I. A new determination of e, N, and related constants

R.A. Millikan

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I. *A new Determination of e, N, and Related Constants.*
By R. A. MILLIKAN *.

1. *Introductory.*

THE only preceding determination of e and N for which a high degree of precision has been claimed was completed in 1912 and published in full in 1913 †. This determination yielded, for the electronic charge and the number of molecules in a gram-molecule, the following values :—

$$e = 4.774 \times 10^{-10} + .0095 \text{ electrostatic unit,}$$

$$N = 6.062 \times 10^{23} + .012.$$

Although these values, as well as the method by which they were obtained, received quite general recognition, it soon became imperative to reopen the problem and to attempt to make a new and, if possible, more convincing determination.

For, first, Professor Ehrenhaft and his pupils began publishing in 1914 ‡ a series of results which, though obtained by a modification of my method, were wholly irreconcilable with the results which I had found. I wished, therefore, to see whether I could check their conclusions and find conditions under which my method failed.

Secondly, there developed a tendency, especially among

* Communicated by the Author.

† Physical Review, ii. pp. 109–143 (1913).

‡ *Ann. der Physik*, xlv. p. 657 (1914), xlv. p. 261 (1915); also *Phys. Zeit.* xvi. p. 10 (1915).

British physicists, to regard the value of e given above as somewhat too high, a value being commonly adopted which was about 2 per cent. lower. As this was much greater than the necessary error in the method, I was anxious to see by entirely new work whether a numerical error could have crept into the former determination.

Finally, the constant e has recently taken on added importance, since not only does it now carry with it, as formerly, the knowledge of the most important molecular, atomic, and radioactive magnitudes (such as the exact number of molecules in a given weight of any substance, the absolute weight of any atom or molecule, etc.), but all of the most significant of the radiation constants as well (such as Planck's h , the Stefan-Boltzmann constant σ , the Wien constant C_2 , all the X-ray constants, *i. e.*, the wave-lengths of characteristic X-rays) have recently been found to depend for their most reliable evaluation* upon the value of e . Further, if electricity exists in nature only in exact multiples of e , then e is in a more complete sense than any other physical quantity a natural unit, having none of the arbitrariness about it which inheres in so-called absolute units like the centimetre, the gram, and the second. In a word, e is increasingly coming to be regarded both as the most fundamental of physical or chemical constants, and also as the one of most supreme importance for the solution of the practical numerical problems of modern physics. It seemed worth while, therefore, to drive the present method for its evaluation—a method which is certainly exceedingly exact if its validity is granted—to the utmost limits of its possible precision.

Accordingly, early in 1914, the work herewith reported was begun.

2. *The Method.*

For the sake of completeness, it may be stated again that the method consists in capturing electrons† on an oil-drop

* Physical Review, Millikan, vii. pp. 353-388 (1916), and Webster, vii. p. 607 (1916).

† The word electron is used with the meaning originally given to it by Dr. G. Johnstone Stoney, viz., "the natural unit of electricity." This use has been consistently followed by the most authoritative writers, like Sir J. J. Thomson, Sir E. Rutherford, O. W. Richardson, N. Campbell, etc., all of whom speak in recent books or articles of positive as well as negative electrons, though the mass associated with the former is never less than that of the hydrogen atom. When an electron is found associated with a mass but $\frac{1}{184}$ of that of the hydrogen atom it may be called "a free negative electron," or following Sir J. J. Thomson "a corpuscle."

situated between the plates of an air condenser between which a constant electrical field may be thrown on parallel to gravity. If v_1 is the constant speed of descent under gravity, and v_2 the speed of ascent under the influence of the constant field F , then it is found by experiment that when the charge on the drop is changed through the capture of electrons, or their loss through the direct incidence of X-rays or ultra-violet light, the series of speeds imparted to it by the field—namely, the series of values of $v_1 + v_2$ constitutes an exact arithmetical progression the greatest common divisor $(v_1 - v_2)_0$ of which is the value of the electron measured in terms of a velocity. This is reduced to apparent electrical units by means of the equation derived from Stokes's law,

$$e_1 = \frac{4}{3}\pi \left(\frac{9\eta}{2}\right)^{\frac{2}{3}} \left(\frac{1}{g(\sigma - \rho)}\right)^{\frac{1}{3}} \frac{(v_1 + v_2)_0 v_1^{\frac{2}{3}}}{F}, \quad \dots \quad (1)$$

in which η is the coefficient of viscosity of air, σ the density of the particle, and ρ that of the air. The radius, a , of the drop is then found to a sufficiently close approximation by inserting an approximate value of e in the equation

$$a = \sqrt[3]{\frac{3Fe}{4\pi g(\sigma - \rho)}} \frac{v_1}{(v_1 + v_2)_0}, \quad \dots \quad (2)$$

this equation being derived from

$$\frac{v_1}{v_2} = \frac{mg}{Fe - mg} \quad \text{or} \quad e = \frac{mg}{Fv_1} (v_1 + v_2)_0$$

$$\text{and} \quad m = \frac{4}{3}\pi a^3 (\sigma - \rho).$$

Such observations are made on a considerable number of drops at various pressures or on the same drop at different pressures, and, for the sake of obtaining a linear relation, the values of $e_1^{\frac{2}{3}}$ are obtained and then plotted against the corresponding values of $\frac{1}{pa}$. This procedure amounts to adding a first-order correction term to Stokes's law, and writing it in the form

$$v_1 = \frac{2}{9} \frac{ga^2}{\eta} (\sigma - \rho) \left(1 + \frac{b}{pa}\right), \quad \dots \quad (3)$$

The relation between e_1 and e then takes the form

$$e^{\frac{2}{3}} \left(1 + \frac{b}{pa}\right) = e_1^{\frac{2}{3}}. \quad \dots \quad (4)$$

From this equation * it is seen at once that the intercept of the $e_1^{\frac{2}{3}} \frac{1}{pa}$ line on the $e_1^{\frac{2}{3}}$ axis is the value of $e^{\frac{2}{3}}$, and that the slope of this line divided by the above-mentioned intercept is the constant b , the significance of which has been pointed out before † and will be more fully discussed in a following paper.

3. The Apparatus.

The apparatus is new throughout, and every constant involved in this method of determining e has been re-evaluated with the aid of improved and refined methods. The old air-condenser ‡ had consisted of ribbed brass plates, tested merely mechanically and found flat to about .01 mm. They were held apart by ebonite posts 16 mm. long. These posts were found to change in length slightly through the absorption of oil. The new plates M and N (fig. 1) were made optically flat by polishing and then testing them with the aid of mercury fringes against a standard optical test-plate. They were nowhere in error by more than two wave-lengths of green mercury light. They were 22 cm. in diameter and were separated by three small pieces of echelon plates about 1 cm. square and 14.9174 mm. thick, placed at points 60° apart about the circumference. These echelon plates, of course, had optically perfect plane-parallel surfaces. The dimensions of the condenser, therefore, no longer introduced an error of more than about 1 part in 10,000 instead of about 1 part in 1000 as in the previous work.

The oil droplets from the atomizer A, blown by a puff of air through r (fig. 1) entered the condenser MN through five minute holes $\frac{1}{4}$ mm. in diameter in the middle of the upper plate, and were observed as in the former work by means of light from the arc, a , filtered through a trough of water, w , and one of cupric chloride, d , for the removal of heat rays. The temperature was held constant to within one or two hundredths of a degree and very close to 23° C. by the oil-bath, G. The charge on the drop, p , was changed by X-rays from the bulb X passing through the window, g . The

* Equations (3) and (4) are perhaps more easily visualizable if the correction term $\left(1 + \frac{b}{pa}\right)$ is written in the form $\left(1 + A \frac{l}{a}\right)$, in which l is the mean free path of the gas molecule. Since the exact value of l is uncertain, I have for simplicity chosen to compute it uniformly from $\eta = .3502nm\bar{c}l$. This gives at 23° 76 cm., $l_{\text{air}} = .000009417$ cm.

† Millikan, Phys. Rev. xxxii. p. 381 (1911).

‡ Millikan, Phys. Rev. ii. p. 122 (1913).

pressure, held absolutely constant during an experiment, was varied from 13 cm. to 76 cm. and was measured to a tenth mm. by the manometer m . The atomizer, A , fed from s with the highest grade of watch-oil, density at 23°C .

Fig. 1.

redetermined as '9199, was blown with carefully dried and cleaned air, let in through the cock r , the bulbs below A being to catch excess oil. The observing optical system was a specially constructed telescope of 30 mm. objective and a magnification of 25 diameters. The objective was about

25 cm. distant from the drop, which was brought into focus by advancing or withdrawing the whole telescope system by means of a nut and screw. The distance through which the drops were timed was 1.0220 cm. It was measured precisely as in the 1912 work, and could be duplicated in successive readings to one part in a thousand. The mean value obtained from ten readings varied from 1.0218 to 1.0223—a maximum difference of one part in two thousand. This factor in the determination of e shares with the coefficient of viscosity of air in introducing the largest uncertainty into the final result.

The velocities of the drops both under gravity and in the field were measured with a most convenient and reliable printing chronograph made by William Gaertner & Co., of 5545 Lake Avenue, Chicago, and kindly loaned to the Laboratory for this determination. This instrument is controlled by a standard astronomical clock, and prints on a tape the hour, minute, second, and hundredth of a second at which the key is pressed as the drop crosses the cross-hair in the eyepiece—the maximum error, so far as the recording mechanism is concerned, being never more than a hundredth of a second. Some slight errors were found in the calibration of the Hipp chronoscope used in the previous determination; but with the Gaertner Printing Chronograph the uncertainty in the time-measurements was reduced to a wholly inappreciable amount.

The electrical field strengths were determined for each drop with the aid of a 750-volt Weston Laboratory Standard voltmeter, and contain no uncertainty larger than 1 part in 3000. For this voltmeter was repeatedly calibrated in the midst of the observations against three different standard Weston cells, with results which never differed by as much as the limit indicated. The volts are then actually measured in terms of a Standard Weston cell, the above limit being merely the limit of certainty in reading the pointer in the part of the scale used. All the other elements of the problem were looked to with a care which was the outgrowth of six years of experience with measurements of this kind. This work was concluded in August 1916, and occupied the better part of two and a half years of time.

4. *The Validity of the Method.*

That portion of the investigation which has had to do with the testing of the general validity of the method and the endeavour to discover the causes of the disagreement

between the results of Professor Ehrenhaft and his pupils and those obtained at the Ryerson Laboratory has been reported in detail elsewhere*. It is sufficient here to say that, although we have worked extensively with droplets of substances other than oil, particularly with mercury, upon which the irregularities are chiefly found in the Vienna work, and with particles of the same order of magnitude as those there used, we have found no indications whatever that the method, when properly used, even remotely suggests the existence of charges which are not equal to or multiples of the electron. We have studied thousands of drops of many different substances in a number of different gases, and have never found one whose charge did not fit into an arithmetic series whose greatest common divisor was the electron. We have definitely disproved Ehrenhaft's contention that this greatest common divisor is a function of the radius of the drop. Further evidence of this independence is given herewith. We have also discovered what seem to us wholly adequate reasons for the irregularities observed by Professor Ehrenhaft.

5. *The Precision of the Method.*

The consistency of the results on different drops is sufficient guarantee of the precision of the method, provided no constant error inheres in the measurement of the dimensions of the condenser, the volts, the time, or the viscosity of air, and provided the speed with which a given drop moves through the gas is strictly proportioned to the force acting upon it, as it is assumed to be. This last point was very carefully studied in the 1912 work, and considerable time has been given to studying it again. Incidentally, since for convenience most of the preceding work was done on drops charged with electrons of one sign only, and since it was thought conceivable that the electron of opposite sign might have a slightly different value and thus account for the discrepancies between different methods of getting e , exact demonstration has here been made that the sign of the charge is wholly immaterial. This means simply that an un-ionized gaseous molecule possesses no residual charge of either sign which is at all comparable with the electron. Some evidence upon these points is contained in Table I. (p. 8), which records experiments in which a given drop was alternately loaded with positive and negative electrons. It will be seen, not only that

* Phys. Rev. Dec. 1916, "The Existence of a Subelectron?"

TABLE I.

Sign of drop.	t_g sec.	t_F sec.	n .	e .	
—	63.118		8	$e_1=6.713$	
	63.050				
	63.186	41.728			
	63.332	41.590			
	62.328		11		
	62.728	25.740			
	62.926	25.798			
	62.900	25.510			
63.214	25.806				
Mean=62.976					
+	63.538	22.694	12		$e_1=6.692$
	63.244	22.830			
	63.114	25.870			
	63.242	25.876	11		
	63.362	25.484			
	63.136	10.830			
	63.226	10.682	22		
	63.764	10.756			
	63.280	10.778			
	63.530	10.672			
	63.268	10.646			
	Mean=63.325				
+	63.642		6	$e_1=6.702$	
	63.020	71.664			
	62.820	71.248			
	63.514	52.668	7		
	63.312	52.800			
	63.776	52.496			
	63.300	52.860	6		
	63.156	71.708			
	63.126				
	Mean=63.407				
—	63.228	42.006	8	$e_1=6.686$	
	63.294	41.920			
	63.184	42.108			
	63.260	53.210			
	63.478	52.922	7		
	63.074	53.034			
	63.306	53.438			
	63.414	12.888			
	63.450	12.812	19		
	63.446	12.748			
	63.556	12.824			
	Mean=63.335				

Duration of experiment 1 hr. 40 min.

Initial volts = 1723.5.

Final volts = 1702.1.

Pressure = 53.48 cm.

Mean $e_1 = 6.697$ $e_1 = 6.700$

the mean values of the positive and negative units are the same within 3 parts in 6700, but that the time t_g under gravity is quite the same when the drop contains 22 electrons as when it contains but 6 (see column n); further, that the value of e , computed by (1) from the speeds when the drop carries from 11 to 22 electrons is the same as that found when it carries 6 or 7 electrons. These numbers show, first, that the speed produced by a given field is an exact measure of the charge; second, that the speed under gravity and hence the apparent resistance of the medium to the motion of the drop through it is independent of the charge; and, third, that the un-ionized atom is strictly neutral, or that the positive and negative electrons are alike in charge.

6. The Coefficient of Viscosity of Air.

The only remaining element of uncertainty is in the coefficient of viscosity of air. In 1913, in view of measurements made in this laboratory upon this constant, in addition to measurements made elsewhere, I published* as the most probable value at 23° C. $\eta = .0001824$, and estimated that this could not contain an uncertainty of more than .1 per cent. The correctness of this estimate was questioned by Vogel † and Gille ‡, who, while reducing the value obtained by the Halle observers by one per cent., still retained a value which was half a per cent. higher than that which I had adopted. It is to be pointed out that if this result were correct, my value of e instead of coming down would go up by three-fourths per cent., but in any case it was obviously necessary to institute further tests as to the correct value. These tests were carried out most thoroughly by Dr. E. L. Harrington, who, using the constant deflexion apparatus designed by Dr. Gilchrist and the author, succeeded, by the introduction of improvements in condition and perfections in detail, in making a determination of η which is, I think, altogether unique in its reliability and precision. I give to it alone greater weight than to all the other work of the past fifty years in this field taken together. For the individual determinations, though made with different suspensions and in such a way as to eliminate all constant sources of error save the dimensions of the cylinders, never differ among themselves by as much as .1 per cent. and the error in the final mean can scarcely be more than one part in 2000. Indeed, the work has since been repeated by another

* *Annalen der Physik*, xli, p. 759 (1913).

† *Ibid.* xliii, p. 1235 (1914).

‡ *Ibid.* xlviii, p. 799 (1915).

observer, Mr. Stacy, and the result found dependable to within that limit of uncertainty. Dr. Harrington's* value is $\eta_{23} = .00018227$. This value is within less than a tenth per cent. of my 1913 value. The constants of the suspensions were determined by taking the periods *in vacuo*, and it is interesting that they differed from the period in air by as much as .2 per cent., because of the moment of inertia of the air which is dragged along with the rotating cylinder.

7. The Observations.

The results of the final series of observations on 25 consecutive drops are given in Table II. and fig. 2. The numbers at the top of the sheet in the figure represent the approximate times of fall under gravity of the drops opposite which they stand. They are inserted to show the reader at a glance that the value of the slope and of the intercept on the $e_1^{\frac{2}{3}}$ axis, that is, the value of the electron, is not in any way a function of the radius of the drop. One can get this slope by comparing only relatively large drops at different pressures (for example, drops falling in from 14 to 20 seconds), or quite small ones (such as those falling in from 44 to 57 seconds), or by comparing drops of different sizes at the same pressure. The starred drops were those taken when the conditions of observation were considered as perfect as possible. No attempt was made to take observations on drops which fell through the fixed cross-hair distance of 1.0220 cm. in less than 14 seconds, since it was desired to keep the timing errors negligible. The value of $e^{\frac{2}{3}}$ —namely, the intercept on the $e_1^{\frac{2}{3}}$ axis—was taken from the graph, as was also the slope divided by the intercept, which is the value of b in equation (4). The values thus found were

$$e^{\frac{2}{3}} = 61.13 \times 10^{-8},$$

$$b = .000618,$$

p being measured in centimetres of mercury at 23° C. and a in centimetres. The value of A (see foregoing footnote) corresponding to this value of b is .864, instead of .874 as found in 1913. The difference is due, I think, to small errors which were then made in the calibration of both the Hipp chronoscope and the voltmeter, which, however, compensated each other in their effect on e , though not in that on A . The numbers given in the last column of Table II. are the values of $e^{\frac{2}{3}}$ obtained algebraically from (4), and the

* Phys. Rev. Dec. 1916.

TABLE II.

No.	Temp. ° C.	P.D. (volts).	t_g (sec.).	v_i cm./sec.	n .	$a \times 10^5$ cm.	p (cm. Hg).	$\frac{1}{pa}$.	$\frac{l}{a}$.	$e_1^{2/3} \times 10^8$.	$e_2^{2/3} \times 10^8$.
1	23.07	6650	16.50	06194	7-13	23.40	74.49	57.45	0.4111	63.21	61.03
2	23.00	6100	16.76	06099	8-11	23.22	75.03	57.5	0.4115	63.204	61.03
3	23.05	5308	19.73	05180	7-15	21.34	74.49	63.0	0.4509	63.51	61.16
4	23.08	4132	37.82	02703	4-6	15.33	75.37	86.7	06205	64.27	60.97
5	23.06	4661	40.09	02521	3-6	14.84	75.00	90.6	06484	64.63	61.21
6	23.12	4111	51.53	01983	3-4	13.05	75.77	101.3	06502	65.02	61.19
7	23.08	5299	51.48	01985	2-5	13.05	74.98	102.4	07329	65.07	61.20
8	23.01	6661	56.06	01823	2-3	12.50	75.40	106.3	07608	65.13	61.11
9	23.00	6082	59.14	01728	1-4	12.17	75.04	109.7	07850	65.19	61.05
10	23.10	4077	57.46	01779	3-8	12.34	75.67	107.3	07680	65.21	61.16
11	23.13	4663	16.58	06165	10-12	22.72	29.26	150.6	1.078	66.70	61.01
12	23.11	4661	29.18	03502	5-7	17.08	36.61	160.1	1.146	67.12	61.07
13	22.98	4687	18.81	05432	8-10	21.26	30.27	155.6	1.114	67.14	61.26
14	23.12	4651	47.65	02145	2-7	13.20	36.80	206.4	1.477	68.90	61.11
15	23.10	4648	32.72	03129	4-6	15.92	31.35	200.7	1.437	68.97	61.39
16	23.15	3393	18.34	05572	12-16	21.11	20.58	227.8	1.630	69.88	61.27
17	23.12	4669	46.82	02294	2-4	13.12	29.10	262.4	1.878	70.85	60.94
18	23.12	4691	26.62	03819	5-7	17.32	20.54	281.4	2.014	71.60	61.98
19	23.10	3389	14.10	07249	15-19	23.00	13.24	321.4	2.297	73.34	61.20
20	23.14	4682	39.24	02605	3-5	14.00	20.72	345.4	2.472	74.27	61.22
21	23.14	3350	18.30	05585	10-13	20.47	13.62	359.1	2.570	74.54	60.97
22	23.00	3370	43.88	02329	3-6	13.17	20.47	371.5	2.659	75.00	60.97
23	23.13	3381	46.90	02179	3-6	13.69	20.74	380.6	2.724	75.62	61.24
24	23.09	3345	19.65	05201	9-12	19.65	13.12	388.5	2.781	75.92	61.24
25	23.15	3344	26.76	03819	6-9	16.57	13.80	483.3	3.137	77.74	61.18

Mean 61.126

Fig. 2.

above value of b taken in connexion with each individual set of values of the observed quantities $\frac{1}{pa}$ and $e_1^{\frac{2}{3}}$. It will be seen that the final mean value of $e^{\frac{2}{3}}$ obtained by this method of analysis—a method which yields the most reliable value of $e^{\frac{2}{3}}$ obtainable from the data at hand—is

$$e^{\frac{2}{3}} = 61.126 \times 10^{-8}.$$

There is but one drop in the table which yields a value of $e^{\frac{2}{3}}$ differing from this by as much as one-third of one per cent., and the probable error of the mean computed by least squares is one part in 4000.

No such precision, however, is claimed for this determination of $e^{\frac{2}{3}}$. It is .07 per cent. higher than the value (61.086) which I published in 1913, both values being computed in terms of .0001824, as the value of the coefficient of viscosity of air. Dr. Harrington's new value of this constant, viz. .00018227, is, however, more reliable than the old one and is .07 per cent. lower than that; so that *the new value of e and N computed solely from the new data obtained in this redetermination is exactly the same as the value published in 1913.* The maximum uncertainty in this value was then estimated as one part in five hundred. This work has reduced it so that that it is no more, I think, than one part in a thousand; for it now contains but two factors which are uncertain by as much as one part in two thousand, namely, the coefficient of viscosity of air and the cross-hair distance. The exactness of the agreement to four places with the 1913 value is, of course, accidental. This is, however, the third time that with independent determinations (one unpublished) I have come out well within one-tenth per cent. of the foregoing result.

The result of this investigation may, then, be stated as follows:—

$$e = 4.774 \times 10^{-10}, \pm .005,$$

$$N = 6.062 \times 10^{23}, \pm .006,$$

the last number being obtained from

$$N \times 4.774 \times 10^{-10} = 9649.4 \times 2.9990 \times 10^{10}.$$

Since this is an attempt at a precise determination, and by far the most carefully carried out of any work which I have thus far done upon the evaluation of the electron, it is perhaps worth while to give more of the original data than would otherwise be justified, in order that others may better form their own estimates of its probable reliability.

Accordingly, the actual records of the observations on all of the 18 starred drops which were chiefly considered in determining the line in fig. 2 are given below. The other seven are omitted merely to save space. As the graph shows, they do not modify in any way the result. With their inclusion the graph becomes the record of 25 consecutive observations without any discards, so that the result is entirely free from the exercise of choice.

8. *The Values of some other Related Constants.*

I have already recorded in the 1913 paper the values of six fundamental but related constants which are at once known as soon as e is found, and with the same precision as that attained in its evaluation. These six are: (1) the electron, e ; (2) the Avogadro number, N ; (3) the number of molecules, n , in an ideal gas at 0° C. 76 cm.; (4) the kinetic energy, E_0 , of molecular agitation at 0° C.; (5) the constant change, ϵ , in molecular energy per degree; (6) the entropy constant, k , or the gas constant applied to a single molecule. A seventh constant which should have been included at that time is the mass m of a hydrogen atom, given by

$$m = \frac{4.774 \times 10^{-10} \times 1.0077}{9649.4 \times 2.999 \times 10^{10}} = 1.662 \times 10^{-24} \text{ gram.}$$

This list may now be extended as follows:—

The constant of the Balmer series of hydrogen is known with the great precision attained in all wave-length determinations, and has the value 3.290×10^{15} . From Bohr's theory it is given by

$$\frac{2\pi e^4 m}{h^3} \quad \text{or} \quad \frac{2\pi e^5}{h^3 \frac{e}{m}} \dots \dots \dots (5)$$

I have shown that h may be determined photoelectrically* with an error in the case of sodium of no more than $\frac{1}{2}$ per cent., the value given by my work on sodium being 6.56×10^{-27} . The value found by Webster† by the X-ray method discovered by Duane and Hunt‡ is 6.53×10^{-27} . Taking the mean of these two results, viz. 6.545×10^{-27} , obtained by wholly dissimilar methods, and substituting

* Phys. Rev. vii. p. 374 (1916).

† *Ibid.* vii. p. 599 (1916).

‡ *Ibid.* vi. p. 166 (1915).

in (1), after introducing Bucherer's value of $\frac{e}{m}$, viz. 1.767×10^7 , we obtain for the Rydberg constant, 3.294×10^{15} , which agrees within one-tenth per cent. with the observed value. This agreement constitutes most extraordinary justification of Bohr's equation, and warrants the use of spectroscopic data, combined with the foregoing data on e , for a most exact evaluation of h . The value of h computed thus from (5) with the aid of my value of e and the foregoing value of $\frac{e}{m}$, which is now known with a precision of one-tenth per cent., is

$$h = 6.547 \times 10^{-27} \pm 0.011.$$

It will be seen that the uncertainty is just $\frac{5}{3}$ the uncertainty in e , since e appears in (5) in a power $\frac{5}{3}$ that of h , while $\frac{e}{m}$ affects h by an amount which is negligible in comparison. The foregoing value of h may be considered the most reliable thus far obtainable, its uncertainty being one part in six hundred. It will be seen, too, that it agrees within just one part in five hundred with the value obtained for my sodium curves, which I estimated correct to only one part in two hundred.

Having thus fixed the value of h to one part in six hundred, we may obtain from Planck's equation the Wien constant, C_2 , with the same precision, for it will be recalled * that

$$C_2 = \frac{hc}{k} = \frac{6.547 \times 10^{-27} \times 2.999 \times 10^{10}}{1.372 \times 10^{-16}} \\ = 1.4312 \pm 0.0030 \text{ cm. degrees.}$$

The estimated error set down above is obtained from the assumption of an uncertainty of one part in six hundred for h and one part in one thousand for k . The latest experimental result on C_2 given out by the Reichsanstalt † is $C_2 = 14300$. Coblenz ‡ gives as the result of his direct experiments $C_2 = 1.4369$, while his combination of total radiation experiment and theory lead him to $C_2 = 1.4322$.

* Phys. Rev. ii. p. 142 (1913).

† Ann. Phys. xlviii. p. 430 (1915).

‡ Phys. Rev. vii. p. 694 (1916).

Again, from Planck's equation,

$$C_2 = \left(\frac{48\pi ak}{a} \right)^{\frac{1}{3}},$$

we can compute the Stefan-Boltzmann constant of total radiation a [or $\sigma = \frac{ac}{4}$] and obtain a result which is uncertain by but six-tenths per cent. The result is

$$\sigma = 5.72 \times 10^{-12} \pm .034 \text{ watt cm.}^{-2} \text{ deg.}^{-4}.$$

This is exactly the value found by Coblentz from his most recent and most thorough experimental work * on σ . The exceedingly close agreement between all these values of h , C_2 , and σ , computed on the one hand from the work on e , and directly observed on the other, is an indication of the exactness of the work on e .

The grating spacing in calcite computed † from the foregoing value of e is $3.030 \text{ \AA}$.

A summary of the most important constants the values of which are fixed by this determination of e is given below, with the uncertainty attaching to each:—

The electron	$e = 4.774 \pm .005 \times 10^{-10}$.
The Avogadro constant	$N = 6.062 \pm .006 \times 10^{23}$.
Number of gas molecules per c. c. at 0° C. 76 cm.	$n = 2.705 \pm .003 \times 10^{19}$.
Kinetic energy of translation of a molecule at 0° C.	$E_0 = 5.621 \pm .006 \times 10^{-14}$.
Change of translational molecular energy per degree C.	$\epsilon = 2.058 \pm .002 \times 10^{-16}$.
Mass of an atom of hydrogen in grams.	$m = 1.662 \pm .002 \times 10^{-24}$.
Planck's element of action	$h = 6.547 \pm .011 \times 10^{-27}$.
Wien's constant of spectral radiation..	$c_2 = 1.4312 \pm .0030$.
Stefan-Boltzmann constant of total radiation.....	$\sigma = 5.72 \pm .034$.
Grating spacing in calcite	$d = 3.030 \pm .001 \text{ \AA}$.

I have to express my hearty thanks to Dr. Yoshio Ishida. for invaluable aid both in observing and in computing the accompanying data.

Ryerson Physical Laboratory,
University of Chicago,
January 12th, 1917.

* Phys. Rev. vii. p. 694 (1916).

† Webster, Phys. Rev. vii. p. 607 (1916).

APPENDIX.—Observational Data.

t_g	t_F	n'	$\frac{1}{n'}\left(\frac{1}{t_F'} - \frac{1}{t_F}\right)$	n	$\frac{1}{n}\left(\frac{1}{t_g} + \frac{1}{t_F}\right)$	
DROP No. 1.						
16.56	97.96			8	.008852	$V_i = 6658$ volts.
16.46	97.99	1	.008831			$V_f = 6647$ volts.
16.46	52.35			9	.008849	$t = 23^{\circ}.07$ C.
16.48	52.70	1	.008816			$p = 74.49$ cm.
16.54	97.84			8	.008854	$v_1 = .061940 \frac{\text{cm.}}{\text{sec.}}$
16.58						$a = .0002340$ cm.
16.50						$\frac{1}{pa} = 57.45$
16.45	51.26	1		9	.008901	$\frac{l}{a} = .04111$
16.46	51.30	1	.009136	8	.008872	$e_1^{2/3} = 63.21$
16.46	96.48	1	.008948	9	.008880	
16.50	51.68	2	.008856			
16.44	624.39		.009016	7	.008886	
16.55	94.18	5	.008951	8	.008903	
16.52	18.01			13	.008921	
16.44	18.10	5	.008930			
16.63	93.27	1	.008924	8	.008916	
16.44	50.90			9	.008917	
16.46						
Mean 16.50			.008934		.008895	$e^{2/3} = 61.03$
DROP No. 2.						
16.75	32.45			11	.008225	$V_i = 6107$ volts.
16.72	32.49	3	.008211			$V_f = 6091$ volts.

t_g	t_F	n'	$\frac{1}{n'}\left(\frac{1}{t_F'} - \frac{1}{t_F}\right)$	n	$\frac{1}{n}\left(\frac{1}{t_g} + \frac{1}{t_F}\right)$	
DROP No. 2 (<i>cont.</i>).						
16.74	162.22	1	.008294	8	.008231	$t=23^{\circ}.00$ C.
16.77	69.16			9	.008267	$p=75.00$ cm.
16.77	69.54	1	.008163	10	.008222	$v_1=.061022 \frac{\text{cm.}}{\text{sec.}}$
16.66	44.35					$a=.0002322$ cm.
16.72	163.51	2	.008218	8	.008225	$\frac{1}{pa}=57.5$
16.76	69.93			9	.008220	$\frac{l}{a}=.041154$
16.74	162.59	1	.008184	8	.008229	$e_1^{2/3}=63.204$
16.78	69.71	1	.008194	9	.008225	
16.76	44.26	1	.008249	10	.008223	
16.74	44.26					
16.75	69.65	1	.008234	9	.008227	
16.92	69.63					
Mean 16.756			.008218		.008227	$e^{2/3}=61.03$
DROP No. 3.						
19.70	13.06	2	.007814	15	.007800	$V_i=5319$ volts.
19.65	15.12					$V_f=5303$ volts.
19.77	15.08			10	.007799	$t=23^{\circ}.05$ C.
19.72	36.80					$p=74.49$ cm.
19.70	36.53					$v_1=.051799 \frac{\text{cm.}}{\text{sec.}}$
19.66	36.54					$a=.0002134$ cm.
19.74	36.59					$\frac{1}{pa}=63.0$
19.72	85.45					$\frac{l}{a}=.04509$
19.77	85.11			8	.007801	$e_1^{2/3}=63.54$
19.70	51.07					

t_g	t_F	n'	$\frac{1}{n'}\left(\frac{1}{t_F'} - \frac{1}{t_F}\right)$	n	$\frac{1}{n}\left(\frac{1}{t_g} + \frac{1}{t_F}\right)$	
DROP No. 3 (cont.).						
19.69	51.01			9	.007812	
19.72	50.82					
19.75	50.92	1	.007807			
19.73	84.52					
19.72	83.53			8	.007827	
19.62	83.39	2	.007816			
19.75	36.38					
19.77	36.02			10	.007835	
19.79	36.16					
19.69	36.05	3	.007819			
19.70	233.47	1	.007845	7	.007852	
19.82	82.45	2	.007872	8	.007851	
19.76	35.88			10	.007856	
19.75	23.09			11	.007844	
19.76	228.15			7	.007866	
19.78						
Mean	19.73		.007829		.007831	$e^{2/3}=61.16$
DROP No. 5.						
39.95	23.77			6	.009957	$V_i=4665$ volts.
39.99	28.70	2	.009890			$V_f=4659$ volts.
39.87	66.43			4	.010015	$t=23^\circ.06$ C.
40.06	66.24					$p=75.00$ cm.
40.14	65.78	1	.010056			$v_1=.025213 \frac{\text{cm.}}{\text{sec.}}$
39.93	39.59					$a=.0001484$ cm.

t_g	t_F	n'	$\frac{1}{n'}\left(\frac{1}{t_{F'}} - \frac{1}{t_F}\right)$	n	$\frac{1}{n}\left(\frac{1}{t_g} + \frac{1}{t_F}\right)$	
DROP No. 5 (<i>cont.</i>).						
39.89	39.65	2	.010094	5	.010037	$\frac{1}{pa} = 90.6$ $\frac{l}{a} = .06484$ $e^{2/3}_1 = 64.63$
40.09	28.64			6	.010026	
40.19	28.16			4	.010067	
40.01	65.26					
39.99	39.26	1	.010148	5	.010083	
40.05	39.25			4	.010120	
40.22	64.67					
40.10	64.04			2	.009995	3
40.13	64.38					
40.51	182.40					
40.41						
Mean	40.09		.010037		.010056	$e^{2/3} = 61.21$
DROP No. 7.						
51.13	30.55	2	.013015	4	.013040	$V_t = 5306$ volts.
51.25	21.86			5	.013034	$V_f = 5295$ volts.
51.19	50.72	1	.012961	3	.013047	$t = 23.08$ C.
51.32	148.63	1	.013138	2	.013092	$p = 74.98$ cm.
51.53	147.46			3	.013120	$v_t = .019852 \frac{\text{cm.}}{\text{sec.}}$
51.69	50.29	1	.013150			$a = .0001305$ cm.
51.55	50.25			2	.013128	$\frac{1}{pa} = 102.4$
51.54	50.39	1	.013150			$\frac{l}{a} = .07329$
51.98	49.70			2	.013128	$e^{2/3}_1 = 65.07$
51.64	146.41	1				
Mean	51.48		.013066		.013084	$e^{2/3} = 61.20$

t_g	t_F	n'	$\frac{1}{n'}\left(\frac{1}{t_F'} - \frac{1}{t_F}\right)$	n	$\frac{1}{n}\left(\frac{1}{t_g} + \frac{1}{t_F}\right)$	
DROP No. 8.						
55.69	29.97					$V_i = 6669$ volts.
55.90	29.83			3	.017094	$V_f = 6657$ volts.
56.18	29.69					$t = 23^{\circ}.01$ C.
55.91	60.89	1	.017251	2	.017131	$p = 75.40$ cm.
56.12	29.76	1	.017179			$v_1 = .018230 \frac{\text{cm.}}{\text{sec.}}$
55.90	29.49					$a = .0001250$ cm.
56.29	29.51			3	.017211	$\frac{1}{pa} = 106.3$
56.26	59.49	1	.017072			$\frac{l}{a} = .07608$
56.33	59.44			2	.017327	$\frac{2}{3} = 65.13$
						$e_1^{2/3} = 65.13$
Mean 56.06			.017167		.017191	$e^{2/3} = 61.11$
DROP No. 9.						
58.92	21.23	2	.015934	4	.016002	$V_i = 6091$ volts.
58.92	65.52			2	.016071	$V_f = 6077$ volts.
59.78	65.75					$t = 23^{\circ}.00$ C.
59.04	31.85	1	.016222			$p = 75.04$ cm.
58.69	31.72			3	.016121	$v_1 = .01728 \frac{\text{cm.}}{\text{sec.}}$
58.36	20.98	1	.016202			$a = .0001217$ cm.
59.50	64.45	2	.016074	4	.016002	$\frac{1}{pa} = 109.7$
59.50	64.45			2	.016211	
59.34	$t_{g+F} = 1802.40$	1	.016071	1	.016351	
58.85	20.90	1	.016134			$\frac{l}{a} = .07850$
59.95				4	.016188	$e_1^{2/3} = 65.19$
Mean 59.14			.016107		.016151	$e^{2/3} = 61.05$

t_g	t_F	n'	$\frac{1}{n'} \left(\frac{1}{t_F'} - \frac{1}{t_F} \right)$	n	$\frac{1}{n} \left(\frac{1}{t_g} + \frac{1}{t_F} \right)$	
DROP No. 10.						
57·35	28·10	2	·010714	5	·010598	$V_i = 4091$ volts.
57·24	70·62			3	·010567	$V_f = 4071$ volts.
56·92	69·26	2	·010731			$t = 23^\circ 10$ C.
57·18	27·84					$p = 75·67$ cm.
57·20	27·97			5	·010624	$v_1 = .017787 \frac{\text{cm.}}{\text{sec.}}$
57·35	28·49					$a = .0001234$ cm.
57·05	27·71					
57·26	28·00	2	·010543			$\frac{1}{pa} = 107·3$
57·48	68·35			3	·010678	$\frac{l}{a} = .07680$
57·30	68·53					$e^{2/3} = 65·21$
57·76	68·20	5	·010661			
57·84	14·72			8	·010669	
57·18	21·32			6	·010781	
57·33	20·97					
57·74	27·67	2	·010557	5	·010709	
57·78	66·55			3	·010810	
57·80						
Mean	57·46		·010661		·010670	$e^{2/3} = 61·16$
DROP No. 11.						
16·43	74·69					$V_i = 4668$ volts.
16·50	74·31			11	·006701	$V_f = 4660$ volts.
16·50	74·22	1	·006869			$t = 23^\circ 13$ C.
16·51	49·19			12	·006721	$p = 29·26$ cm.
16·52	72·98	1	·006630			$v_1 = .061651 \frac{\text{cm.}}{\text{sec.}}$

t_g	t_F	n'	$\frac{1}{n'}\left(\frac{1}{t_{F'}} - \frac{1}{t_F}\right)$	n	$\frac{1}{n}\left(\frac{1}{t_g} + \frac{1}{t_F}\right)$	
DROP No. 11 (cont.).						
16.54	72.55			11	.006739	$a = .0002272$ cm.
16.56	72.35					$\frac{1}{pa} = 150.6$
16.58	71.85	1	.006625			$\frac{l}{a} = 1.078$
16.59	137.12	1	.006865	10	.006762	
16.55	70.58	1	.006898	11	.006772	$e_1^{2/3} = 66.70$
16.58	47.52			12	.006782	
16.59	47.43	2	.006734	10	.006792	
16.54	131.68	1	.006945			
16.63	68.78	1	.006810	11	.006811	
16.70	46.84	1	.006723	12	.006806	
16.58	68.37	1	.006799	11	.006813	
16.65	127.77	2	.006816	10	.006815	
16.75	46.60	1	.006692	12	.006815	
16.65	67.72			11	.006826	
16.59						
Mean 16.577			.006783		.006781	$e^{2/3} = 61.01$
DROP No. 12.						
29.00	34.59					$V_1 = 4668$ volts.
29.30	34.65					$V_f = 4660$ volts.
28.99	34.59			7	.009037	$t = 23^{\circ}.11$ C.
29.15	34.42					$p = 36.61$ cm.
29.18	34.50					$v_1 = .035023 \frac{\text{cm.}}{\text{sec.}}$
29.12	34.38	2	.009020			$a = .0001708$ cm.
29.19	90.53			5	.009075	$\frac{1}{pa} = 160.1$

	t_g	t_F	n'	$\frac{1}{n'}\left(\frac{1}{t_F} - \frac{1}{t_F'}\right)$	n	$\frac{1}{n}\left(\frac{1}{t_g} + \frac{1}{t_F}\right)$	
				DROP No. 12		<i>cont.</i>	
	29.35	89.61	2	.009071			$\frac{l}{a} = .1146$
	29.32	34.14			7	.009081	$e_1^{2/3} = 67.12$
	29.14	34.12	2	.009043	5	.009097	
	29.11	89.17	1	.009184	6	.009108	
	29.32	49.07	1	.009086	5	.009113	
	29.23	88.55					
	29.27						
Mean	29.18			.009081		.009085	$e^{2/3} = 61.07$
				DROP No. 13.			
	18.62	51.82					$V_i = 4703$ volts.
	18.67	51.52			10	.007263	$V_f = 4679$ volts.
	18.71	50.83					$t = 22^{\circ}.98$ C.
	18.79	51.23	1	.007209			$p = 30.27$ cm.
	18.68	81.23					$v_1 = .05432 \frac{\text{cm.}}{\text{sec.}}$
	18.62	80.34			9	.007291	$a = .0002126$ cm.
	18.81	80.29					$\frac{1}{pa} = 155.6$
	18.82	79.07	2	.007353	11	.007319	$\frac{l}{a} = .1114$
	18.71	36.56			10	.007325	$e_1^{2/3} = 67.14$
	18.84	49.90					
	18.82	49.66	2	.007325			
	18.87	182.28	1	.007427	8	.007330	
	18.87						
	18.80	77.44			9	.007347	

t_g	t_F	n'	$\frac{1}{n'}\left(\frac{1}{t_F} - \frac{1}{t_g}\right)$	n	$\frac{1}{n}\left(\frac{1}{t_g} + \frac{1}{t_F}\right)$	
DROD No. 13 (cont.).						
18.95	76.76	1	.007187	10	.007345	
18.84	49.47					
18.86	49.23					
18.81	49.15	1	.007212	9	.007364	
18.85	76.14	1	.007451			
18.89	175.98	1	.007333	8	.007355	
18.83	76.89	1	.007468	9	.007365	
18.87	48.82	1	.007352	10	.007352	
18.96						
Mean 18.813			.007325		.007331	$e^{2/3} = 61.26$
DROD No. 14.						
47.34		1	.012150	3	.011986	$V_i = 4660$ volts.
47.63				2	.011905	$V_f = 4447$ volts.
47.78	66.79			3	.012053	$t = 23^{\circ}.12$ C.
47.40	354.00	1	.012350	4	.012053	$p = 36.80$ cm.
47.57	65.91	1	.012053			$v_1 = .021448 \frac{\text{cm.}}{\text{sec.}}$
47.70	36.70	1	.012010	3	.012051	$a = .0001320$ cm.
47.38	36.75					$\frac{1}{pa} = 206.4$
47.90	65.72			7	.012006	$\frac{l}{a} = .1477$
47.79		4	.011959	4	.012083	$e^{2/3} = 68.90$
47.90	15.86	3	.011903			
47.55	36.57	1	.012181	3	.012050	
47.66	66.20					

t_g	t_F	n'	$\frac{1}{n'}\left(\frac{1}{t_F'} - \frac{1}{t_F}\right)$	n	$\frac{1}{n}\left(\frac{1}{t_g} + \frac{1}{t_F}\right)$	
DROP No. 14 (<i>cont.</i>).						
47.81	65.70					
47.70						
Mean 47.65			.012050		.012040	$e^{2.3} = 61.11$
DROP No. 15.						
32.50	34.73					$V_i = 46.56$ volts.
32.57	34.36			6	.009928	$V_f = 46.45$ volts.
32.75	34.57					$t = 23^\circ.10$ C.
32.71	34.49	1	.009859			$p = 31.35$ cm.
32.44	52.25					$v_1 = .03129 \frac{\text{cm.}}{\text{sec.}}$
32.78	51.85			5	.009994	$a = .0001592$ cm.
32.65	51.51					$\frac{1}{pa} = 200.7$
32.78	51.47					$\frac{l}{a} = .1437$
32.88	51.27	1	.010111			$e_1^{2/3} = 68.97$
32.84	33.71			6	.010032	
32.97	33.91	2	.010011			
32.50	104.92					
32.94	104.40			4	.010042	
Mean 32.72			.009994		.009999	$e^{2/3} = 61.39$
DROP No. 16.						
18.20	29.66					$V_i = 34.03$ volts.
18.23	29.76					$V_f = 33.89$ volts.
18.14	29.58			16	.005521	$t = 23^\circ.15$ C.
18.29	29.55					$p = 20.58$ cm.

t_g	t_F	n'	$\frac{1}{n'}\left(\frac{1}{t_F'} - \frac{1}{t_F}\right)$	n	$\frac{1}{n}\left(\frac{1}{t_g} + \frac{1}{t_F}\right)$	
DROP No. 16 (cont.).						
18.28	29.33	2	.005592	14	.005552	$v_1 = .05572 \frac{\text{cm.}}{\text{sec.}}$
18.30	43.55					$a = .0002111 \text{ cm.}$
18.42	42.98					$\frac{1}{pa} = 227.8$
18.38	42.95					$\frac{l}{a} = .16304$
18.32	42.90	2	.005452	12	.005586	$e_1^{2/3} = 69.88$
18.26	80.60					
18.48	80.74					
18.46	80.42					
18.38	79.55	2	.005600	14	.005587	
18.29	42.07					
18.48	42.35					
18.36	79.80					
18.32	79.32	3	.005541	12	.005583	
18.27	34.16					
18.48	34.26					
18.36						
18.45		3	.005556	15	.005584	
Mean 18.34			.005557		.005569	$e^{2/3} = 61.27$
DROP No. 17.						
46.60	63.00	1	.012461	3	.012439	$V_i = 4670 \text{ volts.}$
46.87	62.42					$V_f = 4669 \text{ volts.}$
46.79	35.11					$t = 23^\circ.12 \text{ C.}$
46.60	61.85		.012314	4	.012463	$p = 29.10 \text{ cm.}$

t_g	t_F	n'	$\frac{1}{n'}\left(\frac{1}{t_F'} - \frac{1}{t_F}\right)$	n	$\frac{1}{n}\left(\frac{1}{t_g} + \frac{1}{t_F}\right)$	
DROP No. 17 (<i>cont.</i>).						
46·81	62·04			3	·012513	$v_1 = .022940 \frac{\text{cm.}}{\text{sec.}}$ $a = .0001312 \text{ cm.}$ $\frac{1}{pa} = 262.4$ $\frac{l}{a} = 1.878$ $e_1^{2/3} = 70.85$
46·81	61·65	1	·012559			
46·85	273·08	1	·012540	2	·012516	
46·97	61·76			3	·012524	
46·72	61·68	1	·012402			
46·99	34·94					
46·95	35·13			4	·012494	
46·88	34·83					
46·84	34·92					
Mean	46·82		·012475		·012492	$e^{2/3} = 60.94$
DROP No. 19.						
14·01	89·54			16	·005136	$V_i = 3342 \text{ volts.}$ $V_f = 3327 \text{ volts.}$ $t = 23^\circ 10 \text{ C.}$ $p = 13.24 \text{ cm.}$ $v_1 = .072488 \frac{\text{cm.}}{\text{sec.}}$ $a = .0002300 \text{ cm.}$ $\frac{1}{pa} = 321.4$ $\frac{l}{a} = .2297$ $e_1^{2/3} = 73.34$
14·07	88·32	1	·005218			
14·06	60·46			17	·005146	
14·11	60·37	2	·005146			
14·11	31·27			20	·005149	
14·16	31·15	2	·005228			
14·10	60·90					
14·09	59·90			17	·005151	
14·16	59·78					
14·11	59·79	2	·004996			
14·11	146·35			15	·005184	
14·09						
14·14	86·20			16	·005157	

	t_g	t_F	n'	$\frac{1}{n'}\left(\frac{1}{t_F'} - \frac{1}{t_F}\right)$	n	$\frac{1}{n}\left(\frac{1}{t_g} + \frac{1}{t_F}\right)$	
				DROP No. 19 (cont.).			
	14.11	86.38	3	005127	19	.005153	
	14.12	37.22					
	14.08	36.94					
	13.94		2	.005153	17	.005153	
	14.05	60.01					
	14.18						
Mean	14.10			.005161		.005154	$e^{2/3}=61.20$
				DROP No. 22.			
	43.71	173.98	1	.009571	3	.009512	
	43.99	65.28			4	.009563	$V_i=3384$ volts.
	44.11	64.58					$V_f=3369$ volts.
	44.22	64.07	1	.009428	5	.009533	$t=23^{\circ}00$ C.
	44.27	64.74					$p=20.47$ cm.
	43.75	40.20			4	.009543	$v_1=.02329 \frac{\text{cm.}}{\text{sec.}}$
	44.02	65.01	1	.009493			$a=.0001317$ cm.
	44.19	40.13	1	.009537	5	.009542	$\frac{1}{pa}=371.5$
	43.95	65.12	1	.009563			$\frac{l}{a}=0.2659$
	43.68	65.76	1	.009593	3	.009484	$e_1^{2/3}=75.00$
	43.98	65.55					
	43.75	176.57			5	.009465	
	43.56	40.76	2	.009436			
	43.83	66.81	1	.009566	4	.009451	
	43.71	66.39	2	.009458			

30 *A new Determination of e, N, and Related Constants.*

	t_g	t_F	n'	$\frac{1}{n'}\left(\frac{1}{t_F} - \frac{1}{t_g}\right)$	n	$\frac{1}{n}\left(\frac{1}{t_g} + \frac{1}{t_F}\right)$	
DROP No. 22 (cont.).							
	43.61	29.43			6	.099461	
	43.55						
Mean	43.88			.009516		.009510	$e^{2/3} = 60.97$
DROP No. 25.							
	26.74						$V_i = 3347$ volts.
	26.59	31.09					$V_f = 3342$ volts.
	26.60				9	.007733	$t = 23^{\circ}.15$ C.
	26.84	31.06					$p = 13.80$ cm.
	26.66	30.94	3	.007706			$v_1 = .038191 \frac{\text{cm.}}{\text{sec.}}$
	26.85	108.66			6	.007768	$a = .0001657$ cm.
	26.70	107.86	1	.007829			$\frac{1}{pa} = 438.3$
	26.74	58.48					$\frac{l}{a} = .3137$
	26.65				7	.007784	$e^{2/3}_1 = 77.74$
	26.70	58.37					
	27.02	58.42	1	.007766			
	26.92	106.87			6	.007792	
	26.96	106.32	1	.007694			
	26.65	58.47	1	.007785	7	.007782	
	26.80	107.32					
	26.74	107.21			6	.007777	
	26.82	108.48	1	.007668			
	26.80	59.22			7	.007782	
	26.70						
Mean	26.76			.007742		.007774	$e^{2/3} = 61.18$