CRITICAL REVIEW OF THE BIBLIOGRAPHY ON UNBALANCED MAGNETIC PULL IN DYNAMO-ELECTRIC MACHINES

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ABSTRACT OF PAPER

The purpose of this paper is to serve as an introduction to the succeeding comprehensive article by Mr. E. Rosenberg, on the subject of Magnetic Pull in Electric Machines. The Maxwell equation, upon which all of the formulas for unbalanced magnetic pull are based, is first established. A critical review is then given of the articles pertaining to this subject, which have appeared in the technical press to date. It is pointed out wherein the various expressions which have been proposed differ from one another and to what extent they are consistent.

ALL of the formulas are based on Maxwell's equation:

$$\text{Pull, in dynes} = \frac{B^2 A}{8 \pi}$$

where B = flux density, in lines per sq. cm.

A = cross section, in sq. cm. of the flux path in the gap between the pulling surfaces.

This formula may be established in a simple manner for the case of a saturated magnetic circuit, as follows:

The magnetic circuit shown in Fig. 1 is symmetrical about the axis OO' , and therefore the action of each pole may be considered separately. With n turns per pole and e volts across each coil, the excitation ni produces a magnetic flux of ϕ lines.

In Fig. 2, Oa shows the magnetization curve for each half of the complete magnetic circuit of Fig. 1 when the air gap clearance between the armature and each pole-face is g cm. If now, the armature is allowed to move by a virtual displacement Δg under the action of the magnetic pull P , so as to reduce the gap, then the curve for the new magnetic circuit becomes Ob (Fig. 2).

In order to establish the initial flux ϕ at an excitation ni , the total energy supplied to the coil after closing the switch ss , is

$$\int_0^t e i dt = \int_0^t \left(n \frac{d\phi}{dt} \cdot 10^{-8} \right) i dt + \int_0^t i r i dt$$

In this expression, the stored energy

$$\int_0^{\phi} ni \, d\phi \cdot 10^{-8} = \text{Area } Oac \text{ (Fig. 2), in watt-sec.}$$

If, now, the armature is allowed to move under the action of the pull P , so as to reduce the gap by an amount Δg , the excitation ni being maintained constant, the flux changes from ϕ to $\phi + \Delta\phi$. The increase in flux $\Delta\phi$ induces a counter e. m. f. in the winding and since, by assumption, the current is held constant, the applied voltage must temporarily increase. The amount of energy so brought into the electromagnet is

$$\int_{\phi}^{\phi + \Delta\phi} \left(n \frac{d\phi}{dt} \cdot 10^{-8} \right) idt = ni \times \Delta\phi \cdot 10^{-8} = \text{Area } cabd$$

FIG. 1

FIG. 2

The energy which is thus supplied by the source goes partly to increase the stored energy and the rest lifts the armature.

The energy stored in the magnetic circuit under the new conditions, with flux $\phi + \Delta\phi$,

$$= \text{Area } Obd.$$

Hence, mechanical work done = initial stored energy + energy supplied from the line — final stored energy

$$\begin{aligned} \text{or } P \cdot \Delta g &= \text{Area } Oac + \text{Area } cabd - \text{Area } Obd \\ &= \text{Area } Oab \\ &= \text{Area } Oa'b' \end{aligned}$$

where Oa' and Ob' in Fig. 2 represent the magnetization curves for the air gaps g and $g - \Delta g$ respectively.

But, for very small values of $\Delta \phi$,

$$\begin{aligned}
 \text{Area } Oa'b' &= \frac{1}{2} \cdot \text{Ampere-turns for gap } \Delta g \times \phi \\
 &\quad \cdot 10^{-8} \dots \text{watt-sec.} \\
 &= \frac{1}{2} \left(\frac{10}{4\pi} \cdot \frac{\phi}{A} \cdot \Delta g \right) \times \phi \cdot 10^{-8} \\
 &= \frac{\phi^2}{8\pi A} \times \Delta g \times 10^{-7} \dots \text{watt-sec.} \\
 &= \frac{\phi^2}{8\pi A} \times \Delta g = P \times \Delta g \dots \text{ergs.}
 \end{aligned}$$

or, Pull in dynes,
$$P = \frac{\phi^2}{8\pi A} = \frac{B^2 A}{8\pi}$$

This formula is thus true whether the magnetic circuit is saturated or not. It will also be seen from the above what is meant by A in the formula, and how problems involving slotted armatures are to be solved.

The subject of magnetic pull in dynamo-electric machines has not received much attention in recent years; in fact, it appeared as if the whole field had been covered by 1911.

Fisher-Hinnen¹, in his "Dynamo Design" published in 1899 derives the following formula:

$$\text{Pull, in dynes} = \left(\frac{B^2 A}{25 \times 10^6} \cdot \frac{x}{g \alpha} \right) C$$

where B = density of flux in the air gap, in lines per sq. cm.

A = section of the pole face, in sq. cm.

x = rotor displacement

g = air-gap clearance

α = ratio of the reluctance of the total magnetic circuit to that of the normal air gap alone.

C is a calculated coefficient, given in the following table:

Value of C	Number of poles	Remarks
2	4	Displacement along axis of field magnets.
2.8	4	Displacement along neutral line.
6	6	Displacement along axis of magnet.
9	8	Displacement along neutral line.
15	12	Displacement along neutral line.
20	16	Displacement along neutral line.

B. A. Behrend² in 1900 gives a mathematical discussion of the case of a machine with a very large number of poles and derives the formula

$$\text{Pull} = \frac{B^2 S'}{8\pi} \cdot \frac{2x}{g}$$

where $S' = \pi \times R \times l = \frac{1}{2} \times \text{total air-gap area}$, and shows that this is exactly one-half the value obtained by assuming that half of the poles have a gap $g - x$ and the other half a gap $g + x$.*

If we substitute for $S' = \frac{1}{2} \times A \times \text{poles}$, Behrend's expression becomes

$$\text{Pull} = \frac{B^2 A}{8\pi} \cdot \frac{x}{g} \cdot \text{poles.}$$

Edgar Knowlton,³ about the same time, derives an expression for a machine with a definite number of poles and obtains the formula

$$\text{Pull in lb.} = \left(\frac{B^2 A}{77,134,000} \cdot \frac{x}{g\alpha} \right) C$$

where B = normal gap density in lines per sq. in.

A = area of pole-face in sq. inches

x = displacement

g = normal air gap

α = ratio of the total reluctance of the magnetic circuit to that of the normal gap alone.

C = a coefficient, obtained by calculation

= 2 for 4 poles

= 4.7 for 6 poles

= 7 for 8 poles

= 9.6 for 10 poles

Above 10 poles, C = number of poles, and the formula then checks with that of Behrend.

Knowlton's expression is of exactly the same form as that of Fisher-Hinnen, but the constants given by the two authors for 6 and 8 poles differ considerably. Knowlton, moreover, states that it makes but little difference whether the plane of deflection is taken through a pole or between two poles.

Hans Linsenmann⁴, in 1902, uses the saturation curve directly

* Note that in the following paper Rosenberg formulates a similar statement.

(as is done in Fig. 3 of the following article by Rosenberg) for determining, at a given excitation, and with sine distribution of flux under the pole, the variation of the average magnetic pull per pole with the length of airgap. He expresses this variation by an approximate equation and introduces it into the general equations connecting the bending moments of the deflecting forces in large alternators.

J. Rey⁵, in 1904, states that Behrend's and Fisher-Hinnen's formulas are not consistent and sets out to derive a new formula especially applicable to induction motors with eccentric rotor. Rey's formula is

$$\text{Pull in dynes} = \frac{1}{8\pi^2} (B_{eff})^2 S \times C$$

where $B_{eff} = B_{av} \times 1.1$ for a sine distribution of flux

S = rotor surface = $2\pi Rl$

$C = f(\epsilon)$

ϵ = eccentricity, expressed as a fraction, in terms of the single air gap = x/g

and for

$$\epsilon = x/g = 0.05 \quad 0.1 \quad 0.15 \quad 0.2 \quad 0.25 \quad 0.3 \quad 0.4 \quad 0.5$$

$$C = 0.157 \quad 0.319 \quad 0.488 \quad 0.688 \quad 0.866 \quad 1.084 \quad 1.63 \quad 2.415$$

For an eccentricity less than 20 per cent, this formula reduces to

$$\begin{aligned} & \frac{(B_{eff})^2}{8\pi^2} \times S \times \frac{0.488}{0.15} \times \frac{x}{g} \\ &= \frac{(B_{eff})^2}{8\pi} \times S \times \frac{x}{g}, \text{ approximate within one per cent,} \end{aligned}$$

which is identical with Behrend's formula, because with a sine distribution of flux, as in an induction motor, $(B_{eff})^2$ is the average value of $(B)^2$, where B is the density at any point along the gap.

J. K. Sumec⁷ derives a similar formula to that of Behrend and Rey, but by certain transformations reduces it to the form

$$\text{Pull in dynes} = \frac{B^2}{8\pi} \cdot S \cdot \frac{x}{g} \cdot \frac{1}{[1 - (x/g)^2]^{3/2}}$$

This is the same as Behrend's formula except for the last term which for an eccentricity as large as 25 per cent, is equal to 1.1 or a difference of only 10 per cent, while for eccentricities below

10 per cent, the correction term differs from unity by only about one per cent.

Sumec's formula, as might be expected, gives the same values as that derived by Rey. It is to be noted that in most of the formulas derived mathematically, the effect of the reluctance of the iron part of the magnetic circuit is neglected; the assumption made that the flux density is inversely as the airgap is therefore only true in unsaturated machines

In 1905, B. Soschinski⁸ gives an account of some tests made to check the formulas of Rey and Sumec. Very good agreement was obtained for small airgaps and iron feebly saturated, taking the area and flux density at the top of the teeth but with increasing saturation, the calculated results (iron reluctance neglected) were higher than experimental ones. With larger values of air gap, however, the test values were throughout higher (up to 100 per cent). The latter result was accounted for by the fact that as the armature moves to one side the lines become concentrated at the tooth tip on the side of the reduced gap and spread out on the side of the increased gap.

In the same year (1905), Niethammer⁹, by a transformation of Sumec's expression, obtains the following formula which permits taking into account the reluctance of the iron path by determining the flux density from the saturation curve, as is done by Rosenberg. This expression is regarded by him as giving the most reliable results.

$$\text{Resultant pull, in kg.} = \frac{1}{2} \left\{ \left(\frac{B_{max}}{5000} \right)^2 - \left(\frac{B_{min}}{5000} \right)^2 \right\} p A$$

where p = number of pairs of poles
 A = mean effective cross sectional area of gap per pole in sq. cm.

In 1907, Picou¹⁰, starting with the relation for stored energy, derives the expression for magnetic attraction, and gives a modification of the formula due to Sumec for unbalanced pull.

C. R. Moore¹² in 1911, gives a graphical method for studying the unbalanced magnetic pull, by using Maxwell's formula and the given saturation curve of the machine, and summing up the pull for various adjacent halves of adjoining pole faces. He takes the airgap density for these half arcs as corresponding to the average airgap lengths across these faces and to the given excitation. Upon plotting the unbalanced magnetic pull for

various field excitations he establishes the important fact noted by Rosenberg that, for low saturations the pull increases with the excitation; at a critical density, however, corresponding to about the knee of the magnetization curve, the unbalanced pull reaches a maximum for all eccentricities, and for larger excitations then decreases again. Therefore a machine, which normally operates at high saturation, may be subjected to greater stresses while it is building up than under normal field excitation.

Miles Walker¹⁴ in his book on "Specification and Design of Dynamo-Electric Machinery" (1915), points out that the amount of the unbalanced pull for a given displacement will depend on the extent to which the iron parts are saturated, and that the effect of increased saturation is to reduce the pull for a given airgap clearance and magnetic induction. He first assumes that all the ampere-turns are expended in the airgap and by the usual method derives an expression for the unbalanced pull exactly similar to that of Behrend (see above). He then shows how the saturated magnetic circuit may be replaced by an equivalent airgap obtained by means of a graphical construction on the saturation curve of the machine, and then uses this equivalent airgap in the formula for the unbalanced pull.

R. E. Hellmund¹¹, Miles Walker¹⁴ and others have pointed out the effect of series and parallel windings on the unbalanced pull and have also discussed how the unbalanced magnetic pull in the induction motor may be reduced to a minimum, by using in the stator winding two paths in parallel which lie on opposite halves of the frame and thus make it impossible for the flux on these two halves to be very unequal.

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