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GEOMETRY TEACHING: THE NEXT STEP.

BY C. GODFREY, M.V.O.

I DRAW your attention to two dates of some importance in the teaching of Geometry. In 1903 Cambridge ruled that "Any proof of a proposition will be accepted that appears to the examiners to form part of a systematic treatment of the subject"; in 1909 the Board of Education issued Circular 711. These two dates determine three periods of time.

Before 1902, Geometry teaching was ruled by Euclid's sequence; we may describe this as the ice-age; teaching was frozen. What we were compelled to teach was called Euclid, but of course it was not really Euclid, for we did not teach Book 5. It was Euclid with the backbone removed; Euclid filleted.

I suppose that most of us were at school then. Personally I have no complaint to make of the teaching I received in these conditions; perhaps I was unusually fortunate, for my teacher was Mr. R. Levett, a name very well known to members of this Association. Presumably everyone present in this room was very far from being an average mathematician at school; we were all gifted mathematicians, otherwise we should not now be teaching the subject. What suited us may not have suited the average boy; in fact, when it became my duty to teach Geometry, it was very obvious to me that the average boy could not be taught properly in such conditions as were then possible. No far-reaching reconstruction could be planned while we were restricted to Euclid's sequence. Well, that is all over now; we need not revive the controversies of the nineteenth century. But we must remember that similar controversies would come to life again if any attempt were made to impose any one fixed sequence on teachers; happily there seems to be no immediate prospect of any such proposal being accepted.

What was the Mathematical Association doing during this period? Under the name of the A.I.G.T., and afterwards as the M.A., it did much valuable work; it did all that was possible to alleviate the situation. But it must have been disheartening work, for no fundamental cure was feasible until the Universities released us from bondage. On the eve of our liberation the M.A. published a report on Geometry teaching; a very conservative report, as it was considered impracticable to secure the abolition of the sequence. This report became obsolete in 1902, and no report on Geometry has emanated from your Teaching Committee since that date. This may seem strange, but our inaction has not been due to slothfulness. We have felt that during the period of rapid change it was undesirable that the Association should attempt to crystallise teaching. Until matters have settled down, it is for individuals to urge new ideas; an Association cannot easily do more than register the average opinion of sound teachers; and at the present moment a report on these lines would delay rather than advance the movement of reform. I do not consider that the time has yet come for the Association to support with its authority any particular method of teaching Geometry.

The next period is that between 1903 and 1909. What was the immediate effect of the Cambridge decree? Cambridge gave liberty of sequence, and at the same time published a list of propositions, explicitly not in order of treatment; other Universities have taken similar action.

The effect seemed to us at the time to be very great; but I suppose that as a matter of fact a great majority of schools went on teaching in the old way, and no doubt many to this day have been quite unaffected by the new spirit. But in those schools which moved with the times the immediate effect was probably rather chaotic. This was quite natural and inevitable; if the former period may be compared to an ice-age, this was the break-up of the ice. We did not at once realise how to make the best use of our liberty.

One mistaken development was a great abuse of drawing and measurement. These methods have their proper and permanent place in the teaching of Geometry, but many of us did not understand at the outset their proper limitations. These limitations have been explained very clearly by Mr. Carson, to whom teachers of mathematics owe a great debt. He has insisted on the *role* of intuition in learning Geometry. Some geometrical properties are intuitively obvious to children; for instance it is obvious that vertically opposite angles are equal. Probably these intuitive perceptions are the result of numberless unconscious experiments of everyday life; they are lying in the mind, more or less hidden, ready to be brought into the light and expressed in clear language. It is a mistake to attack such properties as these by drawing and measurement; the result may possibly be a weakening of geometrical intuition; it certainly will be a waste of time, and will lead to boredom.

On the other hand, there are many other properties that are not obvious to intuition; consider the property connected with the product of the segments of chords of a circle through a fixed point. It is inappropriate to *approach* such properties by a purely logical method; they must be proved logically as soon as they are believed to be true, or at any rate likely; but it is not superfluous to secure assent by a preliminary experiment.

After we had fought our way for some years towards reconstruction, we received Circular 711 from the Board of Education, and the third period of which I speak is that dating from this publication to the present time. You are familiar with the proposals of Circular 711. Briefly is it suggested that we should recognise three stages in our teaching.

Stage 1. To be concerned with the fundamental concepts and generally entitled Introductory Practical Work. This stage will not take up very much time; possibly it will not take up more than a term.

Stage 2. To deal by intuitive methods with a few fundamental propositions. These propositions are the groups concerned with angles at a point, parallel lines, congruence of triangles, angles of triangle and polygon. The "angle at a point" theorems are too obvious to a boy of 12 for him to feel the need of a formal proof. The "parallel lines" propositions involve us in grave controversy as to sequence; by appealing to intuition we outflank these troubles. The congruence theorems are proved mainly by superposition. It is difficult for a young boy to understand what he is getting at. To his mind, the triangles fit clearly enough, and he does not see that he is proving anything substantial. In fact, is he proving anything? Hilbert found it necessary to assume Euclid 1.4 as an axiom, or at any rate to assume a part of the proposition; why should a boy of 13 be expected to prove what Hilbert failed to prove? We are to substitute for the group of congruence propositions a discussion of the data needed for the determination of a triangle; in this the boy finds something substantial and real.

The "angle-sum" theorems are not quite on the same footing; they are quite good propositions for a boy to prove; the general argument for taking them without formal proof is that they are involved, together with theorems of parallels, in the sequence difficulty. The proposals of 711 in this particular have not found universal acceptance, but after all this is a comparatively unimportant detail.

Stage 3 was to include the main part of the subject, the general deductive development; and it is with Stage 3 that I have to deal in the proposals I shall lay before you to-day.

On the whole, it may be said that the plan advocated by Circular 711 has been widely accepted in this country. This acceptance depends to an important degree on the assent of the examining bodies, especially as regards examinations for boys of 13 or 14. I believe that the starred propositions are omitted from the schedules governing the Common Entrance Examination

and the examination for Naval Cadetships. The same proposals are sanctioned in the mathematical curriculum drawn up by a joint committee of the H.M. Conference and the Association of Preparatory Schools; the syllabus covers the ages of 9-16. They are also accepted in the regulations of the Oxford Local Examinations, and it is to be expected that in the course of the next year or so the assent will be obtained of all the chief examining bodies whose activities affect boys of public and secondary schools; perhaps London University will lag behind. The Board of Education—or rather the Schools Examinations Council, a body representing the Board in this matter—has recently conducted an investigation into the conduct of the seven examining bodies awarding School Certificates in England; the report of the mathematical investigators concurs with the main proposals of Circular 711.

This question then is approaching solution, and it now remains to discuss the next step. You may say, why is a further step necessary? What remains to be reformed? My quarrel is with Stage 3 of the circular. I must for a moment ask you to consider the two separate subjects of logic and geometry. Geometry has traditionally been represented in a dress of formal logic; or perhaps we may describe logic as the medicine and geometry as the jam. I have not studied logic, but I suspect that the particular form we meet with in geometry by no means exhausts the subject. Still, I am not prepared to say that we ought not to study this sort of logic; what I want to ask you is, at what age ought we to study it? Stage 3 will naturally be entered at about the age of 13; is there any natural break in the development of a child's mind which justifies a sudden step from the quite informal methods of Stage 2 to severely formal methods in Stage 3? I say that a boy of 13 is not ripe for the full rigour of the game: I am sure that he is not ready for it before the age of 15, and I expect that even 15 is too early. I should like to recast the work of Stage 3 and to relegate the strict Euclidean method to a Stage 4, which should certainly not be taken up before 15 or 16. The natural place for Stage 4 would be after the School Certificate age, *i.e.* after 16; but until the examining bodies have been converted—that is the next campaign—it may be necessary to attack Stage 4 a year or so before the School Certificate is taken.

The heart of my discourse is my new Stage 3. Now I want to be free at this stage to adopt whatever methods seem appropriate in each instance. There are many propositions which can perfectly well be dealt with by Euclidean methods in this stage; an instance is the group of propositions dealing with the angle properties of a circle. The things proved are striking and interesting; the boy feels that they are worth proving; there are no philosophical subtleties in the proofs. On the other hand, there are properties of such a character that the boy does not at this age appreciate the need of formal treatment. Of these I instance two classes: properties connected with Symmetry and properties connected with Similarity.

If any one will take the trouble to analyse the exercises in an ordinary text-book, he will find that a very high percentage require the proof of the congruence of two triangles. *And practically all of these exercises are proofs of properties depending on nothing but symmetry.* Now, in the first place, why do we not recognise explicitly in our teaching the existence of a fundamental geometrical fact like symmetry? As far as I know Euclid nowhere speaks of symmetry. Secondly, are not all these symmetry properties obvious to the boy? and is it not very tedious for him to spend all this time in proving the obvious? There comes a time in the development of the mathematician when he delights in proving, or disproving, the obvious; that time is certainly not before the age of 15. If you interrogate any of your non-mathematical friends, they will tell you that this continual proving of the obvious was one of the unpleasant features of their mathematical work at school; probably they gave up the study of mathematics some time before they left school, and this feature of the work was one of the causes that cooled their affection.

I would abolish all these congruence exercises. I would present to boys once and for all the two types of plane symmetry—Symmetry about a line and Symmetry about a centre—and after that I would use the principle of symmetry fearlessly. You will not find more than one standard proposition in which symmetry cannot be used to replace congruence; that proposition is the theorem of Pythagoras, and this can be proved much more shortly by similarity.

Traditionally, similarity comes rather late in the book, and the reason for this is historical; Euclid could not deal with similarity to his satisfaction until he had developed the theory of incommensurables in Book 5. We do not teach the theory of incommensurables, and there is every reason for putting similarity quite early in the course. It is as easy and patent a fact as symmetry. A boy is appreciating the existence of similar figures whenever he makes a drawing from nature, whenever he takes a photograph, whenever he goes to a cinema. Similarity must come in quite early; but for this purpose it must be presented as a matter for intuition. There are three fundamental theorems connected with similar triangles, analogous to the three fundamental theorems of congruence. There can be no question of teaching at the age of 13 or 14 these fundamental theorems of similarity; their formal proofs must be postponed to Stage 4; but the facts involved can be made clear by intuitive methods.

Stage 3, then, is to cover the whole of elementary Plane Geometry (and some Solid Geometry, I hope) by eclectic methods—sometimes a formal proof, sometimes an appeal to intuition. The amended course may now be summarised—and I am persuaded that it is a natural and inevitable development of Circular 711, and by no means an antagonistic proposal.

Stage 1. Introductory practical work.

Stage 2. Presentation by intuitive methods of the fundamental propositions.

Stage 3. Plane and Solid Geometry, with free choice of method, sometimes informal and sometimes formal: what modern language teachers call *méthode mixte*.

Stage 4. Recapitulation of Plane Geometry, the attention being directed to a formal chain of propositions and the interest being logic rather than geometry.

You will notice that I am proposing to convert a 3-speed gear into a 4-speed gear: in practice the change from the second to the third speed in the old motor has been found too abrupt.

I will not apologise for discussing before an audience of practical teachers the bearing of examinations on this new method. The only examinations that restrict a school are external examinations. The external examinations that concern preparatory and public schools are the Common Entrance Examination and the School Certificate Examination. The managers of the C.E.E. are a progressive body, and it should not be difficult to convert them to a policy that is so unquestionably sound for boys of 13. The seven university bodies that grant School Certificates cannot be expected immediately to concur with new methods of teaching: naturally they follow rather than lead. For some years they will continue to examine on familiar lines; how will the course of teaching I have described fit in with these examinations? Clearly it will fit admirably; Stage 4 will be an excellent preparation for the formal part of these examinations. But I have not concealed my opinion that, ideally, Stage 4 should come after, not before, the School Certificate. As a matter of fact, I believe that a boy who has mastered my third stage and postponed Stage 4 has nothing to fear from the School Certificate ordeal. He may fail to write out some of the theorems set: but what he may lose on the swings he will gain on the roundabouts; no paper consists solely of

theorems, and no boy has to do the whole paper in order to pass with credit. I do not think that external examinations need deter an enterprising teacher from teaching as he thinks best; at any rate, his teaching will be alive, and the effect on his pupils' work will be unmistakable.

In the course of the subsequent discussion the following observations were made by Prof. T. P. NUNN.

Some who are present this morning will remember that at the corresponding meeting held in this room three years ago I gave an address on the subject—"The Curriculum in Geometry"—upon which we have just heard so interesting and valuable a paper. What I then said was not accessible to Mr. Godfrey, for I was remiss enough not to print even an abstract of my argument. It is, therefore, with all the more pleasure that I note a very close congruence between the recommendations we offer.

Like Mr. Godfrey, I divided the curriculum into four phases: (1) A preliminary stage, in which the learner acquires, under guidance, his fundamental spatial experiences and ideas. (2) An "intuitional" stage, in which he reduces those experiences and ideas to clearness, and is made aware of some of their more important connections and consequences. I referred to this stage comprehensively as "boy scout geometry," and again, like Mr. Godfrey, gave an important place in it to the principle of similarity. (3) A stage in which the essential theorems of geometry are derived deductively from a wide basis of assumptions by a free use of the principles of congruence, similarity, parallelism and symmetry. (Also, I should have added, of the principle of projection, which, as I showed by practical illustrations, could very profitably be used to enrich the scope of elementary geometry by inclusion of the salient properties of the conic sections.) (4) A stage of "strict" geometry, based upon a minimum number of carefully examined postulates.

Like Mr. Godfrey, I urged that Stage 4 should be postponed until after the age of 16 (*i.e.* until after the General School Examination), and that it should be made more rigorous and philosophical than would be suitable for the non-specialist. Mr. Godfrey seems to contemplate that arguments based on symmetry should not be reduced to written "propositions." Since my proposals imply that General School and Matriculation Examinations in geometry should be based on Stage 3, I assumed that our pupils *would* be required to reduce their arguments to written form. I see no difficulty here, for there are excellent models in several standard French geometries. I agreed with Mr. Godfrey in discarding from the earlier stages both the Euclidean proofs of the fundamental propositions with regard to congruence and similarity, and their modern substitutes. On the other hand, I suggested that there were proofs of these propositions which are quite appropriate to Stage 3, and are in a line with the modern rigorous treatment—more rigorous than Euclid's—to be aimed at in Stage 4. I may add that I have since developed the treatment I proposed for the conditions of similarity, and have worked it up into what seems to be a satisfactory shape. I also urged that tri-dimensional geometry should, from the outset and all through the course, receive a greatly more important place than at present. I contended that the principles of congruence, symmetry and similarity should be taught by the study of tri-dimensional as well as plane instances; the principle of symmetry, in particular, being introduced by means of models of right and left-handed houses, etc.

As far as we differ, it is mainly in matters of detail, not of principle. I conclude by re-emphasising my cordial agreement with the general tenour of Mr. Godfrey's proposals, and by expressing the hope that the Association will shortly take steps to get them incorporated in the practice of the schools and the syllabuses of public examinations.