581. The Sound-Ranging Problem

Author(s): W. Hope-Jones

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MATHEMATICAL NOTES.

581. [s. 6.] *The Sound-Ranging Problem.*

In his surmise that a method of gun-location by drawing circles "should prove rapid, and of greater accuracy than any approximation where the hyperbola is replaced by an asymptote," I think that Sir G. Greenhill is underrating the ease and accuracy of the asymptote-method actually used.

In assuming three microphones only, he is also underrating the difficulties of the method in which circles are drawn with the microphone-stations as centres. Six microphones are used in practice; and the labour of drawing six circles, and increasing their radii until they all cut, would be a formidable job even if there were no possibility of mistaking the many odd noises of a battle-front for the report of the gun in question. As a dozen wrong sounds have often to be tried and rejected before the right one is found, circle-plotting would be intolerable in the rough-and-tumble of real war.

If A and B are two consecutive microphones at a distance a apart, and G an enemy gun at a distance r from X , the middle point of AB , in a direction

making an angle θ with AB , then the sound-ranging apparatus, by recording the moments of arrival of the sound at A and B , gives us $GA - GB$, which

$$= \sqrt{r^2 + ar \cos \theta + \frac{a^2}{4}} - \sqrt{r^2 - ar \cos \theta + \frac{a^2}{4}}.$$

This "interval" being known (call it c),

$$\sqrt{r^2 + ar \cos \theta + \frac{a^2}{4}} - \sqrt{r^2 - ar \cos \theta + \frac{a^2}{4}} = c$$

can be solved as an equation in $\cos \theta$, whence

$$a \cos \theta = c \sqrt{1 + \frac{a^2 - c^2}{4r^2}}.$$

r being much larger than a and c , c is a good first approximation to $a \cos \theta$. The remarkable accuracy of this first approximation, and the rapidity with which a small correction (the "asymptote-correction") can be applied to it so as to get exact results, deserve especial notice.

The graduation of the edge of a map, mounted on a board, so that in two seconds or less a string attached to X may be laid down along the line XG (or rather an approximation to it, got by taking $a \cos \theta$ as c) can be so easily thought out that I will not use up your space by describing it. The inter-

section of five such strings gives a good first approximation to the gun's position—so good, in fact, that it is often used uncorrected.

r being now known, and values of the asymptote-correction

$$c\left(\sqrt{1+\frac{a^2-c^2}{4r^2}}-1\right)$$

having been tabulated in advance for all likely values of c and r , this correction

can then be added to the observed value of c so as to give $c\sqrt{1+\frac{a^2-c^2}{4r^2}}$,

the exact value of $a \cos \theta$. The strings are slightly moved to the exact directions (the work of a few seconds only), the intersection noted, and "That's him."

As an instance of the sort of size to which asymptote-corrections commonly run, take the case $a=350$ (the unit of length is always 3·376 m., the distance travelled by sound in 0·01 sec.), $c=100$, $r=2000$.

Then $\frac{a^2-c^2}{4r^2}=0\cdot0070$; and $c\left(\sqrt{1+\frac{a^2-c^2}{4r^2}}-1\right)$, which can always be taken as $\frac{c(a^2-c^2)}{8r^2}$ except for unusually small values of r , is only 0·35 of a

unit, the unit being 0·01 of a second. From this it is evident that even when the asymptote-correction is not itself negligible, errors in it due to imperfect knowledge of r before its application are far below the limits of measurement, readings being commonly taken correct to the nearest half-unit, and only under exceptionally good conditions to the nearest tenth.

In all cases the corrections for wind and temperature are applied before the asymptote-correction. The wind-correction is of some interest, being independent of the position of the gun.

W. HOPE-JONES.

582. [v. 2. 9.] The following sayings of Napoleon are quoted from R. M. Johnston, *The Corsican*:

Oct. 29, 1784 [at Brienne]. "... every one said of me, 'That boy is no good except at Geometry'" (p. 3).

Aug. 7, 1792. "I have been working a good deal at astronomy during my stay here [in Paris]. It is a splendid instrument and a superb branch of science. With my knowledge of mathematics it required very little effort to learn. It is a great acquisition" (p. 9).

June, 1817 [speaking of his memory]. "As a boy I knew the logarithms of thirty or fifty numbers" (p. 494).

Jan. 29, 1818. "To be a good general, a man must know mathematics: it is of daily help in strengthening one's ideas. Perhaps I owe my success to my mathematical conceptions" (p. 499).

Can any one give references to the French sources from which these sayings are quoted? Johnston gives no references.

C. G.

583. [v. 1.] *The Census and the Teaching of Mathematics.*

The filling up of the decennial Census forms is an opportunity which must not be lost to us for teaching purposes, and the mathematical lessons deduced from it are as important as the civic. We may take it that the forms will include age, sex, birthplace, *état civil*, occupation, infirmity (if any), number of children, and number in each house. The chief points for us seem to be the following:

(1) *The idea of number.* It is important that some appreciation of the numbers involved should be obtained. One of the most effective ways of doing this is by letting the population file past at the rate of, say, one per second; massing the people in a solid square conceals the hugeness of the