Transactions of the Royal Society of Edinburgh

<http://journals.cambridge.org/TRE>

Additional services for *Transactions of the Royal Society of Edinburgh*:

Email alerts: [Click here](#)

Subscriptions: [Click here](#)

Commercial reprints: [Click here](#)

Terms of use : [Click here](#)

VII.—Rupture Stresses in Beams and Crane Hooks.

Angus R. Fulton

Transactions of the Royal Society of Edinburgh / Volume 50 / Issue 01 / January 1914, pp 211 - 223
DOI: 10.1017/S0080456800017300, Published online: 06 July 2012

Link to this article: http://journals.cambridge.org/abstract_S0080456800017300

How to cite this article:

Angus R. Fulton (1914). VII.—Rupture Stresses in Beams and Crane Hooks.. Transactions of the Royal Society of Edinburgh, 50, pp 211-223 doi:10.1017/S0080456800017300

Request Permissions : [Click here](#)

VII.—**Rupture Stresses in Beams and Crane Hooks.** By **Angus R. Fulton**,
B.Sc., A.M.Inst.C.E., Engineering Department, University College, Dundee.
Communicated by Professor A. H. GIBSON, D.Sc., A.M.Inst.C.E.

(MS. received November 4, 1913. Read February 16, 1914. Issued separately April 15, 1914.)

I. BENDING MOMENT AS IN A BEAM.

In connection with a research into the failure of timber under stress, carried out by the author, and on which a paper was read before the Royal Society of Edinburgh and published in their *Transactions*,* many of the tests were made by cross-breaking.

The probable development of the stress diagram of a beam of rectangular section, supported at two ends and loaded in the middle where the tensile and compressive strengths of the material are of different values, was illustrated in fig. 38 of that paper. It was there laid down that when the fracture point was ultimately reached either (a) the tension and compression stress areas both assume the rectangular form and are equal to each other, the stress ordinates respectively being equal to the ultimate breaking stresses of the material in direct tension and compression; or (b) the cohesion between adjacent fibres measured from the neutral axis outwards is not sufficient to withstand the shear induced by the resisting moment of the beam, a shear which is at a maximum along the neutral axis of the beam.

These assumptions made for the case of beams of rectangular sections might be said to have found verification in the experiments on such beams of various timbers supported at two ends and loaded in the middle, the results of which were tabulated in that paper.

Experiments on Cast-Iron Beams.

Since then the author has carried out a number of similar tests on rectangular beams made of cast iron, a material which differs from timber in that it apparently possesses no plastic stage such as characterises the latter.

The specimens used for the purpose of obtaining direct tensile and compressive stresses were cast from the same melting as the beams and so were directly comparable.

The average values obtained for these direct stresses were :—

Cast Iron.

Tension stress, $T = 9.5$ tons per sq. in.

Compression stress, $C = 45.5$ „ „ „

The dimensions of test pieces used in compression were $2\frac{5}{8}$ in. long and $1\frac{1}{8}$ in. $\times \frac{7}{8}$ in.

The formula as derived in the paper referred to, and presumably applicable to all

* *Trans. R.S.E.*, vol. xlviii., 1912, pp. 417–440.

rectangular beams in which an ultimate rectangular distribution of stress is reached, gives :—

$$t = 2.3 \frac{M}{bd^2} \frac{T+C}{C}, \quad c = 2.3 \frac{M}{bd^2} \frac{T+C}{T} \quad (1)$$

where t = the ultimate extreme tensile fibre stress in beam,

c = „ „ „ compressive fibre stress in beam,

M = „ external bending moment,

T = „ ultimate direct tensile stress of the material,

C = „ „ „ compressive stress of the material,

b = „ breadth, and d = depth of the beam.

Substituting for $M = \frac{WL}{4}$, since the beams were loaded in the centre and supported at the two ends, and for T and C the average values obtained, we have

$$t = .70 \frac{WL}{bd^2} \quad \text{and} \quad c = 3.33 \frac{WL}{bd^2} \quad \text{for cast iron.}$$

Table I. gives the results of the beam experiments, and it will be seen that the calculated values of the extreme fibre stresses t and c agree very well with the

TABLE I.
CAST-IRON BEAMS.

No.	Span.	b .	d .	Breaking Weight in Tons.	Tons per sq. in.		Elastic Formula.	$\times 10^7$.	
					$t = .70 \frac{WL}{bd^2}$.	$c = 3.33 \frac{WL}{bd^2}$.	$t = c = 1.5 \frac{WL}{bd^2}$.	E_b .	E_n .
1	36	2.00	1.02	.8	9.7	46.17	20.77	1.48	...
2	12	1.02	2.00	4.43	9.14	43.50	19.57	...	...
3	36	1.00	1.95	1.45	9.65	45.93	20.67	...	1.85
4	12	1.00	1.95	4.29	9.45	45.00	20.25	...	...
5	12	1.95	1.00	2.17	9.31	44.33	19.95	...	...
6	36	1.05	2.05	1.6	9.17	43.67	19.65	...	1.53
7	12	1.05	2.05	4.34	8.30	39.50	17.77	...	...
8	12	2.05	1.05	2.2	8.61	41.10	18.45	...	...
9	36	2.03	1.07	.65	7.07	33.67	15.15	1.35	...
10	12	2.03	1.07	1.93	7.00	33.33	15.00	...	...
11	12	1.07	2.03	4.43	8.47	40.33	18.15	...	...
12	36	2.00	1.05	.72	8.26	39.33	17.7	1.48	...
13	12	2.00	1.05	2.1	8.01	38.17	17.17	...	...
14	12	1.05	2.00	4.4	8.75	41.67	18.75	...	...
15	36	1.03	2.05	1.34	7.77	37.00	16.65	1.51	1.51
16	12	1.03	2.05	4.74	9.18	43.87	19.74	...	...
17	12	2.05	1.03	2.43	9.41	44.83	20.17	...	...
18	36	1.05	2.04	1.55	8.94	42.57	19.15	1.48	1.67
19	12	2.04	1.05	2.33	8.71	41.47	18.66	...	...
20	12	1.05	2.04	4.75	9.14	43.50	19.57	...	...
21	36	2.06	1.03	.76	8.50	40.50	18.22	1.45	1.61
22	12	2.06	1.03	2.38	8.88	42.30	19.03	...	...
23	12	1.03	2.06	4.32	8.30	39.53	17.79	...	...

ultimate stresses obtained in direct tension and compression. The last two columns of the table give YOUNG'S modulus when the beam is placed on its broad or narrow face.

General Case for Beams of any Section, but with the Ultimate Distribution of Stress a Rectangular One.

The general case may be deduced as follows:—

Let fig. 1 denote the section of the beam, and let

A = total sectional area,

A_t = „ area ultimately subjected to tensile stress,

A_c = „ „ „ „ compressive stress.

t, c, T, C , and M have the same meaning as before.

Also let NN = ultimate position of neutral axis, the plane of which extends along the length of the beam and divides each cross-section of the beam into areas A_t and A_c ,

y_t = distance of centroid of area A_t from NN ,

y_c = „ „ „ „ „ A_c „

and let the ultimate distribution of stress be a rectangular one.

From the condition (α) that the tension and compression stress areas are equal to each other, we have

$$tA_t - cA_c = 0. \quad (1a)$$

If the ultimate fibre stresses reached at the point of rupture be equal to the ultimate direct stresses T and C in tension and compression respectively, then

$$TA_t - CA_c = 0,$$

and

$$\frac{A_t}{A_c} = \frac{C}{T}; \quad \frac{A_t}{A_t + A_c} = \frac{C}{T + C} \quad \therefore A_t = A \frac{C}{T + C},$$

and similarly

$$A_c = A \frac{T}{T + C}.$$

We therefore obtain the position of the neutral axis of the beam at fracture point by dividing the total area A into areas A_t and A_c in the ratio of $\frac{C}{T}$.

Further, since the resisting moment of the section just up to the point of rupture equals the external bending moment,

$$\therefore M = tA_t y_t + cA_c y_c = tA_t (y_t + y_c) = cA_c (y_t + y_c) = tA \frac{C}{T + C} (y_t + y_c) = cA \frac{T}{T + C} (y_t + y_c) \quad (I.)$$

$$\therefore t = \frac{M}{A(y_t + y_c)} \frac{T + C}{C} \quad (I.A)$$

$$\text{and } c = \frac{M}{A(y_t + y_c)} \frac{T + C}{T} \quad (I.B)$$

Fig.1

If T and C are equal, the final position of the neutral axis will divide each section of the beam into two equal areas and the value of the fibre stress

$$t \text{ or } c = \frac{2M}{A(y_t + y_c)} \quad (1)$$

In such a case, if the section is symmetrical about the original neutral axis, the position of the latter does not change.

Experiments.

In order to verify the correctness of such a formula, some tests were made by the author on wrought-iron bars. These bars were stressed as beams, and since the

Fig. 1a

material used here was ductile, no definite rupture took place. The ultimate load was therefore taken as that load at which the beam failed to stand up to the applied load and buckled up. Portions were cut out and tested to failure in direct tension. No tests, however, were made in direct compression, the fracture value of the material under compression being assumed to be equal to that under tension.

The bars were of various sections, and are illustrated in fig. 1a.

The sections A and B, which are circular, one solid and the other hollow, show an entire agreement between the direct tensile stress and the induced tensile stress.

Of the rectangular specimens C and D agree fairly well, but E, which is square, falls short in the induced stress.

F, G, and H are comparable as a group, and the results in the cases F and G are quite satisfactory. H, however, shows a deficiency in the induced stress which is somewhat marked on the 36-inch span, but less so on the 15-inch span. This may be explained by the fact that it is impossible for the web portion of the section as it is laid to be stressed to its full rupture value before the flanges give way. But if we consider this section as made up of the flanges only, as shown by the shaded portions of H, then the results are much nearer agreement, and are given in the table as $_1H_1$ and $_2H_2$. Section K was also tested for two lengths of span, the results of which differ somewhat. If the average of the induced stresses be taken, it agrees with the direct tension result.

Table II. gives the results of these tests.

TABLE II.

BEAMS.						TENSION TESTS.		
Section.	Area in sq. in.	$y_t + y_c$	Central Load. Tons.	Span. Ins.	Fibre Stress, t or $c = \frac{2WL}{A(y_t + y_c)4}$ Tons per sq. in.	Area in sq. in.	Total Load. Tons.	Tensile Stress per sq. in. Tons.
A	·7854	·425	·428	36	23·08	·7854	18·75	23·87
B	·463	·755	·485	36	24·98	·463	11·62	25·10
C	·805	·805	·805	36	22·36	·805	18·8	23·35
D	1·8	·45	·925	36	20·55	·163	3·28	20·1
E	·93	·48	·477	36	19·23	·93	23·6	25·34
F	·26	·445	·135	36	21·00	·123	3·09	25·1
G ₁	·81	·39	·383	36	21·82	·161	3·75	23·3
G ₂	·81	·39	1·055	15	25·04	·161	3·75	23·3
H ₁	·448	·266	·109	36	16·43	·108	2·64	24·46
H ₂	·448	·266	·285	15	17·88	·108	2·64	24·46
$_1H_1$	·285	·315	·109	36	21·86	·108	2·64	24·46
$_2H_2$	·285	·315	·285	15	23·81	·108	2·64	24·46
K ₁	·67	1·31	·87	36	17·84	·137	2·99	21·76
K ₂	·67	1·31	2·725	15	23·29	·137	2·99	21·76

II. BENDING MOMENT IN CASES OF ECCENTRIC LOADING, AS HOOKS, ETC.

Another and important case is where a section is subjected to non-axial loading, as in crane hooks.

Under such conditions a section as AB of fig. 2 is generally said to experience:—

- (1) A uniformly varying stress due to the bending moment of a couple of magnitude WL, the intensity at any point varying directly as the distance of that point from a fixed axis N in the plane of the section known as the neutral axis of stress;

- (2) A direct uniformly distributed stress due to the force W acting at the centroid of the section AB.

Thus if the strain produced was entirely elastic, the stress diagram of (1), when the material has approximately the same ultimate strengths in compression and in tension, would be as represented by $AaNbB$ in fig. 3, Aa being the tensile stress of the extreme fibres AA, and Bb being the compressive stress of the extreme fibres BB of the section shown in fig. 2.

The neutral axis NN passes through the centroid of the section. This diagram would be modified by the direct tensile stress AA_1 of (2) to the form $Aa_1N_1b_1B$, this

having the effect of changing the neutral axis from N to N_1 . If $NN_1 = x$, this increases the external bending moment to the extent of Wx , and correspondingly the resisting moment of the section must be increased to the same extent.

If t and c represent the values of the extreme stresses, then, by the ordinary elastic formula for determining these stresses,

$$t = \frac{My}{I} + \frac{W}{A} = W \left(\frac{Ly}{I} + \frac{1}{A} \right) \quad (2a)$$

$$c = \frac{My}{I} - \frac{W}{A} = W \left(\frac{Ly}{I} - \frac{1}{A} \right) \quad (2b)$$

Therefore the ratio of $\frac{t}{c} = \text{constant}$ and the line a_1b_1 , though its slope varies, passes through a fixed point N_1 so long as the condition of elastic stress is maintained.

The very approximate nature of the results obtained by this theory of combined tension and bending moment is dealt with in a paper * by Mr E. S. ANDREWS, B.Sc., and Professor KARL PEARSON, F.R.S., the magnitude of the error being there proved to depend upon the radius of curvature to which the hook is bent. They too arrive at the conclusion that the stress at any point in any fixed section under a given load is only a function of the distance of that point from the centroidal line, the maximum stress being obtained at the maximum distance from that line. Since the stress, then, depends directly on the load and this function, it follows that there will be a true neutral axis. An expression for its distance from the centroidal line is obtained, and this is a constant for a given section. The stresses, however, do not vary directly as the distance from this neutral line.

If the section, which is that of a 15-ton hook, is as shown in fig. 2*a*, with the centroid at N, then the diagram $Aa_1N_1b_1B$ of fig. 3*a* represents the diagram of stresses to some scale, as got by ANDREWS' theory, right up to the true elastic limit on the tension side, and N_1 is a fixed point.

If it were possible for the material to behave elastically right up to the point of fracture, then our ultimate stress diagram in accordance with the two theories would be as in figs. 3 and 3*a*, but of greater magnitude, and would be represented by $Aa_1N_1b_1B$ in figs. 4 and 4*a*.

But the extreme stresses Aa_1 on the tensile side, as calculated from (2*a*) and by ANDREWS' formula and as shown on these diagrams, are much greater than the tensile strength of the material. Let AT and BC represent the fracture value of the material in direct tension and compression respectively. (While the fracture value in direct tension is easy to determine, that of a ductile material in direct compression cannot be determined absolutely. The term is here used to mean that value of the stress at which the strain exceeds a certain fraction, having regard to initial section and length of test piece.) Through T and C draw lines parallel to AB to intercept a_1b_1 in T_1 and C_1 . Then the areas Ta_1T_1 and Cb_1C_1 , where the stresses would exceed the fracture value, cannot exist. Note the area Cb_1C_1 is a minus quantity in fig. 4*a*.

But the resisting moment of the internal stresses just before rupture must be equal to the external bending moment, and therefore areas within the stress limits, the summation of whose moments is equal to the summation of the moments of these excess areas, must be found to replace them. These can be represented by $N_1T_1N_2$ and $N_1C_1C_2N_2$, possibly involving another alteration of the neutral axis to N_2 .

The conditions involved in this redistribution of stress areas are (a) that the summation of the moments of the complete resisting stress areas CC_2N_2 and TT_2N_2 should be equal to the moment of the external load W about an axis through N_2 ; and (b), that the algebraic sum of the tensional and compressive stress areas should be equal to the external load W.

* *Drapers' Co. Research Memoirs*, Technical Series I., London, 1904.

It is now possible to proceed to establish a connection between the extreme fibre stresses at fracture point of a non-axially loaded body and the moment of the external load.

Let the symbols A , A_t , A_c , T , C , t , and c have the same meaning as in the case of beams, and whatever may have been the elastic distribution of stress, let the ultimate distribution be a rectangular one.

If the resultant force W was very small and L very great, then only the bending

moment stress need be considered. For this special case, if A_T and A_C be the tensile and compressive areas respectively, then

$$\frac{A_T}{A_C} = \frac{c}{t} \text{ as before, for } tA_T - cA_C = 0.$$

If the ultimate stress reached be equal to the ultimate direct strength of the material, then

$$\frac{A_T}{A_C} = \frac{C}{T}; \text{ and } A_T = A \frac{C}{T+C}; \text{ and } A_C = A \frac{T}{T+C}.$$

In order to obtain the position of the neutral axis NN under this condition, we

divide up the section of the hook in the above ratio as shown in fig. 5. Let y_t and y_c be the distances of centroids of A_t and A_c from the neutral axis NN.

Then we have, since the external moment equals the internal resisting moment,

$$\begin{aligned} WL &= tA_t y_t + cA_c y_c \\ &= tA \frac{C}{T+C} y_t + cA \frac{T}{T+C} y_c \end{aligned}$$

and at the fracture point, when t and c equal T and C respectively,

$$WL = tA \frac{C}{T+C} (y_t + y_c) = cA \frac{T}{T+C} (y_t + y_c) \quad . \quad . \quad . \quad (II.)$$

Now, when the resultant tensile force W becomes important and is taken into consideration, we have

$$tA_t - cA_c = W.$$

We may imagine that this is the previous case, with L diminishing as W increases. Since the stress limits at fracture are T and C as before, these cannot be altered

Fig. 5.

to meet the resultant W , which can only be balanced by a transfer of part of the compression area to the tension side. Of course, the moment of these rearranged areas must be equal to the external moment. Let the shaded area in fig. 5 be the part so transferred from compression to tension, and let the actual final position of the neutral axis be N_2 at a distance x from N , and z_t and z_c be the respective distances of the centroids of the areas A_t and A_c from those of the areas A_t and A_c of the special case.

From the equation of moments we have

$$W(L+x) = tA_t(y_t + x - z_t) + cA_c(y_c - x + z_c).$$

Now

$$A_t = A_t + a = A \frac{C}{T+C} + a$$

$$A_c = A_c - a = A \frac{T}{T+C} - a$$

$$z = a \frac{\left(y_t + \frac{x}{2}\right)}{A} \quad \text{and} \quad z_c = a \frac{\left(y_c - \frac{x}{2}\right)}{A}.$$

$$\therefore W(L+x) = tA \frac{C}{T+C} y_t + cA \frac{T}{T+C} y_c + x(tA_t - cA_c) - a \frac{x}{2}(t+c).$$

But

$$aT - a(-C) = W. \quad \therefore a(T+C) = W; \quad \text{and } a = \frac{W}{T+C}$$

$\therefore$ when the ultimate stresses are T and C we have

$$W(L+x) = tA \frac{C}{T+C} (y_t + y_c) + Wx - \frac{W}{T+C} \frac{x}{2} (T+C).$$

Eliminating balancing moments, this equation becomes

$$cA \frac{T}{T+C} (y_t + y_c) = tA \frac{C}{T+C} (y_t + y_c) = WL + \frac{Wx}{2}.$$

This means that what was the resisting moment of the section before this alteration of the neutral axis has, by reason of the rearrangement, become equal to the original external bending moment plus the moment of the weight about half the displacement of the neutral axis.

Since x is usually small, $\frac{Wx}{2}$ may be considered to be negligible, and therefore for all practical purposes the relationship between the external moment and the resisting moment of the section just at the point of rupture may be taken as the same as when there was no resultant, that is :

$$WL = tA \frac{C}{T+C} (y_t + y_c) = cA \frac{T}{T+C} (y_t + y_c) \quad \dots \quad (III.)$$

$$t \text{ or } T = \frac{WL}{A(y_t + y_c)} \frac{T+C}{C} \quad \dots \quad (III.A)$$

$$c \text{ or } C = \frac{WL}{A(y_t + y_c)} \frac{T+C}{T} \quad \dots \quad (III.B)$$

Now, if $T=C=f$, as is approximately the case for W.I. and mild steel, then

$$f = T = C = \frac{WL}{A} \frac{2}{(y_t + y_c)} \quad \dots \quad (3)$$

Experiments.

The results of experiments undertaken by the author with hooks made from various materials show that the values obtained for " f ," as calculated from formula (3) when the ultimate stress-point is reached, agree more or less accurately with the direct stress values. The first tests were carried out on galvanised wrought-iron hooks used for suspending ship's boats. The hooks were of circular section, and had ends long enough to enable ordinary tension test pieces to be cut off.

Afterwards a series of tests were made on hooks specially manufactured from round bars of various materials. In addition to the length necessary for the preparation of the hook, two other lengths were cut off each bar, one being tested in direct tension cold, and the other was first annealed before being so tested. In this way a direct comparison was made between the stress induced in the hook by bending and the ultimate tensile stress as found after heating.

A hook was also made from a bar of rectangular section, and also another prepared with an approximately rectangular section by flattening the sides of a round bar.

In addition an examination was made of the investigation * by Professor GOODMAN into the strength of drop-forged steel hooks, which was published in *Engineering*, and also of the results contained in his paper † on "Crane Hooks," which followed on the lines of ANDREWS' theory and which was published in the *Proceedings of the Institution of Civil Engineers*.

The values of $(y_t + y_c)$ in terms of the depth of section and of the consequent values of " f " in formula (3) is given below for the various sections investigated:—

	Description of Hook.	$(y_t + y_c)$.	f .
(1)	Circular	$\cdot 425d$	$4\cdot 7 \frac{WL}{Ad}$
(2)	Rectangular	$\cdot 5d$	$4 \frac{WL}{Ad}$
(3)	„ with rounded ends . . .	$\cdot 43d$	$4\cdot 65 \frac{WL}{Ad}$
(4)	GOODMAN'S drop-forged steel . .	$1\cdot 43h$	$4\cdot 65 \frac{WL}{Ah}$
(5)	„ wrought-iron 1st set . .	$\cdot 48h$	$4\cdot 17 \frac{WL}{Ah}$
(6)	„ „ 2nd set . .	$\cdot 48h$	$4\cdot 17 \frac{WL}{Ah}$

These were found to justify the conclusions which are set forth in the present paper, and an analysis of these along with the experimental results are given in Table III.

TABLE III.

Description.	Material.	Depth. Ins.	Area. Sq. in.	Load. Tons.	L".	$f = \frac{WL}{A} \frac{2}{y_t + y_c}$ tons per sq. in.	Max. Direct Tensile Stress.		Elastic Formula.
							(Cold).	(Annealed).	
No. 1 round	W.I.	1·1	·95	4·25	1·3	25·97	24·6	...	46·64
" 2 "	"	1·0	·78	3·90	1·1	25·70	25·1	...	48·58
" 3 "	B.B.H.	1·02	·83	4·35	1·0	24·45	23·05	23·41	46·61
" 4 "	Swed.	1·02	·82	3·24	1·15	20·83	20·00	20·12	39·38
" 5 "	S.M.S.	1·00	·78	3·8	1·1	25·20	25·00	25·80	46·97
" 6 "	Mild S.	1·02	·82	3·0	1·3	21·92	24·4	24·17	40·70
" 7 rect.	"	·93	·46	2·5	1·2	27·90	26·8	...	48·86
" 8 "	W.I.	1·02	·82	3·0	1·3	21·7	24·6	...	...
Average value						24·20	24·20		

* See GOODMAN on the "Strength of Drop-forged Crane Hooks," *Engineering*, lxxii. p. 537.

† GOODMAN on "Maximum Stresses in Crane Hooks," *Minutes of Proc. of Inst. of Civil Engineers*, vol. clxvii. p. 296.

TABLE III.—*continued.*

Description.	Material.	Depth. Ins.	Area. Sq. in.	Load. Tons.	L"	$f = \frac{WL}{A} \frac{y_t + y_c}{2}$ tons per sq. in.	Max. Direct Tensile Stress.		Elastic Formula
							(Cold).	(Annealed).	
GOODMAN'S TESTS.									
tons $\frac{1}{8}$ drop-forged	Steel	·82	·33	1·76	·95	27·45	no value given		52·92
$\frac{1}{4}$ "	"	·90	·38	1·85	1·04	26·04	"		44·30
$\frac{1}{2}$ "	"	1·01	·50	2·18	1·21	24·18	"		41·13
1 "	"	1·40	·95	7·12	1·47	36·60	"		68·38
$1\frac{1}{2}$ "	"	1·60	1·26	7·29	1·83	30·69	"		54·49
2 "	"	2·00	1·94	11·49	2·10	28·93	"		52·64
3 "	"	2·32	2·61	18·15	2·52	35·11	"		64·00
4 "	"	2·68	3·34	23·23	2·83	34·14	"		62·27
5 "	"	3·00	4·01	26·87	3·20	33·34	"		59·86
Average value						30·92			
5A 1st set	W.I.	3·49	5·59	32·5	3·35	23·35	23·5	...	41·6
5B "	"	3·47	5·84	34·1	3·42	23·93	"	...	42·6
10A "	"	4·36	9·20	57·3	4·32	25·4	"	...	45·3
10B "	"	4·34	9·76	57·9	4·33	25·4	"	...	45·8
15A "	"	4·78	12·57	62·5	4·65	20·14	"	...	36·1
15B "	"	4·97	12·65	90·0	4·44	26·27	"	...	47·7
Average value						24·08			
5A 2nd set	W.I.	3·79	5·43	31·0	3·54	22·24	23·4	...	39·1
5B "	"	3·36	5·78	33·6	4·18	30·16	"	...	48·2
10A "	"	4·75	10·96	54·0	4·48	19·37	"	...	33·9
10B "	"	4·63	10·80	59·0	4·71	23·17	"	...	42·1
15A "	"	5·27	14·53	76·9	5·00	20·93	"	...	39·9
15B "	"	4·78	13·51	82·9	4·84	25·91	"	...	47·7
Average value						23·63			

CONCLUSIONS.

1. It may be taken as conclusive that the final distribution of stress at rupture point in a member subjected to an external bending moment is a rectangular one, unless where the cohesion of adjacent layers is not sufficient to withstand the shear induced by the resisting moment of the section.

2. That, provided shear does not take place, the neutral axis moves always to the position which reduces the summation of the tensile and compressive stress areas, across a section, to the equilibrant of the external forces. (In the case of

a beam this reduces to zero; in that of a hook, at the principal section to the suspended weight.)

3. That the total resisting moment of these stresses must be equal to the external bending moment as measured to the neutral axis at rupture point, but that these balancing moments do not differ materially from those measured to an axis obtained by dividing the sectional area into tensile and compressive stress areas which are in inverse proportion to the magnitude of their respective ultimate direct stresses.

The advantage of these formulæ are important. It is possible to indicate with certainty the magnitude of the load which will cause rupture in a beam or a hook provided there is known the point of application or the effective arm of the load, the cross-section of the beam or hook, and the breaking strengths of the material when subjected to the different forms of direct loading.