**CLASS-EXERCISE TYPES IN HIGH SCHOOL MATHEMATICS,
WITH NORMS FOR JUDGING THEM.**

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I

To the several different types of thinking that are involved in High School mathematical study, specific types of class exercise correspond more or less closely. For example, to the conceptual type of study we have the class exercise of conceptual type, to the associational type of thinking the associational type of class exercise, to the assimilational type of thinking the assimilational type of class-exercise, etc. To profit highly by the study of any given class exercise the student of teaching practice should, first of all, determine the type of class exercise he is concerned with, and then apply appropriate norms in judging it. It is the purpose of this paper briefly to point out the most significant types of class exercise that high school mathematics teaching exemplifies and to indicate norms for judging the several types. It would seem that such a paper ought to be of interest to at least three classes of readers, viz.: (1) visiting mathematical teachers who employ their free visiting days studying observationally the work of neighboring schools; (2) teachers in training in colleges of educational and normal schools, and (3) those in charge of the training of teachers in training.

The following are the most general as well as the most important types of mathematical class exercise met in high schools:

I. The conceptual type	VII. The problem type
II. The expression type	VIII. The topic type
III. The associational type	IX. The application type
IV. The assimilation type	X. The test type
V. The review type	XI. The research type
VI. The drill type	XII. The appreciation type

The point here is that all these types of class exercise need to be recognized as such, and for any really profitable study of one of them the specific norms appropriate to it must be applied. In the sequel, we shall attempt to specify the appropriate norms, or criteria, for judging each of the foregoing exercise types in order.

The Conceptual Type and Critical Norm.

The conceptual type of class exercise is an exercise whose primary and leading intent is to clarify and establish the fundamental notions of the subject, or topic. Among the various types of exercise it is of first, and perhaps of chief importance.

It requires a certain kind of organization, preparation and presentation. The specifications for judging a particular example of this type of class exercise must get at an appraisal of these three things. The leading specifications, which we have called norms, for judging this type of exercise are as follows:

1. What are properly the fundamental concepts, and were they treated?
2. Were they vividly and graphically presented? Illustrate.
3. Were the pupils prepared for them and interested in them? Illustrate.
4. Was the presentation direct or indirect? Graphic or formal? Illustrate.
5. Were the concepts presented through the agency of formal definitions, or through descriptions, explanations, diagrams, etc.? Illustrate.
6. Cite any strong or weak points in the presentation.
7. Was there any recognition of individual differences? Illustrate.
8. Suggest any ways of strengthening the weak places.

The Expression Type and Critical Norms.

In mathematical subjects generally, and particularly in algebra, it is desirable now and then to devote a class exercise mainly to teaching, or to giving facility in, the language and expression modes of the subject. All who know the facts will agree that the mastery of the symbolism of any mathematical branch is a very consequential part of the problem of mastering that branch. One good writer¹ holds that analysis, the direct use of symbolism, the extended use of symbolism, and the manipulation of symbolism are the characterizing activities of the study of algebra. Three of the four characteristics pertain to symbolism. Most of us would perhaps regard a claim that three-fourths of the problem of mastering algebra is a language and expression problem, as putting the case too strong, nor need it be thought that the author just cited makes such a claim. There is no claim by this author that the four characteristics are entire quantitative equivalents. There can, however, be no difference of opinion that the interests of economic progress in any mathematical subject require at times the focussing of teaching effort for one or more class exercises upon the expressional types of the subject. Pertinent questions for a critical appraisal of the expression type of class exercise for mathematics are the following:

1. Did the teaching emphasize most the form, or the content? Illustrate.
2. Was the pupil trying to communicate trivial, or important ideas?
3. Was the content partly from the field of general human experience, or entirely from the narrow field of the science itself? Illustrate.
4. Was the emphasis upon giving command of the expression tools for everyday purposes, or for purely technical purposes? Illustrate.

¹T. P. NUNN: *The Teaching of Algebra*.

5. Were the standard language and symbolism of the science taught?
6. Were situations interpreted, or was the expressional work only translating from verbal to symbolic language, and the reverse?
7. To what extent did pupils manifest antecedent practice?
8. Were pupils interested and attentive?
9. Were correctness and good form duly impressed?
10. Did the teacher exemplify his precepts in his practice?

The Association Type and Critical Norms.

Most of the objectives sought formerly through memoriter modes are nowadays being sought by good teachers through the association type of memorizing. Resemblances, differences, relationships in both content and form of mathematical matter, are the modern technics that are largely replacing the still common practice of reliance upon a "dead heave" of formal memory. The association type of thought, of which there is much need in mathematics, is greatly facilitated by a teaching technic that derives the necessary memory factors and products of mathematical thought through association. Many class exercises may well be spent on work that is little more than pure association of the particular with the general, as in substitution work, or of the general with the particular, as in generalizing. Deriving a formula for one of the medians of a triangle, and then writing the formulas for the other two medians *by analogy*, and the like procedure for the altitudes, are examples of associational thinking, and class exercises of which this mode of thought is the outstanding feature are of frequent occurrence. Norms for a critical study of the associational type of class exercise are:

1. Were the associations of ideas actually made by the pupils?
2. Were the generalized types employed properly grasped by the pupils?
3. Were the things associated put together properly?
4. Were symbols connected with meanings, or simply with other symbols?
5. Did the teacher initiate wrong habits by suggesting wrong connections?
6. Were any cross-connections (correlations) established, as arithmetic or geometry with algebra, etc.? Illustrate.
7. Were distinctions of known and unknown, variable and constant, duly familiar to the class?
8. Were any means employed for ascertaining whether pupils were making correct associations? Illustrate.
9. Did the work tend to develop flexible or inflexible study habits in pupils? Illustrate.
10. Suggest any ways of improving the procedure.

The Assimilation Type and Critical Norms.

If, as the writer readily concedes, "only such processes as frequently occur in the lives of all persons should be habituated through drill,"² there is very little in either high school algebra

²J. H. MINNICK: The Recitation in Mathematics, Math. Teacher, March, 1920, p. 121.

or geometry that needs drill. But for ready mastery of many topics and processes of the later work, both early algebra and geometry contain many things that need to be *well known*. Partial knowledge of fundamentals is elusive, fleeting and disappointing. Beginning pupils are prone to think they comprehend, when in fact they know only very superficially. This is even more true of the brighter than of the slower pupils. All need some repetition of fundamentally important things that they may really apprehend, not that they may be habituated. Repetition, or reteaching, only to the extent called for by increased stability and firmness of grasp of related ideas is assimilation work, and not drill work. Drill seeks to routinize, to automatize, to reduce reflection to reflex; whereas assimilation seeks only stable understandings, and conclusions of belief. Many class exercises of the assimilation type occur and are needed in high school mathematics, and useful norms for the critical study of this type of class exercise are as follows:

1. Did the teacher seem to distinguish between repetition for assimilation and repetition for drill? Illustrate.
2. Was the assimilation given at the right time?
3. Did the class seem to be in need of it? Illustrate.
4. To what extent were individual differences of need for assimilative work evident?
5. Was the class aware of the fundamental character of what they were being required to assimilate?
6. Did the repetition, or reteaching, seem to be achieving its objectives? Illustrate.
7. Were synopses, resumés, or other devices used to show the class the fundamental character of what they were being required to learn well?
8. To what extent, if at all, did the work merely repeat former treatments in the identical ways first given? Illustrate.
9. To what extent, if at all, were new uses and applications employed for assimilation purposes? Illustrate.
10. Suggest any ways in which the class exercise might have been improved.

The Review Type and Critical Norms.

Next to the drill type of class exercise the easy-going teacher and program administrator favor the review type, if frequency of occurrence as a class exercise and as a topical heading in current texts can be construed as a measure of favored exercise types. Review and drill have long been the shibboleths of the mathematical school-master, and their coercing influence on teaching practice until recently, have had much to do in bringing this practice into its present disrepute. A review that is a new view, a higher view, or a more perspicuous view of a study-unit is both highly valuable and essential to good teaching technics. A review that only repeats the old thing in the old way,

which is far too completely the prevalent type even today, needs perhaps to be judged and criticized out of existence. The second sort of review, which is only a means of trying to supply the lacks of inefficient teaching in the first instance, needs extinction by the shortest possible route, though we still frequently meet with the advocacy of this type of review by the violent critics of current teaching of high school mathematics. A visiting teacher is sure to meet the review type of class exercise frequently and criteria, such as the following, for judging its character, will be found useful:

1. Was the review given at the proper place?
2. What was the scope, or ground covered? Determined by a program or by the needs of the class?
3. Was it a recapitulation, a sort of synoptical study, or did it seem only a repetition?
4. Did it so emphasize the major and minor elements of the study-unit in hand as to help to a firmer grasp of the entire unit? Did it prepare well for the next advance?
5. Were the questions and problems well chosen and skillfully used?
6. Were enough new elements introduced to hold attention? Was there any reteaching?
7. Did the class seem alert and attentive, and did all participate? Were both class and teacher prepared? Was there too much or too little of it?
8. Did the teacher seem to achieve a definite, set objective? Illustrate.
9. Specify any weak or strong features of the teacher's work.
10. Indicate ways in which the class exercise might have been improved.

The Drill Type and Critical Norms.

If the principle quoted at the beginning of the foregoing discussion on the assimilation type of class exercise is sound, there is little, if anything, in high school mathematics that a teacher ought even to seek to make speedily automatic. Certainly the admonition, "take your time, but be sure you are correct before you answer," is much more frequently needed than are, "Hurry up," "Be quick," etc., in high school classes in mathematics. Most surely the speediest students are not the best mathematical thinkers. A very few exceptional pupils are highly accurate but inordinately slow in their work. In general a degree of speed is essentially conditioning as to accuracy. That there is a sort of correlation of speed and accuracy is almost certain. Nor is the correlation, factor either negative or near +1. Some drill for speed is not without warrant, even in the interests of accuracy. Drill for automatism is, however, out of place in algebra and geometry. Granting that the laws of signs, the laws of exponents and a few other elemental matters need drill enough for easy and certain application, we do not concede that the extended drills on factoring and fractions and the

like, that are still too general, are pedagogical, or even justifiable. Such drills as are implied by directions like, "Do as many of the following exercises as you can in fifteen minutes" are to be unqualifiedly recommended. We have, however, granted the claim that drill for facility, for easy mastery, and for economy of time is sound, and very much more drill than is justifiable is met in the observation of current practice. It is, therefore, desirable to be able to apply to drill exercises the appropriate criteria:

1. Was the drill premature, i. e., started before the ideas were well apprehended?
2. Were all pupils engaged all the time?
3. Did pupils go at it with zeal and spirit, and ask for more?
4. Was it concentrated on specific points, or scattering?
5. Were appreciable results gained?
6. Was the device of "working under time-limits" employed?
7. Was it too long continued?
8. Any evidence that it was given to supply the lack of good teaching?
9. Was there any recognition of individual differences and peculiarities?
10. Did it animate or depress the spirit of the class?
11. Were any standard tests employed? If so, what?

We have now discussed six of the standard types of class exercise, and have already, perhaps, transgressed the limit of patience of the editor. There remain, however, the problem, the topic, the application, the test, the research, and the appreciation types of class exercise to be given similar treatment in a later paper. If agreeable to the editors these treatments will be given in a subsequent issue.

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SNAP-EXPOSURES ON A TRIPOD.

The rapid aiming of a hand-camera with a fair degree of accuracy is an accomplishment acquired only after some considerable practice, which, doubtless, many otherwise expert photographers know to their cost in the shape of negatives wasted through this cause. In these days of short-focus lenses this is a matter of far greater difficulty than the focusing of the picture. When making photographs of public ceremonies, processions, etc., it is a good plan, if the photographer is not very expert, for him to find some convenient coign of vantage, such as the upper window of some adjacent building. There he can erect the camera upon a tripod, thus making sure of its accurate focus and direction before the time to make the pictures arrives. In this way it is quite easy to make a number of exposures with the advantage that the photographer knows exactly what is included in the picture, leaving him quite free to watch the subject and give it and its most favorable poses his undivided attention. The limitations of the view may be mentally marked by the vertical lines of buildings, etc., though a slight latitude is also allowed by a turn of the instrument upon its tripod-screw to right or left, should the subject move off the predetermined spot. Photographers are so inclined to think of instantaneous photography as inseparable from hand-camera work that they do not realize the value of a tripod.—[*Photo-Era*.]