LIX. Note on neumann's law of electromagnetic induction

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current damping, which was further shown by contracting the tin-plate cylinder still further, and placing it inside the frame, where it again had the effect of largely reducing the strength of the signals. That damping enters largely into the matter was also proved by placing a large flat copper plate close up to one side of the frame, where it could have no screening, but only a damping effect, which also greatly diminished the strength of the signals.

The result obtained with the frame inside the copper box, in getting signals of equal strength with the open side of the box facing away from the source of the waves, as when the open side faced towards such source, may perhaps be of some interest, from the point of view of theory, in throwing some light upon the mechanism of electromagnetic wave phenomena, inasmuch as it gives evidence that, in addition to what is analogous to a "push" upon an aerial in front of the wave front, there is also something of the nature of a "pull" upon an aerial behind the wave front.

Otherwise the experiments go to show that, at any rate upon the small scale employed, the use of a shielding tube or box does not assist in improving directional reception. It is possible, however, that if tried on a larger scale, with much more space between the frame and the shield so as to avoid damping, the results might be different.

LIX. *Note on Neumann's Law of Electromagnetic Induction.* By Prof. A. ANDERSON *.

WHEN a closed conducting circuit is moved in a magnetic field, a current flows round it, the value of the electromotive force producing the current being given by Neumann's Law. This law is established quantitatively from dynamical principles, but I think one must feel that the proof is not altogether free from objection. For, as it is usually presented, an electromotive force E is supposed to be already in the circuit and then, by the conservation of energy,

$$EC = C^2R + C \frac{dN}{dt}.$$

The circuit, however, may be moved so that C is always zero, in which case nothing can be deduced from this equation. If $C=0$ at any instant and if it varies in value during the motion we must have $E = \frac{dN}{dt}$ at that instant. If

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C does not vanish we have, dividing by C ,

$$C = \frac{E - \frac{dN}{dt}}{R},$$

showing that the current is increased by $-\frac{dN}{dt} R$, or that there is an *additional* E.M.F. in the circuit equal to $-\frac{dN}{dt}$.

If $E=0$, we can only say that if a current *is* produced when the circuit is moved (and experiment can be appealed to in support of this), its value, in accordance with the conservation of energy, must be such that

$$0 = C^2 R + C \frac{dN}{dt},$$

$$\text{or} \quad C = -\frac{dN}{dt} R.$$

The following elementary method of arriving at the law is based on the existence of electrons in matter. It is so simple that one can hardly believe that it has not been already noticed, though I have failed to find any specific mention of it.

If e is a charge of electricity moving with velocity v in a magnetic field, the force on it is $Bv \sin \theta$ perpendicular to the plane Bv in the direction of translation of a right-handed screw turning from v to B the magnetic induction, θ being the angle between B and v . Suppose now, instead of a charge e , we have an element PQ of a linear conductor moving in the same way. There will be forces on its positive atoms and free electrons, displacements will be produced, and a current will flow round a closed circuit of which PQ is an element. The force per unit charge will be $Bv \sin \theta$ and this resolved along PQ is $Bv \sin \theta \cos \phi$, ϕ being the angle between PQ and the normal to the plane Bv . The E.M.F. round a circuit of which PQ is an element ds is, consequently, $\int Bv \sin \theta \cos \phi ds$, taken round the circuit.

Let PQ move into the position $P'Q'$ in the time δt . Then the number of lines of induction cut by PQ in this time is $Bv \cos \chi \sin \psi ds \delta t$, where χ is the angle made by B with the normal to the plane PQr , and ψ the angle that PQ makes with v . But it is an easy problem in spherical trigonometry to show that $\sin \psi \cos \chi = \sin \theta \cos \phi$.

Hence $\int Bv \sin \theta \cos \phi ds$ is the number of lines of induction

cut per unit time by the linear conductor. It follows, therefore, that the E.M.F. round the circuit is $-\frac{dN}{dt}$, and thus Neumann's law follows immediately from that of Ampère.

The magnetic field has been supposed at rest, but the E.M.F. depends only on the relative motion of the circuit and the field. Hence it is given by $\int Bv \sin \theta \cos \phi \, ds$ or $\int Bv \cos \chi \sin \psi \, ds$, where v is the velocity of the element ds of the circuit relative to the line of induction through it, and this is equal to $-\frac{dN}{dt}$, for, the vector B being solenoidal,

there is no way of increasing or diminishing N except by lines of induction cutting the bounding circuit.

The above may be put shortly as follows:—

Let a surface bounded by the circuit move in a magnetic field whose lines of induction are also in motion. The motion of the circuit and the surface of which it is the boundary is the resultant of two motions, the first such that the velocity u of any point of the circuit or any point of the surface is equal to that of the line of induction through it, and the second such that the velocity v of any point of the circuit or any point of the surface is equal to its velocity relative to the line of induction passing through it.

We may thus consider three near positions of the surface S_1, S_2, S_3 . Applying Gauss's theorem to the space traced out between S_1 and S_2 , we have

$$\int_{S_2} B_n dS - \int_{S_1} B_n dS + \delta t \int [u, B] ds = \delta t \int \operatorname{div} B u_n dS + \delta t \int \frac{\partial B_n}{\partial t} dS.$$

But, since B is solenoidal, $\int \frac{\partial B_n}{\partial t} dS = 0$; also $\operatorname{div} B = 0$, and

$$\int_{S_2} B_n dS = \int_{S_1} B_n dS. \quad \text{Hence } \int [u, B] ds = 0, \text{ or there is no}$$

E.M.F. round the circuit. Applying the same theorem to the space traced out between S_2 and S_3 and remembering that in the motion the lines of induction are at rest, we have

$$\int_{S_3} B_n dS - \int_{S_2} B_n dS + \delta t \int [v, B] ds = \delta t \int \operatorname{div} B v_n dS + \delta t \int \frac{\partial B_n}{\partial t} dS.$$

But $\frac{\partial B_n}{\partial t} = 0$, $\operatorname{div} B = 0$, and, consequently,

$$-\frac{d}{dt} \int B_n dS = \int [v, B] ds,$$

which is Neumann's law if $\int [v, B] ds$ be identified with the E.M.F. round the circuit.