



V. On fluid motion relative to a rotating Earth

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Evidence obtained from the Wilson photographs also leads to the same idea, for it has been shown by Shimizu * that the observed number of ray tracks which break up into two branches near the end of the range is much greater than the number deduced from probability considerations based on our present theory of atomic structure.

It is noteworthy that this anomalous behaviour of the α particle occurs at low velocities, where practically no investigation of the scattering of α particles has been carried out on account of the experimental difficulties of dealing with slow α particles. It is at higher velocities, where the theory of scattering put forward by Sir Ernest Rutherford has been so fully verified by experiment, that the most of the theoretical straggling takes place, and this straggling has apparently been accounted for.

In conclusion I wish to express my best thanks to Professor Sir Ernest Rutherford for his kind interest and advice. I also wish to thank Mr. Crowe for the preparation of the radioactive sources.

V. *On Fluid Motion relative to a Rotating Earth.* By GEORGE GREEN, D.Sc., Lecturer in Natural Philosophy in the University of Glasgow †.

THE subject of this paper is at present one of considerable interest to meteorologists. Papers by the late Dr. Aitken and also by the late Lord Rayleigh on the dynamics of cyclones and anticyclones have been followed by more recent papers by Dr. Jeffreys, Sir Napier Shaw, and others. Very few actual solutions of the equations defining atmospheric motions have been obtained. In the late Lord Rayleigh's paper ‡ attention is drawn to certain general hydrodynamical principles relating to the properties of rotating fluid which can be applied to "assist our judgment when an exact analysis seems impracticable." The importance of the theorem regarding the circulation of the fluid in any closed circuit is clearly explained in its application to any actual fluid motion. In applying this theorem to fluid motion in the atmosphere, however, we must bear in mind that the motions with which we are concerned are not the actual motions of the particles in space but their motions relative to the Earth itself at each point of observation.

* Shimizu, Proc. Roy. Soc. xcix. p. 432 (1921).

† Communicated by the Author.

‡ Sc. Papers, vol. vi. p. 447.

One object of the present paper is to investigate the conditions under which the circulation theorem may be applied to atmospheric motions relative to the Earth's surface; or more generally to motions relative to any three rectangular axes which are themselves rotating about each other, with a fixed origin. In the later part of the paper one or two additional cases of motion of the atmosphere are discussed and the system of isobars corresponding to each motion determined.

In view of the problems to be considered, we shall begin by specifying the system of rotating axes most convenient in dealing with fluid motion in the neighbourhood of any point of reference O on the Earth's surface. The axis OZ is drawn upwards along the *apparent* vertical at O, and line OZ continued downwards meets the axis of the Earth at a point O' which is taken as origin of coordinates. Then axes O'X and O'Y are drawn parallel to horizontal lines through the reference point O in directions due East and due North respectively. In the most general case to be considered the reference point O may be in motion relative to the Earth's surface, and this involves also a motion of the origin O' if point O moves either North or South. But the motion of O' corresponding to any moderate motion of O is very small, and for our present purpose we may regard the origin O' as a fixed point, very near to the centre of the Earth. We shall denote by (x, y, z') the coordinates of any point referred to origin O', and by (x, y, z) the coordinates of the same point referred to parallel axes through O. This makes $z' = z + R$, where R represents approximately the radius of the Earth. The components of the velocity of any particle relative to the axes at any instant are represented by u, v, w , and the angular velocities of the axes themselves, that is, of each two axes about the third, are represented by $\omega_x, \omega_y, \omega_z$, respectively. We shall introduce the particular values of $\omega_x, \omega_y, \omega_z$ corresponding to a reference point O fixed in position on the Earth, or moving relative to the Earth, when we come to deal with special problems. Referred to the above system of axes, the equations of motion of any fluid particle take the form:—

$$\frac{Du}{Dt} + \theta_1 = -\frac{\partial V}{\partial x} - \frac{1}{\rho} \cdot \frac{\partial p}{\partial x}, \quad . \quad . \quad . \quad (1)$$

$$\frac{Dv}{Dt} + \theta_2 = -\frac{\partial V}{\partial y} - \frac{1}{\rho} \cdot \frac{\partial p}{\partial y}, \quad . \quad . \quad . \quad (2)$$

$$\frac{Dw}{Dt} + \theta_3 = -\frac{\partial V}{\partial z} - \frac{1}{\sigma} \cdot \frac{\partial p}{\partial z} \quad . \quad . \quad . \quad (3)$$

In these equations, $\theta_1, \theta_2, \theta_3$ represent the terms depending on the rotation of the axes themselves, being given by equations of the type

$$\theta_1 = -2\omega_z v + 2\omega_y w - \omega_x y + \omega_y z' + \omega_x \omega_y y + \omega_x \omega_z z' - (\omega_y^2 + \omega_z^2)x. \quad (4)$$

The function $V(x, y, z)$ represents the gravitational potential function. We have also

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y} + w \frac{\partial}{\partial z}. \quad (5)$$

The equation of continuity of the fluid is then

$$\frac{D\rho}{Dt} + \rho \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right) = 0. \quad (6)$$

In applying these equations we treat the atmosphere as a perfect gas in which viscosity may be neglected.

Circulation Theorem for Relative Motion.

Consider now the theorem relating to the relative circulation. We have

$$\frac{D}{Dt} (u dx + v dy + w dz) = \frac{Du}{Dt} dx + \frac{Dv}{Dt} dy + \frac{Dw}{Dt} dz + d\left(\frac{1}{2}q^2\right), \quad (7)$$

where $q^2 = u^2 + v^2 + w^2$, the square of the resultant relative velocity. By means of equations (1), (2), (3), the above equation may be rewritten in the form:

$$\begin{aligned} \frac{D}{Dt} (u dx + v dy + w dz) \\ = -(\theta_1 dx + \theta_2 dy + \theta_3 dz) - \frac{dp}{\rho} - dV + d\left(\frac{1}{2}q^2\right). \end{aligned} \quad (8)$$

We can now integrate each term of this equation along any curve within the fluid from any point A to any point B. This integration gives the result,

$$\begin{aligned} \frac{D}{Dt} \int_A^B (u dx + v dy + w dz) = - \int_A^B (\theta_1 dx + \theta_2 dy + \theta_3 dz) - \int_A^B \frac{dp}{\rho} \\ - V_B + V_A + \frac{1}{2}q_B^2 - \frac{1}{2}q_A^2; \end{aligned} \quad (9)$$

and, if the integrations are applied to a closed curve

beginning and ending at the point A, we obtain

$$\frac{D}{Dt} \int_S (u dx + v dy + w dz) = - \int_S (\theta_1 dx + \theta_2 dy + \theta_3 dz), \quad (10)$$

where the suffix S indicates that the integration is to be taken along a definite curve S. We have assumed in obtaining (10) from (9) that V is a single valued function of (x, y, z) , and that p is a function of ρ . It now appears that the rate of change of the relative circulation in any closed circuit which consists of the same fluid particles at all times is not zero unless, in addition to the above conditions, we have

$$\frac{\partial \theta_1}{\partial z} = \frac{\partial \theta_3}{\partial x}; \quad \frac{\partial \theta_2}{\partial x} = \frac{\partial \theta_1}{\partial y}; \quad \frac{\partial \theta_3}{\partial y} = \frac{\partial \theta_2}{\partial z}. \quad (11)$$

When these conditions are not fulfilled, the relative vorticity does not move with the fluid itself, and if a velocity potential exists for a certain portion of fluid at a given instant, a velocity potential will not exist for that portion of fluid at a later instant.

The first case of importance of the above conditions in relation to problems relating to the atmosphere is that in which the angular velocities $\omega_x, \omega_y, \omega_z$ of the axes are constants. In this case, the conditions given above take the form

$$\frac{1}{\omega_x} \delta u = \frac{1}{\omega_y} \delta v = \frac{1}{\omega_z} \delta w = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}, \quad (12)$$

where δ represents an operator defined by

$$\delta = \left(\omega_x \frac{\partial}{\partial x} + \omega_y \frac{\partial}{\partial y} + \omega_z \frac{\partial}{\partial z} \right). \quad (13)$$

These equations have a solution of the form

$$\frac{u}{\omega_x} = \frac{v}{\omega_y} = \frac{w}{\omega_z} = f(\omega_x x + \omega_y y + \omega_z z), \quad (14)$$

where f denotes any arbitrary function. If we draw an axis to coincide with the axis defined by the resultant of the three component rotations $\omega_x, \omega_y, \omega_z$, then $(\omega_x x + \omega_y y + \omega_z z)$ is equal to $\Omega R \cos \phi$, where Ω is the resultant of $(\omega_x, \omega_y, \omega_z)$ and R is the line joining the origin to the point x, y, z . That is, u, v, w are functions of p the perpendicular from the origin to a plane through the point (x, y, z) perpendicular to the axis of the resultant rotation.

When the fluid is incompressible, and when a compressible fluid is moving in such a way that $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$ is zero, a solution of a different type obtains. The solution in this case may be written in the form

$$\left. \begin{aligned} u &= f_1 \{ (\omega_y x - \omega_x y), (\omega_x x - \omega_x z) \}, \\ v &= f_2 \{ (\omega_y x - \omega_x y), (\omega_x x - \omega_x z) \}, \\ w &= f_3 \{ (\omega_y x - \omega_x y), (\omega_x x - \omega_x z) \}, \end{aligned} \right\} \quad (15)$$

where f_1, f_2, f_3 are arbitrary functions subject only to the condition $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$. This solution includes as a particular case any motion of rotation of the atmosphere as a solid about the axis of the Earth.

The solutions which we have above obtained make it clear that the fluid motions relative to rotating axes in which the relative circulation moves with the fluid belong to a very restricted type. A relative motion, for instance, similar to that taking place in a free vortex, does not fulfil the conditions required for permanence of the velocity potential, and therefore no steady motion of this type could take place in the atmosphere—as has been assumed to be the case.

The conditions which we have found to be necessary for the validity of the circulation theorem when the fluid motion is relative to rotating axes, may be obtained in a manner different from that employed above. Taking Ξ, H, Z to denote the components of angular velocity of a fluid element, and U, V, W to denote components of linear velocity of the element, each referred to fixed axes which coincide at instant t with the instantaneous positions of the moving axes, we may derive the conditions from the equations employed by von Helmholtz in his papers on vortex motion:—

$$\frac{D}{Dt} \Xi = \Xi \frac{\partial U}{\partial x} + H \frac{\partial U}{\partial y} + Z \frac{\partial U}{\partial z} - \Xi \left(\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} + \frac{\partial W}{\partial z} \right), \quad (16)$$

with two other similar equations. With ξ, η, ζ to represent the components of relative angular velocity of an element of fluid, referred to the moving axes, we have,

$$\Xi = \xi + \omega_x; \quad H = \eta + \omega_y; \quad Z = \zeta + \omega_z;$$

$$U = u - \omega_x y + \omega_y z; \quad V = v - \omega_x z + \omega_x x; \quad W = w - \omega_y x + \omega_x y;$$

and
$$\frac{D}{Dt} \Xi = \frac{D\xi}{Dt} - \omega_z \eta + \omega_y \zeta.$$

By means of these relations we can readily transform (16) and obtain the corresponding equations for the rates of change of the circulation components of an element of fluid referred to the rotating axes ; in this way we find

$$\begin{aligned} \frac{D\xi}{Dt} = & \xi \frac{\partial u}{\partial x} + \eta \frac{\partial u}{\partial y} + \zeta \frac{\partial u}{\partial z} - \xi \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right) \\ & + \omega_x \frac{\partial u}{\partial x} + \omega_y \frac{\partial u}{\partial y} + \omega_z \frac{\partial u}{\partial z} - \omega_x \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right), \quad (17) \end{aligned}$$

with the corresponding equations in η and ζ . Now the hydrodynamical theorem that relates to the permanence of a velocity potential for the motion of a given portion of fluid and the theorem of the permanence of the circulation of an element of fluid depend on equations (16). The equations which we have obtained for the relative circulations reduce to these equations exactly when the conditions expressed in (12) are fulfilled ; and these conditions must accordingly be fulfilled in order that the theorems referred to may apply to the relative motion, in the same way as they apply to the actual motion.

Particular Cases of Motion Relative to the Earth.

We shall now discuss one or two particular cases of fluid motion relative to the Earth, and we shall, to begin with, take the reference point O as a fixed point on the surface of the Earth at latitude ϕ degrees North. The angular velocity components $\omega_x, \omega_y, \omega_z$ have then the values 0, $\Omega \cos \phi$, $\Omega \sin \phi$, respectively, where Ω represents the angular velocity of rotation of the Earth about its axis. If we now let ϖ represent the perpendicular distance from any point x, y, z to the axis of the Earth, we can write the equations of motion (1), (2), (3), in the form

$$\frac{Du}{Dt} - 2\Omega \sin \phi \cdot v + 2\Omega \cos \phi \cdot w = -\frac{\partial V'}{\partial x} - \frac{1}{\rho} \cdot \frac{\partial p}{\partial x}, \quad (18)$$

$$\frac{Dv}{Dt} + 2\Omega \sin \phi \cdot u = -\frac{\partial V'}{\partial y} - \frac{1}{\rho} \cdot \frac{\partial p}{\partial y}, \quad (19)$$

$$\frac{Dw}{Dt} - 2\Omega \cos \phi \cdot u = -\frac{\partial V'}{\partial z} - \frac{1}{\rho} \cdot \frac{\partial p}{\partial z}, \quad (20)$$

where $V' = V - \frac{1}{2} \Omega^2 \varpi^2$. V' is, in fact, the potential function

corresponding to apparent gravity, so that $-\frac{\partial V'}{\partial z}$ at point $(x=0, y=0, z=0)$ is the value of $-g$ at the reference point O. In applying the above equations to motion of the atmosphere, we may take

$$\frac{\partial V'}{\partial x}=0, \quad \frac{\partial V'}{\partial y}=0, \quad \frac{\partial V'}{\partial z}=g, \quad . \quad . \quad (21)$$

in the immediate neighbourhood of O in the region within which the value of apparent gravity may be regarded as constant in direction and amount. If we neglect a change of direction of one degree in g , our equations (18) to (21) would then represent conditions of motion within a radius of about seventy miles from point O. In order to render our equations suitable to represent circulations of air of diameter exceeding, say, 150 miles, we might employ the approximate values

$$\frac{\partial V'}{\partial x} = \frac{gx}{R}; \quad \frac{\partial V'}{\partial y} = \frac{gy}{R}; \quad \frac{\partial V'}{\partial z} = g, \quad . \quad . \quad (22)$$

wherein we neglect the variation of g with height.

If we exclude certain cases of motion relating specially to the tides, very few solutions of the above equations have been recorded. In order to make ourselves familiar with the types of fluid motion possible in the atmosphere it is of interest to examine all solutions which can be obtained having a bearing on meteorological problems. We, accordingly, take first the steady rotational motions of incompressible fluid under the force of gravity alone.

We may take the boundary condition $w=0$ to apply at the surface of the Earth. A simple rotation of all the fluid about a vertical axis through O, with a uniform angular velocity ω , would be represented by

$$u = -\omega y; \quad v = +\omega x; \quad w = 0. \quad . \quad . \quad (23)$$

With these values in equations (18) to (20), it would be impossible to satisfy (19) and (20) simultaneously; but the motion represented by

$$u = -\omega(y - \beta z); \quad v = \omega x; \quad w = 0, \quad . \quad . \quad (24)$$

fulfils all the conditions contained in our equations, provided β is given by

$$\beta = \frac{2\Omega \cos \phi}{\omega + 2\Omega \sin \phi}, \quad . \quad . \quad . \quad (25)$$

and the pressure p is given by

$$\frac{p}{\rho} = -V' + \frac{1}{2}(\omega^2 + 2\omega\Omega \sin \phi) \{x^2 + (y - \beta z)^2\} + \frac{p_0}{\rho}. \quad (26)$$

The approximate value of V' is $+g\left(\frac{x^2 + y^2}{2R} + z\right)$, and p_0 is the pressure at the reference point O. In the motion indicated above each particle of fluid moves in a horizontal circle whose centre lies in the line ($y = \beta z, x = 0$). This line lies in the meridian plane through point O and is inclined to the vertical at O at an angle θ towards the North, where $\tan \theta = \beta$. When ω is very small in relation to Ω this inclined axis is almost parallel to the axis of the Earth; and when ω is large this axis comes almost to coincidence with the apparent vertical at O. With ω very large the motion described above corresponds very closely with a uniform rotation of the fluid about a vertical axis, as in the case of a simple forced vortex.

Another case of motion of incompressible fluid of interest in the same connexion is that represented by

$$\text{with} \quad u = -\omega y; \quad v = \omega x; \quad w = 2\Omega \cos \phi \cdot x, \quad (27)$$

$$\frac{p}{\rho} = -V' + \frac{1}{2}(\omega^2 + 2\omega\Omega \sin \phi)(x^2 + y^2) + 2\Omega^2 \cos^2 \phi \cdot x^2 + \frac{p_0}{\rho}, \quad (28)$$

as the equation showing the distribution of pressure. This motion differs from that first discussed in not being exactly horizontal. The plane of motion of each particle of fluid passes through the line OX, and is inclined to the horizontal plane XOY at an angle θ given by $\tan \theta = 2\Omega \cos \phi / \omega$. When the angular velocity of rotation ω is very large compared with $\Omega \cos \phi$, the plane of motion of each particle is practically horizontal, and the motion then corresponds very closely with that of simple rotation of all the fluid as a solid about a vertical axis. When ω becomes small, on the other hand, the inclination of the plane of motion of each particle of fluid to the horizontal increases. The two motions, represented by (24) and (27) respectively, are almost identical when ω is very large, and they differ entirely when ω is very small. It would be interesting to investigate the manner in which a fluid, such as water, subsides to rest from an initial condition of steady rotation about a vertical axis. The solution represented by (24), (26), would appear to be the exact solution for steady rotation of water in small scale experiments.

The motions considered above are motions of any incompressible fluid and do not indicate, except as approximations, conditions of motion possible in the atmosphere. One or two solutions of a similar type can be obtained which refer to incompressible fluid and accordingly represent motions possible in the atmosphere. Consider now the motion represented by

$$u = -\omega(y - \beta z); \quad v = 0; \quad w = 0. \quad (29)$$

The equations of motion of any element of fluid in this case are

$$0 = -\frac{gx}{R} - k \frac{\partial}{\partial x} \log \rho, \quad (30)$$

$$-2\Omega \sin \phi \cdot \omega(y - \beta z) = -\frac{gy}{R} - k \frac{\partial}{\partial y} \log \rho, \quad (31)$$

$$2\Omega \cos \phi \omega(y - \beta z) = -g - k \frac{\partial}{\partial z} \log \rho, \quad (32)$$

and these equations are satisfied provided the pressure system throughout the fluid is that indicated by

$$k \log \rho = \Omega \sin \phi \cdot \omega(y - \beta z)^2 - g \left(\frac{x^2 + y^2}{2R} + z \right) + k \log \rho_0, \quad (33)$$

where $\beta = \cot \phi$, and ρ_0 is the density of the fluid at the reference point O. The continuity equation is also satisfied provided we can neglect the term $(g\omega/R)x(y - \beta z)$. This condition limits considerably the extent of the region around O to which our solution is applicable, as stated earlier. Within the region to which the above applies the isobars at the surface of the Earth run East and West, being determined by

$$k \log \rho = \Omega \sin \phi \cdot \omega y^2 + k \log \rho_0, \quad (34)$$

which indicates a system symmetrical on the two sides of the East and West line drawn through reference point O. In this case the isobars become closer as we proceed North or South from point O. They are also parallel to the lines of flow of the fluid.

The coefficient $\frac{g}{R}$ has the value 1.5×10^{-6} with the foot and the second as units; while $2\Omega \sin \phi$ has the value 1.03×10^{-4} at latitude 45° . Certain cases of interest arise in which the terms containing $\frac{g}{R}$ may be neglected. For

example, we may take the case of a uniform east or west wind over a considerable region. In this case

$$u=c, \quad v=0, \quad w=0, \quad . \quad . \quad . \quad (35)$$

and the pressure distribution consistent with this motion is represented by

$$k \log \rho = 2\Omega c (\cos \phi z - \sin \phi y) - gz + k \log \rho_0, \quad (36)$$

where ρ_0 refers to the density of air at the reference point O. The isobars at the Earth's surface are in this case a uniform system running due East and West.

A similar case is that given by

$$u=0, \quad v=cx+d, \quad w=0, \quad . \quad . \quad . \quad (37)$$

which represents a wind towards the North, while the corresponding pressure distribution is that represented by

$$k \log \rho = \Omega \sin \phi (cx^2 + 2dx) - gz + k \log \rho_0. \quad (38)$$

The isobars are again a system of straight lines, but running north and south, and uniformly spaced when $c=0$.

In a similar manner we find that

$$u=c_1, \quad v=c_2, \quad w=0, \quad . \quad . \quad . \quad (39)$$

corresponds to a system of straight isobars represented by

$$k \log \rho = 2\Omega \sin \phi (c_2x - c_1y) + 2\Omega \cos \phi . c_1z - gz + k \log \rho_0. \quad (40)$$

The isobars are again lines of flow of the air, as in each case considered above.

The case of motion corresponding most closely to a cyclonic or anticyclonic circulation is that discussed in an earlier paper *, represented by

$$u = -\omega(y - \beta z); \quad v = \omega x; \quad w = 0. \quad . \quad . \quad (41)$$

In this case

$$\beta = \frac{2\Omega \cos \phi}{\omega + 2\Omega \sin \phi}, \quad . \quad . \quad . \quad (42)$$

and the pressure distribution is that represented by

$$k \log \rho = \frac{1}{2}(\omega^2 + 2\omega\Omega \sin \phi) \{x^2 + (y - \beta z)^2\} - gz + k \log \rho_0. \quad (43)$$

The term $\frac{g}{R} \omega x z$ must be small in order that the continuity equation may be fulfilled.

* Phil. Mag. vol. xli. April 1921; vol. xlii. July 1921.

In each case considered we have only to replace Ω by an increased value Ω' to obtain a motion for which the system of isobars travels eastward at a uniform speed ($\Omega' > \Omega$).

Each of the above solutions has been given to apply to an isothermal atmosphere, and in every case considered the fluid moves so that no element of fluid undergoes change of density. That is, $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$ at each point. Provided

this condition is fulfilled, any solution obtained for motion of an atmosphere, all at one temperature, can readily be transformed to suit an atmosphere in convective equilibrium ($p = k\rho^\gamma$), or one in which pressure is any given function of density. Thus, taking the motion represented by (41) above in an atmosphere in convective equilibrium, we have merely

to replace $\log \rho$ and $\log \rho_0$ in (43) by $\frac{\gamma}{\gamma-1} \rho^{\gamma-1}$ and $\frac{\gamma}{\gamma-1} \rho_0^{\gamma-1}$ respectively, all other conditions being unchanged.

VI. *The Breaking Stress of Crystals of Rock-Salt.*

By Prof. G. N. ANTONOFF, *D.Sc. (Manch.)* *.

IN a paper published in *Phil. Mag.* vol. xxxvi. Nov. 1918, I have developed a theory of surface tension under the assumption that the attraction of molecules is due to electrical or magnetic forces, or both. Instead of assuming a uniform field round the molecules as it is generally accepted according to Laplace, I accepted the view that the molecules act as electrical doublets, and from the theory of potential I deduced that the attraction between them must be inversely proportional to the 4th power of the distance, provided the distance between the doublets is large compared with their respective lengths.

It was shown that the attraction between the doublets can be represented by an expression of the type

$$\frac{kl^2}{d^4},$$

where k is a constant, l the length of the doublet, and d the distance between them. In these calculations the magnetic forces were disregarded altogether, as the law of attraction between small magnets would be just the same, so that they could only have an effect on the value of k .

* Communicated by Dr. J. W. Nicholson, F.R.S.