as this group of purpose qualities is alone concerned, neither g nor this new factor is a single general factor within the meaning of our definition; but any large series of sufficiently dissimilar mental tests may, as we have said, be so expressed that g and specific factors enter into all of them while this new factor, like our factor Cleverness, enters as a group factor to an extent which is insignificant in all but a small proportion of them.

Dr Webb demonstrated the existence of the new (group) factor by showing that if, by means of Yule's equation, the partial coefficients of correlation with g constant, between qualities Nos. 34, 33, 18, 20, 21, 4, 11 and 31 in the above Table I, were calculated and entered in a correlation table, the correlation between columns averaged .94, and therefore approximately satisfied Professor Spearman's condition.

If we apply the test afforded by equation (8) to discover whether these eight qualities depend upon two, and only two, independent factors, of which we may take one to be g , we find that all the combinations of the first six qualities three at a time do satisfy all the series of conditions within the limits of probable error (assumed to be .10 in

¹ *Loc. cit.* 58-60. But, as we are about to observe, the factor Dr Webb described, and measured by w , is not the same as the new group factor z , independent both of g and c , that is here in question. We shall, however, see that w and z have much in common.

Dr Webb's table), except that two combinations (34, 18, 20; and 34, 18, 4) of three of the six qualities miss satisfying the condition (within this limit of probable error) by 4° and 2° respectively. In view, however, of the fact that the correlations between qualities 34 and 33 are very high, as are also those between qualities 18, 20 and 21, it would follow that, if equation (8) is satisfied by qualities 34, 20 and 4, the same equation will be satisfied approximately by any other combination of three qualities out of the six. Too much importance must not, therefore, be attached to the conclusion that, within the limits of probable error that we have defined, the six qualities 34, 33, 18, 20, 21 and 4 are dependent upon two independent variables and upon two only. But this result is entirely consistent with Dr Webb's conclusion that apart from g , one and only one (group) factor, independent of g , is contained by these six qualities. It may be added that quality No. 11, in combination with any other pair of the six qualities just named, in most cases satisfies equation (8), and in other cases very nearly does so. Quality No. 31, however, does not satisfy our test in combination with pairs of the remaining seven qualities.

Dr Webb discusses the nature of this new group factor, and recognizes that it is in some close relation to 'persistence of motives.' He adds that "this conception may be understood to mean *consistency of action resulting from deliberate volition, or will*."¹ But will, used in this sense, is liable to be confused with the word as used to describe an effort of will. It would perhaps be better to describe the new (group) factor as being intimately related to Purpose². It may, for example, measure the extent to which thought and action are influenced by interest—the individual's whole system of innate and acquired neurograms—and especially interest in the future.

Dr Webb's new factor, as defined in the next paragraph, is independent of g .³ It is not the same as our new factor Cleverness; for the calculated correlations between the seven qualities in which the new factor has been shown to enter as a general factor, and Cleverness, are respectively:

¹ *Loc. cit.* 60.

² Cf. first footnote on p. 355 above.

³ But, in discussing its nature, Dr Webb was more concerned with the quality which we are going to call Purpose and of which the axis Op in Fig. 2 below is *not* at right angles to Og , than with the quality independent of ' g ' of which the axis is Oz in that figure and is at right angles to Og .

TABLE II.

No. in Dr Webb's schedule	Name of Quality	Correlation with 'Cleverness'
34	Tendency <i>not</i> to abandon tasks from mere changeability ...	-.06
33	Tendency <i>not</i> to abandon tasks in face of obstacles ...	.39
18	Kindness, on principle ...	.40
20	Trustworthiness ...	.07
21	Conscientiousness ...	-.05
4	Readiness to become angry ...	.15
11	Eagerness for admiration...	.18

There is therefore a third factor, independent both of g and c , which enters into the constitution of a number of character qualities. But the character qualities in question may depend on more than these three independent factors. Suppose we take g and c as the first two independent factors. Then we may express the measure q of any of these character qualities by means of equation (11): namely,

$$q = r_{ga} \cdot g + r_{ca} \cdot c + l_3 x_3 + \dots + l_N x_N \dots \dots \dots (11),$$

where $x_3, \dots x_N$ measure the remaining independent factors. We may then define the new factor x_3 , whose existence was proved by Dr Webb, as that which gives the highest average value of l_3 for all the q 's in question.

If, in Fig. 2 below, we represent Dr Webb's new factor, thus defined, by an axis Ox_3 or Oz at right angles to the plane of our former diagram

Fig. 2.

(Fig. 1) the axes corresponding to the seven qualities in which the existence of this new factor has been demonstrated, and of some others—notably No. 32 (Degree with which he works with distant objects in view)

and No. 28 (Extent of mental work bestowed upon usual studies) into which Dr Webb's new factor also enters in a high degree¹—lie in or near the plane zOg , since all these qualities have small correlations, with Cleverness, the axis (Oc) of which is perpendicular to that plane.

Of the seven qualities (Nos. 34, 33, 18, 20, 21, 4 and 11) named in Table II, the first five have high positive correlations with each other and with the two qualities (numbered 32 and 28 in Table I) to which reference has just been made; and all these seven qualities (Nos. 34, 33, 18, 20, 21, 32, 28) have negative correlations with the last two qualities (Nos. 4 and 11) in Table II². Let us now for shortness describe the seven qualities (Nos. 34, 33, 18, 20, 21, 32 and 28) having high positive correlations with each other as 'purpose' qualities: a description suggested by the names of these qualities in Table I above and by the further particulars of the qualities in question given by Dr Webb³. To these seven purpose qualities we may, if we please, add the inverses of qualities No. 4 and No. 11, defined as being measured by the measures of No. 4 and No. 11 respectively with their signs changed. Suppose that p represents the centroid of the points in which the axes of the various purpose qualities meet any sphere described with O as centre. Then, since the average correlation of all these nine qualities with Cleverness is very small ($\cdot 04$), Op lies very near, if not actually in, the plane zOg ; and it lies much nearer to Oz than to Og ⁴.

If now we repeat for our three-dimensional diagram (Fig. 2) the description that we gave for our two-dimensional diagram (Fig. 1) by imagining a small solid element, δV , between the three perpendicular axes Oz , Og and Oc ; and if we consider especially positions of δV near the plane cOp ; we shall find that exceptional men whose mental qualities, in so far as they depend upon the three independent variables measured along Og , Oc and Oz , are represented by points lying inside δV belong to one or other of two well-recognized types, according as δV lies near to Op on the one hand or to Oc on the other.

Take, for example, the following comparison quoted by Dr Webb⁵

¹ See Webb, *loc. cit.* 59.

² Webb, *loc. cit.* Table VI.

³ *Loc. cit.*, Appendix II, pages 84 *et seq.*

⁴ Taking Dr Webb's figures the angle gOp lies between 70° and 80° . Dr Webb found that the correlations of the purpose qualities were very small in the case of schoolboys. This suggests—what is probable on other grounds—that the angle gOp decreases as years go on.

⁵ *Loc. cit.* 61.

as a sample of the observations on types of character made by G. Heymans and E. Wiersma:

<i>Avaricious.</i>	<i>Spendthrift.</i>
1. Man of principle.	Not so.
2. Not emotional.	Emotional.
3. Depressed and gloomy.	More cheerful and lively.
4. Anxious and serious.	Irresponsible.
5. Hard to appease.	Easily appeased.
6. Adheres to his once-formed opinion.	
7. Man of habit.	Change-loving.
8. Works for a far-off goal.	Works for an immediate result.
9. Answers questions in a provisional manner.	Answers positively.
10. Mathematical talent.	No mathematical talent.
11. No artistic ability.	Talent for music, drawing, authorship, dramatic art.
12. Not witty.	Witty.
13. Quiet and reserved.	Not so.
14. Not given to telling anecdotes.	Given to telling anecdotes.
15. Does not speak much in public.	Fond of speaking in public.
16. Never in debt.	Often in debt.
17. Not easy to manage.	Easy to manage.
18. Brings up his children strictly.	Allows much freedom.
19. Conservative.	Radical.
20. Hermit-like.	Pleasure-loving.
21. Not a sophist.	A sophist.
22. Not sport-loving.	Sport-loving.
23. Interested in finding out the means of others.	Not so interested.
24. Morose.	Complimentary.
25. Dignified and precise.	Ironical.
26. Slow and short in speech.	Loud in speech.
27. Seldom or never laughs.	Laughs much.

Dr Webb gives this example to illustrate a greater or less degree of his new factor. But the typical spendthrift differs from the typical avaricious character, not only in possessing less purposefulness, but also, according to the above analysis, in being more witty and more cheerful, in having more artistic ability and less mathematical talent. For all these reasons, the typical spendthrift must be represented by a point which, since he is 'typical,' i.e. an extreme representative of a type, is at some distance from the origin, and which not only lies a long way from the 'Purpose' axis, *Op*, but also lies near the 'Cleverness' axis, *Oc*.

364 *General Ability, Cleverness and Purpose*

Take another comparison, also quoted by Dr Webb:

<i>Harold.</i>	<i>Earl.</i>
More control of voluntary movement.	In free movement quicker, showing impatience and lack of control.
Industrious—more patient and persistent in performing tasks.	Restless and animated in school, given to idle occupation. More easily distracted.
More helpful—dependable—regular in his expenditure of energy and in application of effort.	More competitive spirit. Has periods of feverish activity, alternating with periods of listlessness.
Always willing to try, but never expected to do <i>very</i> well.	Great enthusiasm in attempting a new task, but enthusiasm soon exhausted.
More moral, good-natured, polite, keen to do work well, at times sensitive and morose.	More sunny and laughing; was a tease.
Less self-confident.	Bolder in social relations, always acted as spokesman.
More sympathetic and generous.	More often acting apparently to show off.
More conscientious in his work.	Impatient of detail.
More genuinely pleased at praise.	
Neat and careful writing.	Careless, hasty and irregular work in copy-books.
Superior in literal memory, both when tested after a year and after five minutes study.	Superior in rapidity of complex mental processes.
Association reactions show more clear and more stable mental imagery, and perhaps less in quality.	Reactions much less uniform—a more variable mental process; more frequently caused by recent experiences and objects in the immediate environment.

Earl's superiority in rapidity of complex mental processes indicates that he differs from Harold, not only in having less 'Purpose,' as Dr Webb points out, but also in having more 'Cleverness.'

In fact, the type of temperament which Dr Webb and others¹ have distinguished from the purposeful temperament differs from it, not merely in having less than the average degree of Purposefulness, but also in having more than the average degree of Cleverness.

¹ For example, William James's "explosive Italian" (*Principles of Psychology*, II. 538) possessed more than average (i.e. a positive degree of) Cleverness, as well as less than average (a negative degree of) Purposefulness.

7. *Summary of Conclusions.*

1. It has here been shown that, unless we carefully define beforehand what we mean by general factors and group factors, much confusion is likely to arise in discussions of the question whether the correlations obtaining between a set of mental tests are due, on the one hand, to a single general factor—'general ability'—entering without group factors into all of the qualities tested, or, on the other, to an indefinitely large number of independent factors each of which may enter as a group factor into any two or more of the tests.

2. If Professor Spearman's 'correlation between columns' conditions are satisfied by the correlations between the measures $q_1, q_2, \dots, q_n$ of n tested qualities, the n differences $q_1', q_2', \dots, q_n'$ between the respective q 's and a real multiple of an $n+1$ th variable y , independent of the q 's, will satisfy Mr Burt's conditions for a hierarchy.

3. Mr Burt's conditions for a hierarchy are equivalent to $\frac{1}{2}n(n-3)$ independent relations between the n mental qualities tested.

4. Were Mr Burt's conditions satisfied by the correlations obtained in a set of mental tests, each of the n qualities tested would be expressible in terms of two independent factors of which one was specific, appearing in that quality alone, while the other was a single general factor common to all the qualities. The n qualities would thus be expressed in terms of $n+1$ independent factors, viz. a single general factor and n specific factors.

5. The fact that the correlations between any set of sufficiently dissimilar tests approximately satisfy Professor Spearman's or Mr Burt's conditions is evidence that the measures of the qualities tested (or, if Mr Burt's conditions are not satisfied, the differences between these variables and a real multiple of an $n+1$ th variable independent of them all) may be expressed as made up for the most part of a single general factor and specific factors together with a comparatively insignificant part due to group factors; or, put mathematically,

$$q_s' = (q_s - ky)(1 + k^2)^{-\frac{1}{2}} = l_s \cdot g + m_s \cdot x_s + n_1 \cdot z_1 + n_2 \cdot z_2 + \dots,$$

where q_s is the measure of the s th quality tested, k is a real quantity which vanishes (making $q_s' \equiv q_s$) when Mr Burt's conditions are satisfied, g is the general factor (which would be a single general factor were the coefficients of the group factors all zero instead of only being on the average very small), x_s is a specific factor, and $z_1, z_2, \dots$ are group factors whose coefficients (n 's) tend severally and in the aggregate to

be very small compared with the coefficients (l 's) of g or (m 's) of the specific factors.

6. In addition to g , group factors enter to a significant extent into certain groups of similar qualities.

7. Thus certain intellectual qualities may be regarded as compounded (according to the formula expressed in equation (10)) of g , General Ability, and one other independent factor c , which we described as Cleverness.

8. Into the constitution of a number of purpose qualities having a most intimate connexion with character there enters to a significant extent not only g but also a group factor z that is independent both of g (General Ability) and of c (Cleverness). The new factor described by Dr Webb as 'Will,' but as we have argued better described as 'Purpose,' is not the same as z but is a compound of z with g and possibly, but to an insignificant extent, with other independent factors also.

(Manuscript received 9th March 1919.)