493. Ptolemy's Theorem

Author(s): P. J. Harding

Source: *The Mathematical Gazette*, Vol. 8, No. 126 (Dec., 1916), pp. 329-330

Published by: The Mathematical Association

Stable URL: <http://www.jstor.org/stable/3602782>

Accessed: 31-05-2016 02:54 UTC

Your use of the JSTOR archive indicates your acceptance of the Terms & Conditions of Use, available at
<http://about.jstor.org/terms>

JSTOR is a not-for-profit service that helps scholars, researchers, and students discover, use, and build upon a wide range of content in a trusted digital archive. We use information technology and tools to increase productivity and facilitate new forms of scholarship. For more information about JSTOR, please contact support@jstor.org.

The Mathematical Association is collaborating with JSTOR to digitize, preserve and extend access to *The Mathematical Gazette*

The Philosophical Magazine, Comprehending the various branches of science, the liberal and fine arts (geology), agriculture, manufactures, and commerce. By A. Tilloch. Vol. 1-42. 8°. London, 1798-1813.

The Philosophical Magazine and Journal, etc. By A. Tilloch (R. Taylor). Vol. 43-68. 8°. London, 1814-26.

Annals of Philosophy ; or, Magazine of chemistry, mineralogy, mechanics, natural history, agriculture, and the arts. By T. Thomson. Vol. 1.—New Series, Vol. 12. 8°. London, 1813-26.

After Vol. 12 of the new series, the *Annals* was combined with the *Philosophical Magazine*.

The Philosophical Magazine ; or, Annals of chemistry, mathematics, astronomy, natural history, and general science. By R. Taylor and R. Phillips. New and united series of the *Philosophical Magazine and Annals of Philosophy*. Vol. 1-11. 8°. London, 1827-32.

The Edinburgh Journal of Science (exhibiting a view of the progress of discovery in natural philosophy, chemistry, mineralogy, geology, botany, zoology, comparative anatomy, practical mechanics, geography, navigation, statistics, antiquities, and the fine and useful arts. Conducted by D. Brewster. Vol. 1.—New Series, Vol. 6. 8°. Edinburgh, 1824-32.

After Vol. 6 of the new series the *Journal* was combined with *The Philosophical Magazine*, and continued as

The London and Edinburgh Philosophical Magazine and Journal of Science. Conducted by D. Brewster, R. Taylor and R. Phillips. Vol. 1-16. 8°. London, 1832-40.

It was then continued as

The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science. Conducted by D. Brewster, R. Taylor, R. Phillips, R. Kane, and others. Vol. 17→. 8°. London, 1840→.]

MATHEMATICAL NOTES.

493. [K¹. 8. a.] *Ptolemy's Theorem.*

In the following $\Pi(x, y, \theta)$ denotes the area of a parallelogram of which x and y are two non-parallel sides, and θ one of the angles. It is clear that $\Pi(x, y, \theta) = \Pi(x, y, \pi - \theta)$.

Let $ABCD$ be a quadrilateral inscribed in a circle, and O the intersection of its diagonals. Let $\alpha, \beta, \gamma, \delta$ be angles at the circumference standing on the arcs AB, BC, CD , and DA respectively ; where by the arc AB is meant that arc which does not pass through C and D . It is easily seen that the angle $AOB = \alpha + \gamma$, the angle $BOC = \beta + \delta$, and $\alpha + \beta + \gamma + \delta = \pi$.

(i) Draw through A and C parallels to BD , and through B and D parallels to AC . The resulting parallelogram (omitted in the figure) is $\Pi(AC, BD, \alpha + \gamma)$ or $\Pi(AC, BD, \beta + \delta)$, and is double the given quadrilateral.

(ii) On the arc ADC take the point D' , so that $AD' = CD$.

Then the triangles $AD'C$ and CDA will be identically equal, and the quadrilaterals $ABCD'$ and $ABCD$ equal in area.

Draw BD' , dividing $ABCD'$ into two triangles.

Since the triangles $AD'C$ and CDA are vertically equal, the angle $CAD' = \delta$.

Hence the angle $BAD' = \beta + \delta$,
and the triangle $BAD' = \frac{1}{2} \Pi(AB, AD', \beta + \delta)$
 $= \frac{1}{2} \Pi(AB, CD, \alpha + \gamma)$.

Similarly, the angle $ACD' = \gamma$,
and the triangle $BCD' = \frac{1}{2} \Pi(BC, CD', \alpha + \gamma)$
 $= \frac{1}{2} \Pi(BC, AD, \alpha + \gamma)$.

Hence $ABCD'$, and therefore also $ABCD$,
 $= \frac{1}{2} \Pi(AB, CD, \alpha + \gamma) + \frac{1}{2} \Pi(BC, AD, \alpha + \gamma)$.

(iii) Equating values of double $ABCD$, as given by (i) and (ii),

$$\Pi(AC, BD, \alpha + \gamma) = \Pi(AB, CD, \alpha + \gamma) + \Pi(BC, AD, \alpha + \gamma),$$

from which it easily follows that

$$\text{rect.}(AC, BD) = \text{rect.}(AB, CD) + \text{rect.}(BC, AD).$$

P. J. HARDING.

494. [K¹. 6. a.] *De plagis plani infiniti in quibus apparere potest linea tertii ordinis, tribus asymptotis realibus datis praecipuâque satellite.*

[Positum sit in primis $\frac{Ax' + By' + C}{A}$ vel affirmativum vel negativum esse quemadmodum punctum (x', y') vel ad dextram vel ad laevam sit lineae $Ax + By + C = 0$.]*

Aequatio lineae tertii ordinis asymptotos habentis lineas $x = 0$, $y = 0$, $lx + my = 1$, induet hanc formam

$$xy(lx + my - 1) + H(\lambda x + \mu y - 1) = 0,$$

ubi $\lambda x + \mu y = 1$ praecipua satelles est (id est linea in qua curva asymptotos secat). Duo puncta coincidentia ad infinitionem secundum singulas asymp-

FIG. 1.

totos, valoresque λ , μ , conficiunt octo e novem conditionibus lineae tertii ordinis; et terminus H nonâ constituitur; e.g. puncto ad arbitrium capto per quod curva eat.

Ita scribamus aequationem:

$$xy \cdot \frac{lx + my - 1}{l} = - \frac{H\lambda}{l} \cdot \frac{\lambda x + \mu y - 1}{\lambda} \dots\dots\dots(i)$$

* *Math. Gazette*, vol. viii. No. 120, p. 182, Note 460.