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A FOCAL-LENGTH EQUATION.

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No general equation for calculating the focal length of a lens system appears to have been published. The mathematical equivalent of the author's graphic method of calculating paraxial ray courses* may be stated in the form of such an equation as follows:

$$j_m = j_m - i + i_m \left(1 - \frac{k = m - 1}{k = 1} \cdot \frac{j_k d_k}{n_k + 1} \right)$$

in which

$$i = \text{surface constant} = \frac{n' - n}{r} = n / \text{focal length};$$

$$j = \text{lens constant} = n / \text{focal length};$$

d = separation between the surfaces; and

n = index of refraction.

The focal length of a lens system, stated in another way, is equal to the sum of the lens constants plus the sum of the correction factors for the successive separations.

The following is a calculation of the telescopic objective given in Southall's "Principles and Methods of Geometrical Optics," pp. 319-321:

		$n' = 1.5146$	$\Sigma \frac{jd}{n}$
i_1	.0086053512	$r_1 = 59.8$	
$i_2 \Sigma$	-.0000324321	$d_1 = 1.$	.0056815999
i_2	.0057082640	$r_2 = -90.15$	
j_2	.0142811831	$d_2 = .013$	.0001856554
$i_3 \Sigma$	.0000424008	$n'' = 1.6121$	.0058672553
i_3	-.0072266824	$r_3 = -84.7$	
j_3	.0070969015	$d_3 = 1.$	.0044022713
$i_4 \Sigma$	.0000153317		.0102695266
i_4	-.0014929268	$r_4 = 410.$	
j_4	.0056193064		

* *Proc. Calif. Acad. Sci.*, Vol. i, p. 192.

The procedure is (1) arrange the data as in column two; (2) determine the lens constant for each value of r ; (3) determine first number in column three $= \frac{i_1 d_1}{n_2}$; (4) determine first correction by multiplying the previous number by i_2 ; (5) determine lens constant of the first lens by addition; (6) determine the second number in column three $= \frac{j_2 d_2}{n_3}$; (7) determine second correction by adding the two numbers in column three and multiplying by i_3 ; (8) determine constant for first three surfaces by adding the constants of the three surfaces and the corrections for the first two separations; (9) determine the number corresponding with the third separation $= \frac{j_3 d_3}{n_4}$; (10) determine the third correction by multiplying the sum of the three numbers of column three to i_4 ; and, finally, (11) determine the constant for the whole system by adding the four surface constants and the three correction numbers.

Because of the alternate addition and multiplication, machine calculation will be very much more rapid than the use of logarithms.

The focal length of the lens is $n(=1)/j_4 = 177.9579060$ the distance from the last lens vertex to the principal plane, is the sum of the corrections divided by the lens constant $\Sigma/j_4 = 1.8275835$ and the difference between these numbers the focal distance from the last lens vertex $= 176.1303625$.

The same lens calculated in the opposite direction gives different correction values corresponding with the different relative location of the other principal plane with respect to the plane of the other outer lens surface, but gives, of course, precisely the same final value to the lens constant of the system and its focal length.

The use of this equation accomplishes a considerable saving in the time required for a calculation, since it eliminates some of the non-essential steps in the ordinary methods.

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