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C.V. Raman M.A. & Ashutosh Dey

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whether this intrinsic field is to be treated as altogether a true magnetic field, or if it is in part due to other than magnetic forces which are in operation and which, playing the part of magnetic forces, have been evaluated in terms of a magnetic field.

In conclusion, the ferromagnetic equation does not claim to be a complete statement of the complex facts of magnetism, but rather is to be regarded as a limiting law to which ferromagnetic substances tend to conform when the molecular magnets are free to move under thermal agitation with no other constraints than the external field and the intrinsic field, just as the paramagnetic equation is a limiting law to which paramagnetic substances tend to conform when the molecular magnets are free to move under thermal agitation with no other constraints than the externally applied field.

XXXIII. *On Discontinuous Wave-Motion.*—Part III.

By C. V. RAMAN, M.A., and ASHUTOSH DEY *.

[Plate VIII.]

IN a previous communication under the same title published in this Journal (Phil. Mag. Jan. 1916), it has been shown that by imposing a linear distribution of initial velocities with a discontinuity at one end upon a stretched string in its position of equilibrium, free oscillations may be obtained of identically the same type as the principal mode of vibration of a bowed string. The experiment furnishes a striking confirmation of the analytical theory of the motion of bowed strings. In the present paper it will be shown that some of the more complicated types of vibration of a bowed string may also be investigated by similar methods. Figs. 1 and 2 (Plate VIII.) reproduce a few typical records from amongst those secured in the course of this work.

From the mechanical theory, it appears that the most general form of vibration of a bowed string obtained by applying the bow at a point *not* coinciding with any node of importance approaches *in the limit* a type defined by one, two, three, or more *equal* discontinuous changes of velocity travelling over it. But when the limiting conditions are not attained, the mode of vibration may still to a close approximation be defined by a finite number of such discontinuous changes of velocity, these no longer being all equal to one another or of the same sign. In the present paper we

* Communicated by the Authors.

are concerned only with the cases in which *two* discontinuous changes having the same sign define the form of vibration. These cases may be considered in two broad divisions, the first comprising the limiting types in which the two discontinuities are of the same magnitude and sign; and the second, those in which they are unequal. We may discuss these separately.

Two Equal Discontinuities.

In this case the velocity-diagram of the string must evidently consist of three parallel straight lines, each passing through one of the three nodes of the second harmonic, namely, the two ends of the string and its centre. At particular epochs, however—that is, when the discontinuities pass each other or reach an end of the string,—one or even two of the straight lines may contract and vanish, leaving only two lines or one line on the diagram. For instance, if the two discontinuities are at the two ends of the string simultaneously, the diagram reduces to one line passing through the centre of the string. As the discontinuities move in towards the centre, two new lines appear, each passing through an end of the string, and the third line passing through the centre gradually contracts and finally vanishes when the discontinuities both arrive and pass each other at the centre. The line then reappears and the form of the velocity-diagram then goes back to its initial state, passing through the same stages in the reverse order. It is obvious that in this case the centre of the string remains completely at rest, and that the string vibrates in two segments with double the ordinary frequency, the fundamental component being entirely absent. In general, however, the discontinuities do not cross at the centre of the string, but at some other point; if one such crossing takes place at a point distant $l/2 + b$ from an end (l being the length of the string), a second crossing would take place half a period later at the point $l/2 - b$. The vibration-curve at either of the points $l/2 \pm b$ is readily seen to be a simple two-step zigzag. For, the velocity of either point remains unaltered when the two discontinuities pass over it simultaneously in opposite directions, and changes only at the two epochs at which either discontinuity passes over it *singly*. If $\tan \alpha$ be the slope of the lines of the velocity-diagram, the velocity at either point changes at the first epoch from $b \tan \alpha$ to $(l/2 - b) \tan \alpha$, and changes back again to its original value at the second epoch. The fraction of the total period of vibration during which the larger velocity subsists is given by the ratio $b/2l$, or by

the ratio $(l-2b)/l$, according as b is smaller or greater than $l/4$.

The form of the vibration-curve at other points on the string in the case considered above may also be found by tracing the successive changes of velocity. It is obvious that the curves for points lying outside the limits $l/2 \pm b$ are different in character from those for points lying between these limits. For, the discontinuities pass any point lying outside the limits $l/2 \pm b$ alternately in opposite directions, so that the velocity of such point alternates between two, and only two, constant values. But within those limits the discontinuities pass any given point, first successively in the same direction, and then successively in the opposite direction. The vibration-curves of points lying outside the limits $l/2 \pm b$ are four-step zigzags, in which alternate lines are parallel to each other. The vibration-curves of points within the limits $l/2 \pm b$ are also four-step zigzags, in which two of the lines are parallel to each other, but the other two are not. The most characteristic vibration-curve is that for the centre of the string. This point remains at rest for a considerable interval, twice in each period of vibration, the two positions of rest lying one on either side of the undisturbed position of the string. The smaller the distance b is, the more closely are the two positions of the centre of the string situated. The vibration-curve of the middle point of the string thus consists of two long horizontal lines which take up a considerable fraction of the period of vibration, and are separated by short steep lines representing the motion from one position of rest to the other, and *vice versa*.

Experimental Method.

It is evident that the modes of vibration described above would be perfectly reproduced if a string has initially a uniform angular velocity about one point, and if in the course of this motion two other points on it situated at equal distances on either side of the first are suddenly fixed with the result of isolating the string between them. If A and B are the points thus fixed and C is the point about which the string has initially a uniform angular velocity, so that $AC=CB$, the mode of vibration elicited would evidently depend upon the simultaneity, or otherwise, of the fixation of the string at A and B. If the two points are fixed at the same instant, the discontinuous changes of velocity travelling inwards from A and B meet at the centre, and the string would, as already explained, vibrate in two segments. On the other

hand, if one of the points is fixed earlier than the other by an interval of less than half a period, the discontinuities cross at some point on the line A B other than the centre of the string, the position of such point being capable of adjustment by a suitable alteration of the interval of time between the fixing of the string at the points A and B.

The experiment here indicated may be successfully performed with a steel "string" stretched on a light frame capable of motion round a horizontal axis at right angles to it. The necessary tension is secured and maintained by a spring-balance fixed to one end of the frame, which keeps the string taut. A small load fixed to the frame on one side causes it to swing into a horizontal position with the angular velocity desired. At this position the string comes up against two stops, one on either side of it, situated at equal distances from the horizontal axis about which the frame and string move. If the two stops A and B are exactly in line with the string at this instant, the impacts take place simultaneously. By putting one of the stops out of line, any desired interval between the impacts may, however, be secured. It is found (with the arrangements adopted by us) that a vertical displacement of one of the stops by a millimetre is equivalent to about half a period in the interval between the impacts, smaller displacements securing proportionately smaller intervals. Just before the impacts take place, the dark slide containing the photographic paper is released automatically, and moves in a direction parallel to the string, recording the initial motion and the vibration of the string at any point desired. The optical arrangements are similar to those described in the first paper of the series.

Fig. 1 (Plate VIII.) shows eight records secured by these arrangements for various points on the string, and exhibits a complete agreement with the indications of theory. Two of the records (for $3l/8$ and $5l/8$) show a close approach to the simple two-step zigzag form for the point at which the discontinuities cross. The ratio of the two velocities of this point or of any other point lying outside the limits $l/2 \pm b$ is equal to the ratio of the distances of such point from the centre and the nearer end of the string. The photographs furnish a confirmation of this kinematical law. Numerous records (not published) have also been secured of the vibration-forms obtained for values of b ranging from zero to $l/2$. When $b=l/4$, the two sides of the two-step zigzag for the points $l/2 \pm b$ are equally steep, and a vibration of the type considered here cannot therefore be elicited by the bow with a two-step zigzag motion at the bowed point if $b>l/4$. As b

is increased from $l/4$ to $l/2$, the vibration-curve at every point on the string approaches the simple two-step zigzag form, and finally becomes identical with it.

Two Unequal Discontinuities.

By setting the axis of rotation of the frame elsewhere than at the point bisecting AB , the case of two unequal discontinuous changes of velocity may be realized experimentally. In this case, no line on the velocity-diagram passes through the centre of the string. If the discontinuities cross at the points $l/2 \pm b$, theory shows the vibration-curves for points outside these limits to be four-step zigzags, in which the two lines of small slope are parallel, but the other two lines are not parallel to each other. Between the limits $l/2 \pm b$ the vibration-curves are four-step zigzags in which none of the lines are parallel. If $b=0$, the vibration-curves are everywhere of a symmetrical type. Otherwise they are everywhere unsymmetrical. The vibration-curves at the points $l/2 \pm b$ consist of only three lines in each period unless $b=0$, in which case there are only two. The most interesting forms are those obtained at the centre of the string and those near an end.

Fig. 2 (Plate VIII.) shows some records obtained when the axis of rotation of the frame passed through a point of trisection of the length AB . Here again a complete agreement with theory is shown. Forms of vibration set up by two discontinuous changes of velocity may be obtained by bowing a string near one end with pressure insufficient to elicit a pre-eminent fundamental*. In this case the discontinuities are generally unequal, though in special cases they may approach or attain equality. Generally, the vibration-forms thus obtained are unsymmetrical; but they may closely approximate to the symmetrical types under favourable conditions †.

Summary and Conclusion.

By setting up two discontinuous changes of velocity, either simultaneously or in succession, upon a stretched string which has initially a uniform angular velocity,

* "On the Wolf-note in Bowed Stringed Instruments." By C. V. Raman, M.A. Phil. Mag. Oct. 1916.

† Some of the symmetrical stationary forms with two unequal discontinuities obtained by bowing were observed and figured by Helmholtz ('Sensations of Tone,' English Translation by Ellis, page 85). Helmholtz's drawing is, however, not quite correct, as the shorter descent should appear in the vibration-curve as less steep than the longer.

vibration-forms may be obtained which are identical with: (a) the motion of a string bowed with suitable pressure close to its centre, or at a distance from it not exceeding a quarter of its length; (b) the motion of a string bowed near an end with pressure insufficient to elicit the fundamental pre-eminent; and (c) the successive stages of the vibration obtained by bowing a string near an end at the "wolf-note" pitch, or under certain other circumstances in which *cyclical*, not periodic, vibrations are obtained.

The photographic records are in complete accord with theory, and are an interesting illustration of the experimental analysis of discontinuous vibration-forms.

The experiments described in this note were carried out in the Laboratory of the Indian Association for the Cultivation of Science.

Calcutta,
November 8th, 1916.

XXXIV. *Note on Prof. Anderson's "Focometry of Lens-Combinations."* By ROBERT E. BAYNES, M.A.*

PROF. ANDERSON'S method (p. 157) of determining the focal length of a combination is most interesting, but the degree of its exactness requires examination; for the data given in the paper for a particular case, viz. $x_1=51.0$, $y_1=9.0$, $x_2=24.5$, $y_2=9.5$, $d=26.9$, lead to curious results when treated in different ways.

Prof. Anderson's formula

$$f = d \div (x_1/y_1 - x_2/y_2)$$

gives $f=8.71$, but the distance between the nodal planes when calculated from this value of f with x_1, y_1 is 8.52 and with x_2, y_2 is 6.58, a difference which cannot be ignored.

The fact is that there is a definite relation between the five given magnitudes, viz.

$$d(1 - y_1 y_2 / x_1 x_2) = (x_1 - y_1) - (x_2 - y_2),$$

which is not satisfied in the above case and which requires $d=29.0$ with the given values of $x_1, \dots$. With this value of d we should have obtained 9.39 for the focal length and 5.93 for the distance between the nodal planes.

* Communicated by the Author.

FIG. 1.

Photographs of vibration-curves showing initial motion and subsequent vibration with two equal discontinuous changes of velocity.

FIG. 2.

As in Fig. 1, but with two *unequal* discontinuous changes of velocity.