than has yet been had. Results of experiments on transfer factors between habits and on comparison of two different modes of learning a given pair of mazes are about to be published.

4. The facility with which multiple unit mazes can be altered indefinitely in a few minutes has opened up a field of quite new problems—an intensive study of the many elements and details involved in the lay-out of a maze situation. Publication is to be made shortly of studies carried on in the Oberlin laboratory of the comparative values of different arrangements of cul-de-sacs.

It may be noted in closing that the writers are at present working on further extensions of the multiple unit idea: to the construction of a pencil maze for experimentation on adults, and to the construction of a tri-dimensional maze for the study of the behavior of rats in a more complicated situation than is afforded by labyrinths built upon the one plane only.

TWO MODELS SHOWING THE INTERRELATION OF SEVERAL SIGNIFICANT CORRELATION VARIABLES¹

BY ARTHUR W. KORNHAUSER, FRANKLYN MEINE,
AND BEARDSLEY RUMI,

Carnegie Institute of Technology

With the strong impetus which mental testing has received during the past year or two, the importance of correlation as a psychological instrument has become more commonly recognized than ever before. At the same time, students of statistical method appreciate the dangers involved in the widespread use of correlation coefficients by those who have not sufficiently mastered the significance and limitations of ' r ' which do not appear from the mere use of a handy formula. Our psychological literature contains no end of studies, for example, where thoroughly unjustified conclusions are drawn on the basis of the correlation coefficients alone, no mention being made of the probable error of r , of the standard deviations of the variables, of the regression equations, of the standard error of estimation, or even of the precise nature of the group measured. Correlations obtained on heterogeneous material are used for predicting relationship in more homogeneous

¹The models here presented were constructed in connection with a course in biometric methods given by Dr. Beardsley Rumi at Carnegie Institute of Technology.

material; equal correlation coefficients are presumed to indicate equal relationship without respect to the distribution of the variables involved. In order to make a little more clear the complexity of r and the wide possibilities of variations not depicted by it alone, we have considered it worth while constructing two models of the important variables involved.

MODEL A

Showing Types of Frequency Surfaces and the Meaning of r , σ_x , and s_x .

The model (Plate I.) represents a scatter diagram in three dimensions. The X -arrays (rows) are shown as small frequency surfaces. The sum of all such partial frequency curves is the

FIG. 1

total X -distribution and is shown as the large frequency surface at the top of the model. Evidently the Y -distribution is a surface in which each interval has the frequency of that particular array of X 's. Only one regression line ($X = b_{xy}Y$) is shown and we shall assume in this discussion that the relation dealt with is that of X to Y rather than the reverse. As its most general function the model enables one to appreciate vividly the nature of a correlation table as a three-dimensional relation.

When set with any particular combination of curves the model represents a specific contingency table or frequency surface. The precise shape of this surface may be varied ad infinitum so that it may be made to approximate any contingency distribution actually encountered. The model, in addition to the general aid it gives in understanding the contingency relation, is of value in two directions: (1) it illustrates the innumerable possibilities of frequency surfaces and shows the special significance of each; (2) it indicates the interdependent changes in the values of the three variables— r , σ_x , and s_x .

1. As the model is now arranged the Y -variable is assumed to be distributed in the form of the normal curve and the X -variable in the form shown by the large curve of the total X -distribution. The small curves represent the distribution of X -values for the particular arrays indicated. The following properties may be noted in this regression: the means of the arrays (small curves) fall along a straight line (linear); the standard deviation or σ of the arrays does not change (homoscedastic); some of the array curves are skewed (allikurtic); the skewness is not equal throughout, some arrays being skewed positively and some negatively (heteroclitic); the change in skewness is continuous or regular (nomic). In a similar manner any regression may be completely described.¹

It should be noted that strictly speaking we have not fully described any correlation surface without all these items. Practically, of course, such procedure is ordinarily out of the question. The important point, however, is that we recognize the existence of this tremendous complexity and do, in view of these facts, exercise the utmost caution in drawing general conclusions from correlation figures. The fact, for example, that we do not care to sacrifice the time necessary to apply a test for linearity in the case of every regression does not mean that we should not be ever on the alert to avoid incorrect inferences as to the degree of relationship and reliability of prediction due to unjustified assumptions of linearity. The same applies of course, to all the other less clearly recognized variations in contingency tables.

2. The three important variables mentioned (r , σ_x , and s_x) are easily explained from the model. The rod passing through the

¹ The corresponding opposites of the terms here used are: non-linear, heteroscedastic, isokurtic, homoclitic, and anomic. The terminology is that of Pearson (Draper's Company Research Memoirs, Biometric Series II., 1905).

mean points of all the small array-curves represents the regression line. The slope of this line to the Y -axis is the regression coefficient, b_{xy} . r bears the simple relation to b_{xy} expressed by the equation $b_{xy} = \sigma_x/\sigma_y \cdot r$. σ_x is seen directly as the standard deviation of the large curve at the back of the model. s_x is the weighted average of the standard deviations of the array-curves. s_x is known as the standard error of estimation of X -values. It is readily derived from the formula $s_x = \sigma_x \sqrt{1 - r^2}$. This standard error of estimation is a kind of average of the standard deviations of all the rows of X -values and hence in the usual case of frequency surfaces of more or less uniform scatter it is roughly equal to the standard deviation of each array (each small curve in the model). s_x is the standard deviation of $(X - b_{xy}Y)$, i.e., the standard deviation of the distances of the true X 's from the predicted X 's. The standard error, represented by E , is a pure number and is simply the ratio of s_x to σ_x or of s_y to σ_y , i.e., $E = \sqrt{1 - r^2}$.

The interdependent changes in these variables may be observed in the model.²

By keeping constant any one of the three variables involved we may readily note the relation of changes in the other two. (σ_y is also constant, of course, by the nature of the model.) For example, if the standard error of estimation (s_x) remains constant it may at once be seen that increasing the scatter of the X -variable (σ_x) necessitates a corresponding increase of r by revolving the regression line away from the Y -axis. Similarly, if the standard deviation of the X -variable remains constant, it is evident that decreasing r (revolving the regression line toward the Y -axis, that is) necessitates increasing the scatter in the arrays, i.e., increasing the standard error of estimation. For constant r , finally, note that the larger the standard deviation of the total distribution the larger the standard error. The model enables one by changing these variables to note their interrelations under various conditions of linearity, scatter, and so on. The quantitative relation of the variations is not shown, however, in this model.

MODEL B

The Relation of the Variables r , σ_x , and s_x

The model (Plate II) is a three-dimensional graph of the formula, $s_x = \sigma_x \sqrt{1 - r^2}$. r is measured to the right and left from zero at the center; σ_x is measured from front to rear; s_x is measured vertically upward.

² The mathematical relations are shown graphically in Model B.

The surface for constant r , seen by passing a vertical plane perpendicular to the back of the model, is perhaps the most obvious and least interesting. The relation is linear, indicating that increase in σ_x is accompanied by a proportional increase in s_x .

The surface for constant σ_x , any vertical plane parallel to the one formed by the back of the model, is more important. This indicates that as r decreases from $+1.0$ or increases from -1.0 , the standard error increases with extreme rapidity. Concretely, when r is equal to $.86$, the standard error is half as great as it ever will be; when r equals $.50$, the standard error is seven-eighths its maximum size. The amount of error made in predicting from the regression equation on the basis of a correlation of $.44$ is nine tenths

FIG. 2

as great as the error made in guessing on the basis of no information whatever. The relation, as seen on the back of the model, is a semicircle if σ_x be measured in terms of s_x .

The surface for constant s_x , any horizontal plane such as the one shown (red in the actual model), is also of great importance. If the error of estimation be constant, as it is in homoscedastic regressions, r will change markedly with slight changes in σ_x , the

increase being most striking for those values of σ_x slightly larger than s_x . Concretely, when $\sigma_x = s_x$, $r = 0.0$; when σ_x is increased from this value by one tenth, r is increased from 0.0 to .45. Thus with a constant standard error of estimation, the correlation coefficient increases with the standard deviation of the group and approaches 1.00 as a limit when σ_x approaches infinity. The relation is parabolic.

A SCORING METHOD FOR MENTAL TESTS

BY I. L. THURSTONE

Carnegie Institute of Technology

Much has been said about the weighting of a series of mental tests so as to obtain the highest possible correlation between a criterion and a group of tests. This is accomplished by multiple correlation. Now, why should we not apply the same technique in ascertaining how much to penalize for errors in a single test in order to obtain the highest correlation with the criterion? How many mental test correlations could have been raised if the test had been scored in some other way?

Let us consider the number of right responses and the number of wrong responses as two independent variables which are to be weighted to obtain the dependent variable, the score. The weighting should be such that the score will give the highest possible correlation with the criterion. This is our problem.

We shall work with the general scoring formula $S = R + CW$, in which S is the score, R is the number of right responses, W is the number of wrong responses, and C is a constant which determines how much we shall penalize for errors. The constant C is usually negative.

Our first problem is to obtain a general expression for the correlation between the test score and the criterion. In other words we want to express the correlation $r_{I(R+CW)}$ between the criterion I and the score $(R + CW)$ in terms of the six fundamental constants which we shall assume to be given, namely r_{IR} , r_{IW} , r_{RW} , σ_R , σ_W , σ_I .

Remember that

$$r_{xy} = \frac{\Sigma xy}{n \cdot \sigma_x \cdot \sigma_y}.$$