GRAPHICAL ALGEBRA.

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For more than forty years, students and instructors in the physics department of Washington University have not hesitated to lay off the square or the cube of any determined value as a straight line. If the varied lengths and time of vibration of a simple pendulum are plotted, and a curve is obtained which resembles the familiar parabola, the lines representing the various values t are elongated until their length is t^2 . If the result is a straight line, it gives conclusive evidence that $l = ct^2$.

A similar method may be used in representing graphically the value of $(y+x)^n$, where n , y and x may have any values. If we assume $n=6$, this quantity may be written $(y+x)^3(y+x)^3$. In the adjoining diagram y was assumed to be 3 units and x 2 units. The values of the terms of the expanded binomials $(y+x)^3$ are laid off along the two axes. The terms $3y^2x$ and $3yx^2$ are each

x^3	y^3x^3	y^2x^4	y^2x^4	y^2x^4	yx^5	yx^5	yx^5	x^6
y^3x^2	y^4x^2	y^3x^3	y^3x^3	y^3x^3	y^2x^4	y^2x^4	y^2x^4	yx^5
y^3x^2	y^4x^2	y^3x^3	y^3x^3	y^3x^3	y^2x^4	y^2x^4	y^2x^4	yx^5
y^3x^2	y^4x^2	y^3x^3	y^3x^3	y^3x^3	y^2x^4	y^2x^4	y^2x^4	yx^5
y^3x	y^5x	y^4x^2	y^4x^2	y^4x^2	y^3x^3	y^3x^3	y^3x^3	y^2x^4
y^3x	y^5x	y^4x^2	y^4x^2	y^4x^2	y^3x^3	y^3x^3	y^3x^3	y^2x^4
y^3x	y^5x	y^4x^2	y^4x^2	y^4x^2	y^3x^3	y^3x^3	y^3x^3	y^2x^4
y^3	y^6	y^5x	y^5x	y^5x	y^4x^2	y^4x^2	y^4x^2	y^3x^3
	y^3	y^2x	y^2x	y^2x	yx^2	yx^2	yx^2	x^3

Diagram $(y+x)^6$.

separated into three lengths which are respectively y^2x and yx^2 . Cross-lines were then drawn through the square whose sides had a length $x^3+3y^2x+3yx^2+x^3$, dividing it by means of full lines into 16 rectangular areas, and by additional dotted lines into 64 areas.

At the bottom of the diagram are eight rectangles. They represent the product of y^3 on the vertical axis with the terms laid off on the horizontal axis. The sum of these areas is $y^6+3y^5x+3y^4x^2+y^3x^3$. When the next term along the vertical axis (namely $3y^2x$) is used as a multiplier we have 24 rectangles, the sum of whose areas is $3y^5x+9y^4x^2+9y^3x^3+3y^2x^4$. In a similar way the remaining terms on the vertical axis are used as multipliers, yielding rectangles which fill the remaining part of the square whose area is $(y+x)^6$. The area represented is in all,

$$y^6+6y^5x+15y^4x^2+20y^3x^3+15y^2x^4+6yx^5+x^6.$$

The second term of this expression is represented by two larger rectangles each having an area $3y^5x$. They adjoin the two squares y^6 and $9y^4x$. They make contact with each other only at their corners, where a diagonal cross-line is drawn. The next term is represented by the square $9y^4x^2$ and two rectangles each having an area $3y^4x$, which make contact with the square at its corners. The areas representing the remaining terms will be readily identified.

The full significance of this diagram will perhaps be more fully realized, if the reader will multiply by ordinary algebraic process the two expanded binomials $(y+x)^3$ into each other. Each of the sixteen terms resulting will be recognized as placed in regular sequence in the square $(y+x)^6$.

For lecture purposes this diagram can easily be reproduced on a larger area, and it will then be an advantage to shade the small rectangles in which the exponent of x is odd. These areas must be considered subtractive if the sign of x is negative. The difference between the two sets of areas will then represent the area of a square, the sides of which have a length $(y-x)(y-x)(y-x)$. When the exponent n is odd, the diagram for $(y+x)^n$ will be a rectangle. If for example $n=9$, the sides of the rectangle similar to the square here shown, will have lengths $(y+x)^5$ and $(y+x)^4$. If the diagram becomes too large, the scale can be easily reduced, as is done in drawing a map of the United States.

Recent results published by the author seem to suggest a fairly satisfactory answer to the question, When is a cube not

a cube? The answer will be found in the adjoining diagram. As is well known the difference between the volumes of two cubes whose edges have a length y and x is found by dividing the quantity $y^3 - x^3$ by $y - x$, and multiplying the factor $y - x$ into the resulting terms. We have,

$$y^3 - x^3 = y^2(y - x) + yx(y - x) + x^2(y - x).$$

These three terms are generally interpreted as representing the volumes of three blocks, which will fill the space between the two cubes. The terms of the second member may be written, $(y^3 - y^2x) + (y^2x - yx^2) + (yx^2 - x^3)$.

It is also well known that the difference between any two real quantities is equal to the product of the sum of the square roots of those quantities multiplied by the difference between them. The three terms in the above expression may therefore be written,

$$\begin{aligned} &(\sqrt{y^3} - \sqrt{y^2x})(\sqrt{y^3} + \sqrt{y^2x}) \\ &+ (\sqrt{y^2x} - \sqrt{yx^2})(\sqrt{y^2x} + \sqrt{yx^2}) \\ &+ (\sqrt{yx^2} - \sqrt{x^3})(\sqrt{yx^2} + \sqrt{x^3}). \end{aligned}$$

These terms are represented in the diagram, where y^3 is represented by a square area whose sides have a length $y^{\frac{3}{2}}$, and x^3 is represented by a square area whose sides have a length $x^{\frac{3}{2}}$.

The two factors in each of the three terms represent the width and the combined length of the rectangles which fill the area between these two cubes.

The function $y^2 - x^2$ may be dealt with in a like manner. We have,

$$(y^2 - x^2) = (y + x)(y - x).$$

This second member represents the area of two rectangles forming a strip between the sides of the two squares.

We may also write the second member in the form,

$$\begin{aligned} y(y-x) + x(y-x) &= (y^2 - yx) + (yx - x^2) \\ &= (\sqrt{y^2} - \sqrt{yx})(\sqrt{y^2} + \sqrt{yx}) \\ &\quad + (\sqrt{yx} - \sqrt{x^2})(\sqrt{yx} + \sqrt{x^2}) \end{aligned}$$

Here we have terms representing two strip areas between the sides of the squares y^2 and x^2 . The widths of the two strips are $y - \sqrt{yx}$ and $\sqrt{yx} - x$.

The assumption made in this discussion is that each of the terms which are separated by the minus sign represents a quantity which is the square of some other quantity.

WOULD HONOR WAR HEROES ON ARBOR DAY.

Secretary of Agriculture Advocates Planting of Trees Dedicated to Men Who Died for America.

"Such an observance of Arbor Day as will secure a widespread planting of trees dedicated to those whose lives have been sacrificed in the great struggle to preserve American rights and the civilization of the world" is advocated by the Secretary of Agriculture, David F. Houston, in a letter just sent to the governors of the states.

The Secretary's letter follows:

"The observance of Arbor Day began soon after the Civil War. A distinguished citizen of Nebraska, who later became Secretary of Agriculture, was the prime mover in securing its recognition within his state, where it first took root; and the Board of Agriculture of that state, on his motion, designated the first Arbor Day. From the beginning it has had a civic motive and an association of patriotism.

"Another and greater war has come to its inevitable conclusion. The cause of righteousness, of liberty, of all that Americans hold dear has prevailed. We shall seek many ways to perpetuate the memory of those who made the great sacrifice. The memorials will take many forms. The names of those who have fallen will be perpetuated by costly monuments and inscribed on enduring tablets. Great works that serve the needs of peace also will doubtless be dedicated to them.

"But along with these memorials, we can easily discover ways in which we may simply and spontaneously pay our tribute to them. We can keep fresh our memory of what they gave; and we can perpetuate their names in familiar places. It has been happily suggested that we may do this by adorning with young trees, each named for a fallen soldier, our waysides, our yards, and our pleasure places. And in most of our states Arbor Day is at hand. This year we may give to that day a meaning more profound, a purpose more exalted, yet also an association more personal.

"I conceive that, if the origin of the day be borne in mind, the invitation to our fellow citizens to join in making it a day 'especially set apart and consecrated for tree planting' may appropriately come from this department. I take the liberty, therefore, of suggesting that you commend to the citizens of your state, and particularly to those in attendance upon its schools, such an observance of Arbor Day as will secure a widespread planting of trees, dedicated to those whose lives have been sacrificed in the great struggle to preserve American rights and the civilization of the world."