461. Approximate Formulae for the Perimeter of an Ellipse

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461. [C. 2. j; L¹. 9. c.] *Approximate Formulae for the Perimeter of an Ellipse.*

The perimeter (P) is $4aE(\theta)$, where $E(\theta)$ is obtained from tables of elliptic functions, and θ is $\arcsin e$.

Also the perimeter is given by the series $2\pi a(1 - \frac{1}{4}e^2 - \frac{3}{64}e^4 - \frac{5}{256}e^6 - \dots)$ which converges too slowly for rapid calculation.

The unsatisfactory formula $2\pi a(1 - 0.364e^2)$, found in a text book, led to an investigation into the availability of an expression of this type, or of that commonly used: $2\pi a\sqrt{1 - 0.5e^2}$. If such a type is used, it can only be approximate for a limited range of values of b/a , and two formulae are really necessary; even then it is difficult to obtain two which cover the whole range and give satisfactory results for the middle cases. The formulae suggested for this type are no. (ii) below. A logarithmic formula, however, gives good results except for values of b/a less than 0.15, which would rarely occur. For this I am indebted to Professor J. P. Dalton, of Johannesburg.

The following values for $P/2\pi a$ may be used within the range specified, with an error of less than one per cent.

Formula for $P/2\pi a$.	Range for b/a .
(i) $2E(\theta)/\pi$	0 - 1
(ii) $\begin{cases} 1 - 0.28e^2 \\ 1 - 0.21e^2 - 0.13e^4 \end{cases}$	$\begin{matrix} 0.64 - 1 \\ 0.25 - 1 \end{matrix}$
(iii) $1 + \log_{10}(0.998 - 0.55e^2)$	0.15 - 1
(iv) $\sqrt{1 - 0.5e^2}$	0.7 - 1
(v) $\frac{1}{2}(1 + b/a)$	0.7 - 1
(vi) $\sqrt{b/a}$	0.8 - 1

The following are values of $1000P/2\pi a$, calculated by the different formulae:

b/a	1	0.985	0.940	0.866	0.766	0.707	0.643	0.5	0.342	0.174	0
(i)	1000	992	970	934	887	861	831	771	712	662	637
(ii)	$\begin{cases} 1000 \\ 1000 \end{cases}$	$\begin{cases} 992 \\ 994 \end{cases}$	$\begin{cases} 967 \\ 974 \end{cases}$	$\begin{cases} 930 \\ 939 \end{cases}$	$\begin{cases} 885 \\ 891 \end{cases}$	$\begin{cases} 860 \\ 862 \end{cases}$	$\begin{cases} 836 \\ 833 \end{cases}$	$\begin{cases} 770 \\ 770 \end{cases}$	$\begin{cases} 714 \\ 714 \end{cases}$	$\begin{cases} 675 \\ 675 \end{cases}$	$\begin{cases} 660 \\ 660 \end{cases}$
(iii)	999	992	970	935	887	859	829	768	710	667	651
(iv)	1000	992	970	935	891	866	841	791	747	718	707
(v)	1000	993	970	933	883	854					
(vi)	1000	992	970	931	875						

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REVIEWS.

Plane Geometry, PARTS I. AND II. By G. ST. L. CARSON and D. E. SMITH. Pp. 480. Each Part, 2s. 6d. (Ginn.)

The publication of Part II. completes this work, of which the earlier portion has already been noticed in the *Gazette*. In their preface the authors express the hope "that the spirit of scholarship and the sense of aesthetic value in geometry, as set forth in this work, may appeal to a large number of English-speaking teachers of mathematics." The authors have aimed high; an examination of their work will show that they have not failed. Their book is one which Euclid, if he could get acquainted with it, would probably read with interest and approval. He might regret that the old maxim, "no superfluity," was not held in such regard as was usual in his times;