On the Moving Coil Ballistic Galvanometer

This content has been downloaded from IOPscience. Please scroll down to see the full text.

1913 Proc. Phys. Soc. London 26 75

(<http://iopscience.iop.org/1478-7814/26/1/307>)

View [the table of contents for this issue](#), or go to the [journal homepage](#) for more

Download details:

IP Address: 150.135.239.97

This content was downloaded on 02/10/2015 at 22:04

Please note that [terms and conditions apply](#).

VII. *On the Moving Coil Ballistic Galvanometer.* By
R. LL. JONES, M.A., *The Presidency College, Madras.*

RECEIVED NOVEMBER 3, 1913.

1. In the theory of the moving coil ballistic galvanometer as usually presented, the motion of the coil is assumed to be oscillatory. But when a search coil of low resistance, such as is employed to explore magnetic fields, is connected to the galvanometer, the motion may become aperiodic. The theory as usually given in text-books is not directly applicable to this case. To evade this difficulty the suggestion is sometimes made to insert sufficient resistance in the circuit to make the motion oscillatory. This is unnecessary, for, as the following considerations show, the throw of the galvanometer is proportional to the change in flux, even when the motion is dead-beat, provided the change is quickly effected. As it is far quicker and much more convenient to work with a galvanometer when its motion is aperiodic than when it is oscillatory, the matter is of some practical importance, and the treatment given below may not be without interest.

In the Grassot fluxmeter, with its very weak torsional control, the throw is proportional to the change in flux, and is, within limits, independent of the time taken to effect the change. On the other hand, in an ordinary galvanometer, with its stronger control, the throw will be very small when the rate of change is the slowest permissible with the fluxmeter, and will have a much greater (its maximum) value when the change is instantaneous. Further, we would naturally expect, even when the change is instantaneous, the throw to be greater when the torsional control is weak than when it is strong. This inference is confirmed by the investigation given below.

In this Paper the aperiodic motion of the moving coil galvanometer is first considered; then an account of some observations with a galvanometer is given which confirm some deductions from the theory; and finally the results obtained are applied to determine the exact relation between its throw and the change in the flux through the search coil which produced it.

2. The coil of the galvanometer turns about a vertical axis in the plane of the coil, passing through its centre of gravity and perpendicular to the field. In the position of

Substituting these values in (3) and its first derivative, we get

$$A = -B = \frac{\omega}{2p}.$$

The position and the velocity at any time t are thus given by

$$\theta = \frac{\omega}{2p} e^{-\lambda t} (e^{pt} - e^{-pt}) = \frac{\omega}{2p} \cdot P, \quad . \quad . \quad . \quad . \quad (4)$$

$$\dot{\theta} = \frac{\omega}{2p} e^{-\lambda t} \{ -(\lambda - p)e^{pt} + (\lambda + p)e^{-pt} \}. \quad . \quad . \quad . \quad (5)$$

From (5) we deduce that $\dot{\theta} = 0$ and therefore θ will be a maximum at the time t_1 where

$$t_1 = \log_e \left(\frac{\lambda + p}{\lambda - p} \right)^{1/2p}.$$

The maximum value of θ is θ_0 where

$$\theta_0 = \frac{\omega}{n} \cdot \left(\frac{\lambda - p}{\lambda + p} \right)^{\lambda/2p}.$$

The ratio of the second term on the right-hand side of (4) to the first is e^{-2pt} . This becomes smaller and smaller as t increases. Hence, after a certain time, the position of the coil will be given with sufficient accuracy by

$$\theta = \frac{\omega}{2p} e^{-(\lambda - p)t}. \quad . \quad . \quad . \quad . \quad . \quad (6)$$

4. To test relation (6) observations were made on a galvanometer (N.B. No. 30,532), the moving coil of which was wound on a narrow non-conducting frame, and had a resistance of about 800 ohms. The suspension was phosphor-bronze strip, and the field was approximately uniform. The period of oscillation on open circuit was 12.32 seconds. The air damping was small, and its effect has been left out of account throughout. The observations showed that when the galvanometer was deflected and the circuit then closed through a small resistance the time the deflection took to fall to half its value was always the same and was independent of t (provided the deflection was not observed too near the turning point). In other words, if θ , the deflection, be a function of t , say $f(t)$, the value of $f(t + \tau)$, where τ is a certain interval, was always equal to $\frac{1}{2}f(t)$, no matter what the value of t be. This shows that $f(t)$ must be a simple exponential function of the

time, of the form $Ae^{-\kappa t}$, where $e^{-\kappa\tau} = \frac{1}{2}$. The observations confirm the form of relation (6).

5. Since
$$\theta = \frac{\omega}{2p} \cdot e^{-(\lambda-p)t},$$

when t exceeds a certain value, and

$$\frac{\theta}{2} = \frac{\omega}{2p} e^{-(\lambda-p)(t+\tau)},$$

we have, by combining the two, $\lambda - p = \frac{\log_e 2}{\tau}$.

Expanding p in powers of n^2/λ^2 , and neglecting all terms after the second, we get

$$\frac{1}{2} \frac{n^2}{\lambda} = \frac{0.6931}{\tau}$$

or
$$R\tau = \frac{A^2 H^2}{I} \cdot \frac{0.6931}{n^2} = \text{constant.} \quad . \quad . \quad . \quad (7)$$

To test this relation, observations were made with the galvanometer mentioned in § 4. The values of τ observed with different external resistances are given in the second column of the following table; the third column gives the product $R\tau$.

R=800—	τ .	R τ (approx.).
0 ohms	17.60 seconds	14,100
100 „	15.77 „	14,200
200 „	14.47 „	14,500
300 „ . . .	12.78 „	14,100
400 „ . . .	11.47 „	13,800
500 „ . . .	10.57 „	13,700
600 „ . . .	10.00 „	14,000

The results given in the above table are, on the whole, in fair agreement with the deduction expressed in (7). When the external resistance is more than 600 ohms, relation (7) is no longer sufficiently accurate; higher powers of n^2/λ^2 must in that case be retained in the expansion of p .

6. To illustrate the results obtained in § 3, consider the case of the galvanometer described in § 4, when the outside resistance is negligible. Since its period on open circuit is 12.32 seconds, this gives us $n^2 = 0.2601$, and (7) becomes

$$R\tau = 2.665 \cdot \frac{A^2 H^2}{I} \quad . \quad . \quad . \quad . \quad (8)$$

Since $R=800$ ohms and $\tau=17.60$ seconds, we get from (8)

$\lambda=3.302$ and $p=3.262$. Substituting these values in the expression for t_1 in § 3, we get

$$t_1=0.782 \text{ second.}$$

The coil always comes to rest in this time, 0.782 second, whatever be the angular impulse given to it. The throw is given by

$$\theta_0=0.148 \times \omega.$$

The following table gives the value of P, the variable part of the expression for θ in (4) for different values of t :—

t .	P.	t .	P.	t .	P.
0.1 sec.	0.487	0.6 sec.	0.957	1.5 sec.	0.942
0.2 „	0.723	0.7 „	0.962	2.0 „	0.923
0.3 „	0.848	0.8 „	0.963	5.0 „	0.819
0.4 „	0.912	0.9 „	0.962	10.0 „	0.670
0.5 „	0.943	1.0 „	0.960	...	...

The values are plotted in the accompanying diagram. They show that the galvanometer coil moves comparatively rapidly

to its maximum deflection and then slowly returns towards its position of equilibrium; this is in accordance with observation.

7. Let the flux through the search coil change, and let $E = \frac{dN_2}{dt}$

denote the E.M.F. induced in it. Then we have, as in the theory of the fluxmeter,

$$Ri + AH \cdot \dot{\theta} + L \frac{di}{dt} = E$$

for the current and

$$I\ddot{\theta} - AH\dot{i} + c\theta = 0$$

for the motion of the galvanometer coil.

Substituting for i in the second equation the value obtained from the first and integrating between the limits $t=0$, the time when the flux begins to change, and $t=t_1$, the time when the galvanometer comes to rest at its maximum deflection, we get

$$[I\dot{\theta}]_0^{t_1} = \left[-c \int \theta dt + \frac{AH}{R} (\int E dt - AH \cdot \theta - Li) \right]_0^{t_1}.$$

Now, $\dot{\theta}=0$ and $i=0$ at times $t=0$ and $t=t_1$, while $\theta=0$ at time $t=0$ and $\theta=\theta_0$ at time $t=t_1$. Remembering that $\int_0^{t_1} E dt = N_2$, the total change in flux through the coil, we get

$$N_2 = AH \cdot \theta_0 + \frac{Rc}{AH} \int_0^{t_1} \theta dt. \quad . \quad . \quad . \quad . \quad (9)$$

In the fluxmeter the factor c is so small that the second term on the right-hand side of (9) is negligible compared with the first. In the ordinary moving-coil galvanometer this is not necessarily the case.

The ratio of the second term to the first is $\frac{\int_0^{t_1} \theta dt}{\theta_0} \cdot \frac{Rc}{A^2 H^2}$. If this ratio be constant and independent of N_2 , then N_2 will be proportional to θ_0 . The factor $\frac{Rc}{A^2 H^2}$ depends on the galvanometer and the resistance of the search coil, and is evidently constant with a given search coil. To find the value of $\frac{\int_0^{t_1} \theta dt}{\theta_0}$ we will assume that the change in flux (N_2) has been completely effected before the galvanometer coil has appreciably moved from its position of rest. The effect of this will be to give an angular impulse to the coil; its subsequent motion will then be given by (4). The value of $\int_0^{t_1} \theta dt$ will

depend only on λ , p and ω , and can be determined from (4), while θ_0 also depends on λ , p and ω alone. The ratio $\frac{\int_0^{t_1} \theta dt}{\theta_0}$ depends on λ and p , and does not involve ω .

Hence the ratio is independent of N_2 , and if the flux change be sudden *the throw will be proportional to N_2* .

8. The ratio of the two terms in (9) is readily evaluated in any given case. With the galvanometer used for the observations recorded in §§ 4-5 and a search coil of negligible resistance, the value of $\frac{Rc}{A^2 H^2} (= \frac{n^2}{2\lambda})$ is 0.039. The value of $\int_0^{t_1} \theta dt$ is most easily obtained from the results given in § 6. These give for $\int_0^t \theta dt$ the value $0.63 \theta_0$. Thus the ratio is 0.024.

Hence, with the given galvanometer and a search coil of negligible resistance, the throws are about $2\frac{1}{2}$ per cent. less than they would be with an infinitely weak control.

ABSTRACT.

The author first considers the mathematical theory of a moving coil galvanometer in which the damping is such as to make the motion non-oscillatory; then an account is given of some observations on a galvanometer which confirm some of the deductions from the theory, and the results obtained are applied to find the relation between the galvanometer throw and the change of flux in the search coil which produces it.

DISCUSSION.

Prof. C. H. LEES thought it likely that with a highly damped galvanometer the creeping at the end would create a difficulty as to what reading to take.

Prof. T. MATHER stated that the creep presented no difficulty, since it was easy to get the maximum deflection reached. The real difficulty was in getting the true zero.

Mr. A. CAMPBELL mentioned that although the case of the strongly damped galvanometer when used with a search coil did not appear to be treated in any English text-book, he had found it described in Kohlrausch's "Lehrbuch der praktischen Physik" (10th edition, 1905).