AN ELECTROMECHANICAL DEVICE FOR RAPID SCHEDULE HARMONIC ANALYSIS OF COMPLEX WAVES*

BY FREDERICK S. DELLENBAUGH, JR.

Secretary, Research Division, Electrical Engineering Department,
Massachusetts Institute of Technology.

IMPROVEMENT in an art requires perfection of detail. Investigation of the details is often delayed, even though the procedure may be well understood, by the large amount of labor involved in handling cumbersome formulas or mathematical processes. The fundamentals of alternating-current theory are based upon a pure sine wave. More careful analysis considers waves formed of many multiple frequencies, which are found to exist in most practical applications. Thus not only the fundamental sine, but its higher harmonics, must be dealt with. The process of splitting up any known wave into its various components is well understood, but is slow and laborious if more than one or two harmonics are required. In present methods of analysis, it is at once striking how many combinations must be made to obtain the result. In mathematical methods this involves the calculation of a host of sums and products, elementary in form, but laborious in procedure. Methods of selected ordinates require many readings of ordinates from the curve, different sets being required for each harmonic. Graphical methods require a large amount of constructional detail. Machines involve changes of gears or pulleys, and many tracings of the curve.

The great ease with which electric circuits may be combined by multiple switches, and the accuracy with

* This Electric Analyzer was designed and manufactured in the Research Division Laboratories of the Electrical Dept., Mass. Inst. of Technology, where it has been in successful use for some time.

which electric measurements may be made, at once suggests the use of an electric network with some adjustable members to solve the curve under analysis (Bibl. 44). The device about to be described represents one way in which this may be done, and gives sufficient promise to indicate the possibility of great speed and reasonable accuracy with machines developed along these lines.

THEORY

The electric analyzer is based upon the schedule method of analysis, so called because the mathematical processes may be combined in a schedule and save duplication of operations. The fundamental solution of Fourier's series, upon which it is based, may easily be derived as follows:

The series: $y = c_0 + \sum c_n \sin [n \theta + \psi_n]$ may be transformed to

$$y = c_0 + \sum [b_n \cos n \theta + a_n \sin n \theta]$$

When substitutions are made of:

$$c_n = \sqrt{a_n^2 + b_n^2} \quad \text{and} \quad \psi_n = \tan^{-1} \left[\frac{b_n}{a_n} \right]$$

In all of which expressions a , b and c represent coefficients of the different terms, n represents the order of the term, or harmonic, and is any integer, and θ represents the time function usually expressed by ωt . ψ is the phase angle of the harmonic, eliminated in the second expression by the introduction of both sine and cosine terms (Bibl. 4, 5, 6, 13 & 18).

If the area of the curve under analysis is the same on both sides of the zero line, then the term c_0 vanishes. If the curve is symmetrical with respect to its axis, then the even harmonics must be absent, and the values of n will be odd only. As these conditions are both true in most cases of alternating-current machinery, they will be the only ones herein considered.

By proper manipulation and integration, it is found that the coefficients sought are represented by the integrals:

$$b_k = 1/\pi \int y \cos k \theta d \theta$$

and

$$a_k = 1/\pi \int y \sin k \theta d \theta$$

where k is 1, 3, 5 etc.

In order to evaluate these integrals, an auxiliary curve may be obtained from the one under analysis by constructing curves of $y \sin k \theta d \theta$ and obtaining the area with a planimeter. This is the basis of many of the graphical methods. It also is possible to give a planimeter wheel such a motion that the vertical displacement is proportional to the values of y and the horizontal displacement is proportional to $\sin k \theta$ or $\cos k \theta$. In this case, the planimeter wheel will read the coefficient direct. This is the basis of most of the mechanical analyzers (Bibl. 65 & 82).

Both of these are indirect, however, and in order to obtain a direct mathematical method, further reduction of form must be obtained. It is shown in any text book upon the subject that the summation below represents the integral within certain limitations.

$$b_n/2 = 1/n \sum y_r \cos n/2 \theta_r = 1/n \sum y_r \cos r \pi$$

$$b_k = 2/n \sum y_r \cos k \theta_r$$

$$a_k = 2/n \sum y_r \sin k \theta_r$$

All that is then necessary is to divide the curve into n equal parts, measure the $(n-1)$ ordinates, multiply by the corresponding sine or cosine of the ordinate position, and add the results. The sum divided by $n/2$ will give the coefficient sought. The results are accurate, provided that there are no harmonics present of an order higher than the value of n .

The number of ordinates $(n-1)$ must be at least equal to the order of the highest harmonic, and, for waves containing only odd harmonics, need only be taken over one-half the wave. Thus for analyzing for the 1st, 3rd and 5th harmonics, the curve must be divided into six parts giving five ordinates, and if there are no harmonics present greater than the 5th, the results will be accurate within the error of measurement of the ordinates and of calculation. The method is quite old, and if worked out through all its stages, is quite laborious. It is interesting that Kintner

(Bibl. 7) published a set of tables to help the calculations, but did not seem to see the duplications of effort later pointed out by Carl Runge (Bibl. 6 & 16). It is evident that since a large number of sines and cosines are used as multipliers, and as these are taken for definite recurring angles and multiples of angles, there must be a recurring sequence of functions used as multipliers. Runge thus showed that the total number of multipliers necessary was equal to the sines of the angles of division of the curve between 0 and 90. The data thus developed were put into engineering form by S. P. Thompson (Bibl. 10) and became the now familiar harmonic analysis schedule. An eleven-ordinate schedule capable of giving the 1st, 3rd, 5th, 7th, 9th and 11th harmonic, both sine and cosine terms, is given in the Appendix, Table XI.

Probably one of the most serious errors likely to occur in this method arises from the fact that the cosine terms are obtained from the differences of what may be large numbers, and therefore the condition exists where the differences consist of numbers smaller than the accuracy of the original products, and so may themselves be entirely due to error. Steinmetz (Bibl. 14) has worked out a modification of the schedule method to eliminate this source of error, but while mathematically successful, it increases the labor **very** greatly and so is not generally used. Another difficulty in practical application arises from the fact that higher harmonics are neglected, and a schedule must be adopted which includes a sufficient number of ordinates, so that the higher harmonics may actually be negligible. With an unknown curve, this is more or less guess work, although the highest harmonic present may sometimes be fairly accurately estimated, or the type of curve may be sufficiently familiar for the general magnitude of results to be known. Thus with the magnetizing current of transformers, it is generally known that unless saturations are carried to very high values, the harmonics above the 11th are extremely small, while with alternator wave shapes, a tooth harmonic may be observed as a ripple whose order may easily be estimated. If no ripple is observed,

it is probable that no harmonics higher than the 13th exist. The labor required to solve a curve by the schedule method rises rapidly as the ordinates or harmonics increase. The rate of increase of labor is at least proportional to the square of the order of the highest harmonic. Schedules as high as 35 ordinates are given in the California Railroad Commission Report (Bibl. 21). F. W. Gover (Bibl. 18) has also furnished some interesting data upon the error to be expected from neglecting higher harmonics which are present. His results seem to indicate that unless the neglected harmonics are very prominent, the error is chiefly confined to the highest of the harmonics determined by the schedule method, but an analysis of a series such as given by a rectangular wave where the harmonics theoretically extend to infinity, and which requires a relatively large number of terms to approximate the curve, will give an error in all the harmonics determined by the schedule method for eleven ordinates, the error increasing in magnitude as the harmonics determined increase in order. In general, if the schedule method is used intelligently, and with a knowledge of its limitations, it affords a very satisfactory method of analysis.

THE ELECTRIC ANALYZER

The new electric analyzer here described is based directly upon the schedule method, and is in substance an electrical network arranged to give the same result as that obtained by the mathematical processes. If an e. m. f. proportional to the ordinate y , is impressed upon a conductance proportional to the sine or cosine of the ordinate angle $k \theta$, then the resulting current is proportional to their product, and it will be seen that this corresponds to one of the terms of the summation it is necessary to evaluate. Thus:

Let: $e = K_1 y$ and $R = 1/\sin k \theta$,

Then $I = K_1 y \sin k \theta$.

If a number of such circuits are connected in parallel, then the total current flowing will be the sum of the currents in each circuit. Thus, if a five-ordinate schedule is used as a basis, it will be necessary to have

five adjustable voltages representing each of the measured ordinates, and a succession of resistances representing the various values of sines or cosines of the various angle multiples. As the ordinates remain the same for each curve, and only the angle functions change, it is evidently possible to have one setting of the adjustable voltages hold for the whole of one analysis, and the required values of resistance may be thrown in by multiple switches. In the use of the simpler schedules, dial switches may be used, but for more complicated schedules, gang switches are necessary to eliminate circulating currents. It will be understood that the current used for obtaining the solution of the schedule is direct current and bears no relation to the alternating-current wave under analysis.

FIG. 1

The analyzer does not analyze an alternating-current wave directly, but an oscillogram must first be taken of it and then ordinates measured at the proper intervals, exactly as would be done for the mathematical schedule method.

The elementary principle is shown in Fig. 1, where three slide wires are used to provide the variable voltage and the sliders are connected from the sliding contacts through proper resistances to a common bus, to which is connected the indicating ammeter. Slide wires are the simplest means of obtaining variable voltage, but of course are subject to the objection that if current is drawn off at the sliding contact, the voltage distribution is no longer linearly proportional. The

error involved in the assumption that the voltage is proportional is given by the equation:

$$\text{Error} = \left(\frac{l-x}{x} \right) \frac{R x}{r l}$$

where R is the resistance of the slide wire, r the resistance connected in series with sliding contact, x the position of contact and l the length of slide wire.

Thus it will be seen that if the series resistance is 500 times the slide wire resistance, the error is 0.1 per cent. As the accuracy of any harmonic analysis seldom exceeds 2 per cent, an error of 0.1 per cent is entirely negligible. Another point that brings in a

FIG. 2—WIRING DIAGRAM FOR 5-ORDINATE ANALYSER GIVING 1ST, 3RD, AND 5TH HARMONICS

The dial-switch is turned to proper point and coefficient read on meter. The switches marked S, S, S, are mechanically coupled so that they move in unison and are thrown to the right for fundamental components and the left for 5th harmonic. Slide wires 2 ohms each.

difficulty is the fact that some of the angle functions are negative and some positive. Therefore, either the voltage or resistance must have two signs. Since negative resistance cannot be obtained for this purpose, it is easier to use a middle tap for the ammeter return circuit on each slide wire. As these middle taps will all be of the same polarity, there will be no current flowing and the position of the slider to the left or

right of the middle tap will give an e. m. f. of opposing signs. Fig. 2 shows the connections for a five-ordinate analyzer, the slide wires being folded back at the middle point and two sliders being used, so that they may both be attached to the same member of the device, and the same scale setting used for either

FIG. 3

positive or negative values, the change being made by switching in the resistance connected to the proper contact, leaving the other one idle. The values of resistance are calculated so that the maximum inherent error due to voltage distortion will be 0.1 per cent. The maximum will only occur in some of the ordinates and so the average error for the instrument will be less than this. The greater the number of slide wires,

FIG. 4

the less this average error will be. In order to calculate the values of the resistances, the lowest one is taken as 1000 ohms and the others designed to bear the same proportion to this as the sine or cosine desired. Thus

the smallest sine is 0.50. The largest is 1.00. Therefore, if the smallest resistance corresponding to the sine value of 1.0 is 1000 ohms, the largest resistance will be:

$$\frac{1.0}{0.5} \times 1000 = 2000 \text{ ohms}$$

The analyzer shown in Fig. 2 will analyze any wave not containing higher than the fifth harmonic and one where the wave does not cross the axis within a half cycle, or, in other words, which has no negative ordinates, as no provision is made for ordinates of more than one sign. This may be taken care of very simply, however, by introducing a reversing switch in each slide wire, which is thrown over when a nega-

FIG. 5

tive ordinate setting is required. The majority of curves met in ordinary practise do not require this additional refinement.

CONSTRUCTION OF ELEVEN-ORDINATE ELECTRIC ANALYZER

The eleven-ordinate analyzer based upon this scheme is shown complete in Fig. 3, and connected for operation in Fig. 4. The double slide wires, similar to those in the five-ordinate diagram, are wound upon maple plywood disks, six inches, (15.25 cm.) in diameter, with a projecting flange for manipulation. The edge of the disk carries a scale divided into 100 parts for the length of the slide wire, which is a little less than the periphery of 18.8 (47.8 cm.) inches. The three connections to each slide wire are made by flexible cables and carried to busbars along the under

part of the face plate, as seen in the front part of Fig. 5, showing the slide wire disks assembled upon the face plate. Each disk projects through the face plate, and the scale reading against a setting line, painted on the face plate, is read through a one-inch slot beside the flange, as may be seen from Fig. 3. The sliding contacts are mounted upon a bakelite platform on the bottom of the face plate just behind the busbars, and consist of two phosphor-bronze springs per disk, bearing upon the two parts of the slide wire. The slide wires are made of No. 25 B. & S. resistance wire having 1.65 ohms resistance each, and, being connected in parallel, give the whole slide wire system a resistance of 0.15 ohms. The resistances representing the angle functions are placed along the inside of the containing box. The resistances are wound from high-resistance wire upon fiber spools $2\frac{1}{2}$ inches (6.35 cm.) long, with $\frac{1}{2}$ -inch (1.27-cm.) cores and $1\frac{1}{4}$ -inch (3.18-cm.) flanges. Each spool consists of two sections, the bottom section for connection to the negative contacts and the top section for connection to the positive contacts. For making the proper connections, six gang switches are mounted in a separate housing upon the front of the box, as shown in Fig. 3. These consist of a brass rod with five crossarms which fit between five sets of jaws upon each side. The jaws are all insulated from each other, so that closing the switch to either side, closes ten circuits and connects them to a common bus.

Each switch represents one harmonic, closed to the left for the sine component and to the right for the cosine component. In the position shown in Fig. 3, it is set for the fifth sine component. In this particular machine, two dial switches are also provided for further isolation of resistances. These are shown upon the front edge of the face plate in Fig. 3. These are entirely unnecessary for the operation of the machine and were added purely for experimental purposes to determine the errors resulting from circulating currents, with the point in view of further simplification. The third dial switch on the left is a resistance in the battery circuit for adjusting the slide wire current.

The complete wiring diagram of this instrument is shown in Fig. 6, and the resistance data are given in the appendix.

METHOD OF OPERATION

The current chosen for the slide wires depends upon the instruments available. In Fig. 4, the battery is connected through an auxiliary resistance for adjustment and about 0.20 ampere per slide wire was used in the tests given below. The meter for reading the harmonics was a Paul unipivot milliammeter, the scales of 20 and 2 milliamperes maximum being used. It does not make any appreciable difference what the resistance of the meter is, so a meter of this type is very convenient, as the sensitivity may be increased by multiples of ten by simply turning a switch; and thus

FIG. 6

the smaller harmonic components read with greater accuracy on the meter than is warranted by the method. If the current in the slide wires is increased five times, which is allowable, the meter required would be a 200-milliamperé maximum reading instrument which is of commercial size. The slide wire current will still not be in excess of that easily furnished by a portable storage battery, so the auxiliary devices required are of easily obtainable nature.

To make the analysis, the instrument is first cali-

brated. This is done by setting the disks for a pure sine wave. Red marks are provided upon the scales, so that this can be rapidly done without reference tables. The first gang switch is then thrown to the left, or fundamental sine position, and the resistance in the battery circuit adjusted until the meter reads 100, or any other convenient amount. All other components should be zero. The switches are then turned successively through the other positions, and if the meter reads zero for all of them, it indicates that all connections are correct. This is a very rapid and invaluable check, for it can be made quickly any time and is an almost infallible indication of any open or short circuits, or poor contacts. All the resistances used in the sine fundamental position are used again in some other position, so that it includes them as well. Of course, it will not indicate poor connections, etc., that may develop for different ordinate settings but this has proved to be a very rare occurrence.

The eleven disks are next set to the values read from the curve, which should be done with some scale which will give the maximum ordinate a value of 80 to 100, or some decimal multiple of it, in order to insure the greatest range of slide wires, and thus obtain the maximum accuracy. If the battery current has remained the same, the readings obtained upon the meter will then give the values of the harmonic components directly, in terms of the fundamental sine wave used for calibration. Thus knowing the relative values of the scales used, the actual values of the coefficients may be easily determined. The readings are obtained by simply throwing one switch after the other, first to the left and then to the right, then returning it to the disconnected position and passing to the next switch. No change has to be made in the slide wire disk settings until another curve is analyzed.

Usually the relative magnitude of the harmonics is of more interest than the actual magnitude. This may be obtained directly from the instruments by adjusting the slide wire current, and this appears to be the only type of harmonic analyzer where this can be accomplished. After the disk settings have been made,

instead of leaving the battery current the same as for the calibration and check, the fundamental switch is thrown to the sine position, and then the resistance in the battery circuit adjusted until the meter reads 100. Further manipulation of the switches then gives the harmonic coefficients directly in percentage of the fundamental sine. This machine also eliminates the chance of mathematical errors which creep in so easily with the multitude of operations necessary in the usual analysis by schedule.

TESTS UPON ELECTRIC ANALYZER

The tables give a series of tests made to determine the accuracy of the network analyser. A very simple curve was first taken, as given in Table III. The only error will be seen to be the 11th sine coefficient. This is due to an error in the value of one of the resistances connected for this combination, and it will be noticed that this error is repeated in all readings. Increasingly trying combinations of terms were analyzed, and it will be seen that the maximum error occurs in Table IX, and amounts to 3.7 per cent. The value for comparison was obtained by working out the schedule method upon a calculating machine and thus obtaining the best possible accuracy by this method. It is interesting to note that in Tables IX and X for the triangular and rectangular waves, neither method gives accurate results, due to the harmonics above the 11th being neglected, and these types of curves having an infinite number of theoretical terms. Tables VII and VIII give the analyses of an ordinary transformer magnetizing current wave, and a peculiar wave made up of a sine with a superimposed discontinuous function obtained from the thermionic conduction in a three-electrode vacuum tube with alternating current impressed upon the plate circuit. These two curves are similar to the types met with in general practise, and show quite good accuracy, the maximum error being 3.32 per cent in the 11th harmonic and 1.88 per cent in the other harmonics. The last harmonic within range of the schedule will always give the largest chance for error as it means that only one point per

cycle is determined, and this is just the requirement for determining the size of the component. Thus a small error in measurement of the ordinate or setting of the machine will introduce relatively large errors in the results.

From these tests and the underlying theory, the conclusion may be arrived at that for the usual type of curve an accuracy of 2 to 3 per cent may be expected from this type of instrument, which is sufficiently accurate for most practical requirements and compares

TABLE I

Data on Resistance Coils for 11-Ordinate Harmonic Analyzer.
Arrangement of Coils:

Coil No.	Resistance Taps.									
	Top Sections.					Plus Connection				
1	0	—	518	—	707	—	1938			
2	0	—	500	—	518	—	577	—	1000	
3	0	—	707	—						
4	0	—	577	—	1000	—	1938			
5	0	—	518	—	707	—	1938			
6	0	—	500	—						
7	0	—	518	—	707	—	1938			
8	0	—	500	—	577	—				
9	0	—	707	—						
10	0	—	500	—	577	—	1000			
11	0	—	518	—	707	—	1938			
	Bottom Sections.					Minus Connection				
1	0	—	518	—	707	—	1938			
2	0	—	500	—	577	—	1000	—	1938	
3	0	—	707	—						
4	0	—	500	—	518	—	577			
5	0	—	518	—	707	—	1938			
6	0	—	500	—						
7	0	—	518	—	707	—	1938			
8	0	—	518	—	577	—	1000	—	1938	
9	0	—	707	—						
10	0	—	500	—	518	—	577	—	1000	— 1938
11	0	—	518	—	707	—	1938			

favorably with other methods of analysis. The error in the highest order of harmonic within range of the machine may be somewhat larger than this if the harmonic is prominent, but this is a characteristic of the method and not the particular instrument. The errors are all calculated against the value of the fundamental sine, as this is the fairest method. The importance of

TABLE II
Connections of Resistances for Different Components in 11-Ordinate Harmonic Analyzer.

1		3		5		7		9		11	
Component:											
Sin.	Cos.	Sin.	Cos.	Sin.	Cos.	Sin.	Cos.	Sin.	Cos.	Sin.	Cos.
Top Coils											
Plus Connection											
1	1938	707	707	518	1938	518		707		1938	
2	1000	500	Inf.	1000				Inf.	Inf.		518
3	707	707					707	707	707	707	
4	577	Inf.		1000	1000	577	1000	Inf.			1938
5	518			1938	518	1938	Inf.		707	518	
6	500		Inf.	500	Inf.		Inf.	500	Inf.		Inf.
7	518		707	1938		1938	518			518	1938
8	577	Inf.	500			577		Inf.	500		
9	707		707	707	707			707		707	707
10	1000	500	Inf.	1000	577		577		Inf.		
11	1938	707		518		518	1938	707	707		518
Bottom Coils											
Minus Connection											
1							1938		707		518
2			Inf.		577	1000	577		Inf.	1938	
3			707	707	707	707		500			707
4		Inf.	500	577				Inf.	500	518	
5		707	707				518	707			1938
6		500	Inf.		Inf.	500	Inf.		Inf.	500	Inf.
7	1938	707			518			707	707		
8	1000	Inf.		577	1000		1000	Inf.		518	1938
9	707			707		707	707			707	
10	577		Inf.			1000		500	Inf.	1938	518
11	518		707		1938						

TABLE III

Curve of Equation: $y = \sin \theta + 0.5 \sin 3 \theta$

Ordinates:

y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9	y_{10}	y_{11}
0.612	1.0	1.06	0.866	0.613	0.50	0.613	0.866	1.06	1.0	0.612

Har- monics	Electric analyser		Calculated		Error in per cent fundamental	
	Sin.	Cos.	Sin.	Cos.	Sin.	Cos.
1st.	+ 1.0	0	+ 0.9986	0	0	0
3rd.	+ 0.5	0	+ 0.4996	0	0	0
5th.	0	0	+ 0.0016	0	0	0
7th.	0	0	+ 0.0016	0	0	0
9th.	0	0	+ 0.0003	0	0	0
11th.	+ 0.02	0	- 0.0013	0	+ 2.0	0

TABLE IV

Curve of Equation: $y = \sin \theta + 0.8 \sin 3 \theta + 0.6 \sin 5 \theta + 0.4 \sin 7 \theta$
 $+ 0.2 \sin 9 \theta + 0.1 \sin 11 \theta$

Ordinates:

y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9	y_{10}	y_{11}
0.978	0.587	0.388	0.297	0.307	0.250	0.307	0.297	0.388	0.587	0.978
Har- monics	Electric Analyser		Calculated		Error per cent fundamental					
	Sin.	Cos.	Sin.	Cos.	Sin.	Cos.				
1st.	+ 1.0	0	+ 0.9987	0	0	0				
3rd.	+ 0.80	0	+ 0.8081	0	0	0				
5th.	+ 0.59	0	+ 0.6040	0	- 1.0	0				
7th.	+ 0.38	0	+ 0.3884	0	- 2.0	0				
9th.	+ 0.195	0	+ 0.1920	0	- 0.5	0				
11th.	+ 0.12	+ 0.005	+ 0.0990	0	+ 2.0	+ 0.5				

TABLE V

Curve of Equation: $y = \sin \theta - 0.8 \sin 3 \theta + 0.6 \sin 5 \theta - 0.4 \sin 7 \theta$
 $+ 0.2 \sin 9 \theta - 0.1 \sin 11 \theta$

Ordinates:

	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9	y_{10}	y_{11}
	0.0015	0.013	0.035	0.048	0.623	1.55	0.623	0.048	0.035	0.013	0.0015

(Actual values twice theses values)

Har- monics	Electric analyser		Calculated		Error in per cent fundamental	
	Sin.	Cos.	Sin.	Cos.	Sin.	Cos.
1st.	+ 1.0	0	+ 0.9998	0	0	0
3rd.	- 0.79	0	- 0.7980	0	- 1.0	0
5th.	+ 0.58	0	+ 0.5890	0	- 2.0	0
7th.	- 0.39	0	- 0.4030	0	- 1.0	0
9th.	+ 0.22	0	+ 0.2120	0	+ 2.0	0
11th.	- 0.12	0	- 0.1200	0	+ 2.0	0

TABLE VI

Curve of Equation: $y = \sin \theta + 0.2 \cos \theta + 0.5 \sin 3 \theta + 0.1 \cos 3 \theta$
 $+ 0.1 \sin 9 \theta + 0.2 \cos 9 \theta$

Ordinates:

y_1 y y_3 y_4 y_5 y_6 y_7 y_8 y_9 y_{10} y_{11}
 0.806 1.073 1.342 0.666 1.130 0.60 0.042 1.066 0.919 0.727 0.560

Har- monics	Electric analyser		Calculated		Error in per cent fundamental	
	Sin.	Cos.	Sin.	Cos.	Sin.	Cos.
1st.	+ 1.0	+ 0.18	+ 1.014	+ 0.189	0	+ 2.0
3rd.	+ 0.49	- 0.10	+ 0.489	- 0.098	- 1.0	0
5th.	+ 0.018	+ 0.01	+ 0.004	+ 0.003	+ 1.8	+ 1.0
7th.	+ 0.023	- 0.028	+ 0.004	- 0.002	+ 2.3	+ 2.8
9th.	+ 0.013	+ 0.21	+ 0.090	+ 0.195	+ 3.0	+ 1.0
11th.	+ 0.011	- 0.023	+ 0.014	- 0.004	+ 1.1	+ 2.3

TABLE VII

Curve of magnetizing current in transformer

Ordinates:

y_1 y_2 y_3 y_4 y_5 y_6 y_7 y_8 y_9 y_{10} y_{11}
 0.30 0.95 1.03 0.90 0.75 0.60 0.55 0.50 0.42 0.35 0.20

Har- monics	Electric analyser		Calculated		Error in per cent fundamental	
	Sin.	Cos.	Sin.	Cos.	Sin.	Cos.
1st.	+ 0.812	+ 0.22	+ 0.8121	+ 0.2165	0	0
3rd.	+ 0.199	- 0.160	+ 0.1933	- 0.1492	+ 0.74	+ 1.23
5th.	- 0.027	- 0.088	- 0.0280	- 0.0886	- 0.12	0
7th.	- 0.041	- 0.020	- 0.0407	- 0.0179	0	+ 2.45
9th.	- 0.041	+ 0.017	- 0.0401	+ 0.0158	+ 0.11	+ 0.12
11th.	+ 0.019	+ 0.038	+ 0.0086	+ 0.0233	+ 3.32	+ 1.85

TABLE VIII

Curve of Sinusoid plus discontinuous peak.

y_1 y_2 y_3 y_4 y_5 y_6 y_7 y_8 y_9 y_{10} y_{11}
 0.60 0.725 0.775 0.775 0.725 0.65 0.75 1.07 1.25 1.15 0.725

Har- monics	Electric analyser		Calculated		Error in per cent fundamental	
	Sin.	Cos.	Sin.	Cos.	Sin.	Cos.
1st.	+ 1.065	- 0.165	+ 1.064	- 0.1630	0	+ 0.19
3rd.	+ 0.425	+ 0.098	+ 0.407	+ 0.0940	+ 1.7	+ 0.38
5th.	+ 0.037	+ 0.085	+ 0.037	+ 0.0833	0	+ 0.15
7th.	+ 0.044	- 0.012	+ 0.040	- 0.0098	+ 0.37	+ 0.19
9th.	+ 0.025	+ 0.005	+ 0.0009	+ 0.0050	+ 1.88	0
11th.	+ 0.037	- 0.020	+ 0.0024	- 0.0088	+ 3.30	+ 1.3

TABLE IX

Rectangular Wave:						
$y = \sin \theta + 1/3 \sin 3 \theta + 1/5 \sin 5 \theta + 1/7 \sin 7 \theta + \dots$						
Ordinates are all equal.						
Har- monic	Electric analyser		Calculated		Error in per cent fundamental	
	Sin.	Cos.	Sin.	Cos.	Sin.	Cos.
1st.	+ 1.0	0	+ 1.000	0	0	0
3rd.	+ 0.325	0	+ 0.318	0	+ 0.3	0
5th.	+ 0.178	0	+ 0.172	0	+ 0.6	0
7th.	+ 0.099	0	+ 0.101	0	- 0.2	0
9th.	+ 0.058	0	+ 0.055	0	+ 0.3	0
11th.	+ 0.055	+ 0.014	+ 0.018	0	+ 3.7	+ 1.4

NOTE. Errors in above table are calculated against the calculated solution of the wave, since the fact that higher harmonics are neglected introduces large errors in the schedule method and the error figures are intended as a criterion of electric machine accuracy.

TABLE X

Triangular Wave: $y = 4/\pi (\sin \theta - 1/3^2 \sin 3 \theta + 1/5^2 \sin 5 \theta - \dots)$											
Ordinates:											
y_1	y_2	y_3	y_4^m	y_5	y_6	y_7	y_8	y_9	y_{10}	y_{11}	
0.262	0.524	0.785	1.05	1.31	1.57	1.31	1.05	0.785	0.524	0.262	
Har- monic	Electric analyser		Calculated		Correct	Error per cent fundamental					
	Sin	Cos	Sin	Cos	Sin	Sin	Cos				
1st.	+ 1.27	0	+ 1.2805	0	+ 1.2732	- 0.3	0				
3rd.	- 0.143	0	- 0.1489	0	- 0.1415	+ 1.2	0				
5th.	+ 0.058	0	+ 0.0588	0	+ 0.0509	+ 0.8	0				
7th.	- 0.034	0	- 0.0346	0	- 0.0259	+ 0.8	0				
9th.	+ 0.025	0	+ 0.0266	0	+ 0.0157	+ 0.9	0				
11th.	- 0.018	- 0.001	- 0.0222	0	- 0.0105	+ 0.7	+ 0.1				

NOTE. It will be noticed that the errors in the calculated values are of the same magnitude as those in the electric machine. The percentages given in the error column are calculated against the correct coefficients and therefore are chargeable in a large measure to the method rather than the electric device for interpreting it.

TABLE XI

SCHEDULE FOR THE ANALYSIS OF A PERIODIC CURVE IN
WHICH ONLY ODD HARMONICS APPEAR UP TO THE
ELEVENTH HARMONIC

Note. The two half-periods will be similar, so that if the mean line be taken between the highest and lowest points in the curve, there will be no constant term. For further simplification the origin should be taken where the curve crosses the zero line.

(1) Divide the half-period into 12 equal parts, and measure the 11 ordinates $y_1 y_2 y_3 \dots y_{11}$; y_0 and y_{12} being each zero.

(2) Then arrange these ordinates as under:

$$\begin{array}{cccccc} y_1 & y_2 & y_3 & y_4 & y_5 & y_6 \\ y_{11} & y_{10} & y_9 & y_8 & y_7 & \end{array}$$

Adding..... $s_1 \ s_2 \ s_3 \ s_4 \ s_5 \ s_6$

Subtracting..... $d_1 \ d_2 \ d_3 \ d_4 \ d_5$

Note. s_1 stands for the sum of y_1 and y_{11} ; d_1 for the difference between y_1 and y_{11} . Great care must be taken as to + and - signs throughout.

(3) Group numbers, to obtain values for use with third and ninth harmonics, as follows:

$$s_1 + s_3 - s_5 = r_1$$

$$s_2 - s_6 = r_2$$

$$d_1 - d_2 - d_5 = e_1$$

(4) Then select from the above numbers and put them in their places the table below, multiplying each by the sine set down in the left-hand column before it is entered.

Angle	Sine-Terms			Cosine-Terms		
	Harmonics			Harmonics		
	1 & 11	3 & 9	5 & 7	1 & 11	3 & 9	5 & 7
Sin 15° = 0.2588	s_1		s_5	d_5		d_1
Sin 30° = 0.500	s_2		s_2	d_4		d_4
Sin 45° = 0.707	s_3	r_1	$-s_3$	d_3	e_1	$-d_3$
Sin 60° = 0.866	s_4		$-s_4$	d_2		$-d_2$
Sin 75° = 0.966	s_5		s_1	d_1		d_5
Sin 90° = 1.000	s_6	r_2	s_6	...	$-d_4$	...
Total first column	...	...	...	...	...	...
Total second column	...	...	...	...	...	...
Sum	$6 B_1$	$6 B_3$	$6 B_5$	$6 A_1$	$6 A_3$	$6 A_5$
Difference	$6 B_{11}$	$6 B_9$	$6 B_7$	$6 A_{11}$	$6 A_9$	$6 A_7$

Check:

$$A_1 + A_3 + A_5 + A_7 + A_9 + A_{11} = 0$$

$$B_1 - B_3 + B_5 - B_7 + B_9 - B_{11} = y_6$$

the harmonic is usually comparable with its relation to the fundamental, and any method of measurements from a curve cannot be expected to give great accuracy upon a harmonic which may have a magnitude of the same general order as the error of the method of obtaining the curve or the measurement of ordinates. When the great saving of time is considered, what might be called the overall effectiveness of the instrument is quite great.

TIME REQUIRED FOR HARMONIC ANALYSIS BY DIFFERENT METHODS

Harmonic analysis may be effected by three general methods:

1. Mathematical
2. Graphical
3. Instrumental

The time required by graphical methods renders them practically useless except for determining one or two

harmonics. They have the advantage of picking out anyone harmonic within the accuracy of the construction, but do not seem to lend themselves readily to large groups of analyses.

The two best known instrumental methods are the Henrici-Coradi machine (Bibl. 55, 65) and the Westinghouse-Chubb polar analyser, (Bibl. 51 and 52). The former consists of five glass spheres, rolling within a carriage so that their displacement is proportional to the ordinates of the curve. The curve in cartesian form is plotted to fairly large scale and followed with a tracer point. Motion of the tracer point in the time-axis direction rotates cages around the glass spheres, the cages carrying planimeter wheels in contact with the spheres. Thus the radius of the circle upon which the planimeter wheel rolls is proportional to $\sin k\theta$ and the motion of the spheres is proportional to y . The resulting displacement of the wheel is the solution of the Fourier integral given in the first part of the paper. Each cage carries two wheels at right angles. Thus for one tracing of the curve, the sine and cosine components of five harmonics may be determined. Resetting of the pulleys upon the top of the cages allows another five to be determined with another tracing of the curve.

The Westinghouse-Chubb analyser requires a curve in polar form, obtained from the polar oscillograph, from which a template must be made of cardboard. This is placed upon a platen, which is given a combined rotational and harmonic translational motion. An arm carrying a roller traverses the edge of the template and guides a polar planimeter in its extremity. After one complete trace of the template the reading of the planimeter gives one component of the harmonic for which the machine was set. Gears are then shifted for another harmonic and the process repeated. Unfortunately, the author has no records of time required for analysis by this machine. If high order harmonics are required, the time is rather long, but this is not fair to the machine, since it may easily be motor-driven and other work done while it is grinding out the analysis. Transferring curves from cartesian form

to polar form requires a good deal of time and introduces a good deal of chance for error, so it is not satisfactory unless used with polar curves.

Some of the times required for analyses are as follows:

Method	Time	No. harmonics determined	Minutes per coeff.	Authority
Steinmetz.....	10 hr.	10	60	D. C. Miller
Schedule.....	3 hr.	8	22.5	D. C. Miller
Schedule.....	1 hr.	8	7.5	F. W. Grover
Schedule.....	2.5 hr.	17	10.6	Author
Schedule.....	15 min.	3	5	D. C. Miller
Coradi Mch.....	13 min.	10	1.3	D. C. Miller
Coradi Mch.....	7 min.	5	1.4	D. C. Miller
Schedule.....	30 min.	6	5.0	Author
Electric Mch.....	3.5 min.	6	.6	Author

The figures are not all comparable. They should be corrected for the number of harmonics determined, as the labor is not proportional with the different methods. The figures for the Coradi machine assume the curve already drawn. If the curve has to be enlarged from an oscillograph film, the added time for this should be included. The last two sets of data are based upon the time for doing the actual calculation, the readings from the curve having already been made, and being the same in each case, both times being for analysis of the same curve. The schedule analysis was performed upon a Marchant calculating machine. It can be done quicker with a slide rule, but the errors in the cosine components are then liable to be very great.

CONCLUDING COMPARISON

In conclusion, it may be of interest to compare the advantages and disadvantages of the different methods.

SCHEDULE METHOD

ADVANTAGES

Simple

Requires only a small chart
for direction

DISADVANTAGES

Very laborious and slow for
more than a few coefficients

Not accurate where har-
monics exist outside range of
schedule.

Fairly accurate within theoretical limitations.

Reasonably fast for a small number of coefficients.

Subject to error unless calculating machine used, particularly in cosine components.

WESTINGHOUSE-CHUBB

Accurate	Requires cardboard template of polar form.
Easy to operate	Difficult to use with curves in Cartesian form.
May be motor-driven and so require little labor.	Requires separate trace for each component.
Semi-portable	Moderately expensive.
Not difficult to obtain	Construction requires much machine work.
Easy to maintain	

HENRICI-CORADI

Accurate	Requires large curve.
Easy to operate	Special table, tracks, etc., must be provided.
Quick	Can only be manufactured by expert instrument makers.
Five harmonics completely determined with one trace.	Expensive.
	Difficult to obtain
	Requires care for maintenance.
	Not portable.

ELECTRIC

Easy and simple to operate.	Subject to limitations inherent with schedule method.
Moderately accurate and eliminates many chances for error.	Subject to usual difficulties of electric networks.
Very quick, as only one group of readings required from curve.	Not extremely accurate.
Reads coefficients either in actual values or per cent of fundamental sine component.	Number of resistances increases rapidly with order of harmonics.
Easy to construct, requiring common materials and little expert workmanship.	Requires storage battery and meter in addition to analyzer itself.
Inexpensive	
Easy to maintain	
Auxiliary apparatus of common form.	

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DISCUSSION ON "AN ELECTROMAGNETIC DEVICE FOR RAPID SCHEDULE HARMONIC ANALYSIS OF COMPLEX WAVES" (DELLENBAUGH), NEW YORK, N. Y., FEBRUARY 18, 1921.

V. Karapetoff: This device is of considerable interest, not only for the solution of the specific problem for which it has been designed, but also as opening new possibilities in the solution of other electrical problems. Now and then we have to solve a system of linear differential equations, or algebraic equations of the first degree, maybe two or five or more, depending upon the nature of the problem, and this is a tedious process. If we know of a physical phenomenon, which mathematically, is reducible say to a system of linear equations of the first degree, then we can set that apparatus and let the laws of nature solve the equations. To illustrate—in an electrical network with given sources of constant voltage and given resistances, the Kirchoff equations for the unknown currents are simultaneous equations of the first degree. So if you want to solve the network, as a rule you have to solve a system of such simultaneous equations.

Now, conversely, suppose that in some problem in engineering or in physics, we have to solve a number of simultaneous equations of the first degree; this may not be an electrical problem at all. Then, if we could arrange an experimental network of conductors with ammeters in various branches and set the e. m. fs. and the resistances of these branches to represent the constant coefficients of the given equations, the ammeter readings would give us directly the values of the unknown quantities. In this way we can make the laws of nature do work for us, that in pure mathematics would be tedious.

Mr. Dellenbaugh does not claim that his device is a universal substitute for all other methods and devices for harmonic analysis. As he himself states, it is only one of the methods which has certain advantages and certain disadvantages. Perhaps the most evident disadvantage is that of a limited number of disks and therefore of a limited number of ordinates, to be taken on the given curve. But I hope that he will continue to work on this idea, and will develop a device that will enable us to analyze harmonics with the highest possible precision, and also will enable us to take automatically into account smaller irregularities, due to much higher harmonics. For instance he mentions Prof. D. C. Miller's work on curve analysis which has been largely in acoustics, where harmonics of higher order are of prime importance. This problem

of curve analysis will never be considered as completed until we develop an instrument by means of which the amplitude and the phase position of a desired harmonic can be obtained almost automatically with a comparatively simple setting of the device.

A. E. Kennelly: When in laboratory practise only an occasional oscillogram calls for analysis, it is ordinarily dealt with by some schedule method, and recourse to mechanical apparatus for deriving the harmonic components is not important. When however, many and frequent oscillograms have to be analyzed, the schedule routine becomes burdensome and tedious. In such cases, a device like that which Mr. Dellenbaugh has described in his paper becomes very useful, not merely because of the time saved in its use, but also because of the greatly reduced liability to accidental arithmetical errors. The device does nearly all of the arithmetic automatically, and indicates the answer on the dial of a milliammeter. If there is any question as to the accuracy of the results in a given recorded case, it becomes an easy matter to repeat the analysis with the device, and obtain a check on the numerical work; whereas the repetition of the work of arithmetical schedule analysis, when carried to a number of harmonics, is often a sufficiently serious task to make the attempt unlikely.

F. S. Dellenbaugh, Jr.: I do not wish to claim undue credit of originality for applying electricity to algebraic solutions of equations. Mr. Arthur Wright, (Bibl. 44.) has done some beautiful work along these lines and built a machine which was not only capable of the solution of ordinary equations but which could also introduce empirical curves. In this machine logarithmic resistances were built by winding wire around a logarithmic shaped form, which then was moved along a scale, forming a sort of electric slide rule. A carriage supported other wires making sliding contact with a bank of these logarithmic resistances. The position of the resistance determines the coefficients of each term and the slope of the sliding wire determines the exponents. By bending the sliding wires to special curves, variable exponents, and thus empirical curves, can be introduced.

As Dr. Karapetoff remarked, the ultimately most desirable machine for harmonic analysis is one which will give all the desired coefficients of the various terms of the Fourier series, after one trace of the curve. This is the ideal which I still hold before me, and undoubtedly some day it will be accomplished. One method of doing this which has occurred to me would be to have a rolling carriage which moves in the

direction of the abscissas. This would carry a stylus moving in the direction of the ordinates. The method upon which it is based is the Kennelly or Fischer-Hinnen method of selected ordinates. With this type of analysis it is only necessary to add and subtract selected ordinates, but special ordinates must be selected in different places for each harmonic to be determined. Thus the rollers of the carriage could carry contact drums closing a circuit at the proper values of ordinates. The stylus could impress voltages in this circuit proportional to the ordinates by a drop wire and sliding contact moved by the stylus. If a fixed resistance and meter, of the Grassot Fluxmeter type, were connected in the circuit, then the reading of the meter after a trace of the curve would give any harmonic coefficient for which the parts were designed. A multiplicity of contact drums, circuits and meters would enable the determination of a multiplicity of harmonics. However, there are a number of practical drawbacks to this scheme, notably that the meter must be one with a large moment of inertia and zero restoring moment, which is a difficult thing to build, and maintain, and one of these would be required for each harmonic to be determined.

There are innumerable ways in which harmonic analysis can be done, all more or less successful and practical, but all having some features of cost, time, or inaccuracies which renders them undesirable. It is extraordinary that a process which can be done in so many ways has no quick and immediate solution.