ART. VI.—*Electrometry with the Displacement Interferometer**; by CARL BARUS.

1. *Introductory.*—The possibility of compensating any displacement of the mirror N by a corresponding displacement of the micrometer M , i. e. the fact that the ellipses are never lost in a properly adjusted displacement interferometer, for any amount of motion of the mirror N , however sudden it may be, suggests the use of this method for electrometry. Some time ago I made† experiments of this kind, using the device of the absolute electrometer for the purpose. A mirror was attached to the disk and the force of restitution obtained from a pendulum suspension for parallel motion. It appeared that the apparatus is very insensitive for small potentials, even with the use of the interferometer.

A cylindric or lamellar form, such as is usually referred to in constructing the theory of the instrument, seemed to be specially available; but the sensitiveness here also, if torsion systems are excluded, is unexpectedly small, unless (as in Thomson's original apparatus) excessive charges are given to the needle, a procedure which introduces its own difficulties. Nevertheless I constructed the apparatus in a variety of forms, each of which will be described, in turn, in order to ascertain in how far the sensitiveness, which may be estimated from theory, may be actually realized in practice; i. e., to actually confront the instrumental difficulties of the problem, with a view to the use of the method in absolute measurements of potential.

2. *Cylindric electrometer. Movable cylinder without.*—Figs. 1 and 2, both sectional elevations, show the first form of electrometer in which the movable cylinder (usually highly charged) is external to the fixed cylinders h and i carrying opposite charges. The latter are supported on hard rubber pillars, u and v , rising from the adjustable feet q , of which r and s are the leveling screws and the set screw. The whole arises from a narrow base of brass plate AA . On this base the braced scaffolding of very thin brass pipe efg is also mounted, grasped above by the light cross rod d . This makes a light but very firm support for the two parallel horizontal rods of hard rubber cc , secured at b and carrying the revolvable brass rollers or reels aa snugly. From aa the threads of very thin copper wires yy .007^{cm} in diameter depend, their lowest point being attached to the cylinder kk , making of it a pendulum. To obviate torsional vibration the wires yy con-

* Abridged from a Report to the Carnegie Institution, Washington, D. C.

† Physical Review, iv, p. 400, 1897.

verge on their front elevation from aa to the top tt of the cylinder, while in their side elevation they are rigorously parallel and equally long. They may be lengthened or shortened by turning the close fitting rollers aa , so that the cylinder kk may be placed accurately concentric to the cylinders ih . The latter may be mounted with a cylinder of wood passed quite through them, which is withdrawn after they are

FIG. 1.

FIG. 1. Cylindrical electrometer, vertical section. Front view.

fastened. The clamp screws U and those at the end of a are convenient for charging, the wires passing out of the light tin plate cover $CCDD$ through hard rubber cylinders at Z . The top DD may be lifted off. When there is no danger of induction, DD may be removed. The ellipses of the interferometer, in a room free from draft, are quite adequately stationary even without DD , a surprising result.

The cylinder kk is either of very thin aluminum tubing, or preferably of gilt paper. The wires yy pass through small holes at the top ends. To secure adequate damping, thin circular rings of mica or of paper, BB , surround the ends, and the front one carries the light mirror w , which reflects one of

FIG. 2.

FIG. 2. Cylindrical electrometer, vertical section. Side view.

the component beams of the interferometer, as indicated at the glass window p . If liquid damping is desired, in the case of a heavy cylinder, wires ending in a vane may be dropped from one of these disks with the vane in a box of oil below. This

was tried without apparent disadvantage, but a light paper cylinder and air damping is of course preferable, being far more sensitive. The mica B should be somewhat adjustable so that the beam p may be given the proper direction, roughly. The methods for doing this will be described below.

The whole apparatus, covered at least as far as C , was then mounted on one arm of the interferometer by the aid of a large clamp of the ordinary pattern, the clamp having been screwed into the base A at F . As the arm is of gas pipe, F is an ordinary nipple of suitable length. It is advisable to further support the tin case with wood uprights from the table to avoid the effect of tremors due to the vicinity of an active laboratory; but no great difficulty was here encountered. In fact, the ellipses of the interferometer were obtained in all cases after a preliminary rough adjustment of the mirrors with surprising ease, and they were quite as stable and available for measurement as if the mirrors had been rigidly fixed. Naturally this was quite contrary to our anticipations, but it is shown by the large number of experiments made, each of which required a new adjustment.

The measurement consists merely in a compensation of the displacement of the movable mirror on the electrometer, by the micrometric displacement ΔN of the mirror belonging to the other component beam. The center of the displaced ellipses in the field of the telescope is thus brought back to the fiducial sodium line. The compensation admits of an accuracy of about $\Delta N = .0001$ centimeter without resorting to the interference rings. Other remarks on the optical method are given in the next paragraph.

3. *Cylindrical electrometer; movable cylinder within. Interferometry.*—The frame-work and the mounting here is in general the same as in the preceding case and the movable cylinder kk is the same, but differently suspended. Details must be omitted here and other forms of the instrument (disk pattern, closed field pattern) will be described below.

The general treatment of the displacement interferometer has been given in my earlier papers. The field of the telescope after the appropriate two (of the four) undeviated images of the slit, from front and rear of the grating plate, have been put into coincidence horizontally and vertically (the images being usually bluish and yellowish), contains but three superposed spectra, although there are four white images, two from the front and two from the rear of the plate. For the component beam reflected from the rear face of the grating is not diffracted. It is thus advantageous to eliminate the third spectrum also, and to keep only the two which are superimposed for interference in the field. This may be easily done as follows: the two component beams reflected from the mirror

facing the ruled side of the grating plate usually appear as separated images on the mirror, about $1/8$ inch apart, and the one yellowish in color may be blotted out by a small screen near the mirror. As a result of this the stationary interferences which are due to the two beams from the same mirror also vanish. The spectrum is now clear and contains only the interfering superpositions, giving the ellipses their maximum definition.

In other respects the mounting on the interferometer is the same as before, and very little difficulty was experienced in obtaining the ellipses. The top of the case *DD*, here necessarily a square box, may usually be removed without disadvantage. If liquid damping is necessary in case of a heavy cylinder *kk*, the wire with vane may be dropped into a vessel of oil from the rear end of *tt*, the front carrying the mirror.

The usual method of connecting was used for potential measurements, one pole of the experimental cells being alternately earthed as well as one pole of the charging battery. A Mascart key was found very useful. When the needle swings to and fro the ellipses are visible at the extreme elongations of the deflection (if not too large) of the vibrating cylinder. They flash into the field of the telescope at the two ends of it, but are invisible between. The distance apart of the positions of visibility gradually diminishes until they coincide and the cylinder is stationary. If the ellipses are clear, slight motion does not interfere with the measurements, as the mean position is readily determinable in the displacement method. This would not, however, be the case had the evanescence of rings been made the basis of measurement.

4. *Equations. Cylindrical electrometer.*—The equations may be found in the usual manner, and as so little of the field is displaced during deflection they apply with acceptable accuracy. Virtually a small cylindrical shell of uniform field is supposed to disappear on one side to reappear on the other. Hence, let V_3 , V_2 , and V_1 be the potentials of the two fixed and a movable cylinder (needle) respectively. Let f_{12} denote the electric force between cylinders (2) and (1), and f_{13} the force between cylinders (3) and (1), and R the common radius of the fixed cylinders, r that of the movable cylinder. Hence, if $R-r$ is small,

$$f_{12} = (V_1 - V_2)/(R-r); f_{13} = (V_1 - V_3)/(R-r)$$

If the displacement of the cylinder is dx the changes of potential energy on the two sides are

$$dW_{12} = \frac{R+r}{R-r} \frac{V_1 - V_2}{8} dx, \quad dW_{13} = -\frac{R+r}{R-r} \frac{V_1 - V_3}{8} dx$$

and hence the force acting in the direction x if $W = W_{13} + W_{12}$ is after reduction

$$X = \frac{1}{8} \frac{R+r}{R-r} (V_3 - V_2) \left(2V_1 - (V_2 + V_3) \right)$$

On the other hand, the restoring force is for small displacements

$$X = Mgdx/l$$

where M is the mass of the cylinder, l the length of the pendulum suspension; whence if $dx = \Delta N$, in electrostatic units,

$$\left(V_3 - V_2 \right) \left(V_1 - \frac{V_3 + V_2}{2} \right) = \frac{4}{l} \frac{Mg}{(R+r)} \Delta N$$

which is the required equation. Since, if t is the thickness, L the length, and ρ the density of the movable cylinder,

$$\left(V_3 - V_2 \right) \left(V_1 - \frac{V_3 + V_2}{2} \right) = \frac{8\pi L t \rho}{l(1+R/r)} \Delta N$$

the appurtenances, like mirror, etc., being disregarded. A given deflection ΔN will therefore correspond to smaller $V_3 - V_2$ as $R-r$, L , t , are smaller and l and V are larger.

If the instrument is used idiostatically (electrostatic units),

$$\left(V_3 - V_2 \right)^2 = \frac{8}{l} \frac{Mg}{(R+r)} \Delta N$$

If one pole of the cell $V_3 - V_2$ is earthed, $V_2 = 0$, and the deflections on commutation are ΔN and $-\Delta N'$,

$$V_3 = \left(300 \right)^2 \frac{4}{V_1} \frac{Mg}{l(R+r)} \frac{\Delta N + \Delta N'}{2} = 300^2 \frac{8\pi L t \rho}{V_1 l(1+R/r)} \frac{\Delta N + \Delta N'}{2} \text{ volts.}$$

If the cylinder V_1 is made of paper, the following dimensions seem to be reasonably convenient limits, so that for $\Delta N = .0001$ centimeter, $L = 5$ cm., $t = .2$ cm., $\rho = 1$; $l = 100$ cm., $V_1 = 300$ volts, whence

$$V_3 = 3.8 \times 10^{-4} \text{ volts,}$$

or about 10^{-4} volts per ring.

One should thus be able to measure voltages even within 4×10^{-4} ; but on trial the actual limits fall much within this. The use of wide apparatus has the advantage only of diminishing the effect of appurtenances like the mirror, etc. In the practical work below, however, these limits could not be reached for reasons which will appear.

The idiostatic method under the same circumstances, $\Delta N = .0001$ centimeter, should just measure

$$V_3 = .5 \text{ volt,}$$

so that alternating currents much within 1 volt will apparently be measurable. This method is naturally much less sensitive than the other, but its absolute character and its adaptation to alternating currents are of interest. The sensitiveness of the idiostatic electrometer as here designed, and in the above case of the absolute electrometer, would thus appear to have the same practical limit. In the way of absolute measurement, nothing has been gained.

5. *The Same. Disk electrometer.* In this case a charged disk with a guard ring (the apparatus to be described hereafter, figs. 5 and 6) at potential V_1 is suspended bifilarly, as above, between the two concentric parallel disks of the condenser, at potentials V_2 and V_3 . The adjustment may be regarded as the extreme case of the cylinder electrometer with closed ends and the equations will be, necessarily, nearly the same. Let d be the distance apart of the plates of potential V_2 and V_3 , D the distance between the plates of potential V_1 and V_2 , r the radius of the movable disk. The total energy W of the variable system is then, after reduction

$$W = \frac{r^2}{8\pi} \left\{ \frac{(V_2 - V_1)^2}{d} + \frac{(V_1 - V_3)^2}{D - d} \right\}$$

and the corresponding mechanical force

$$X = \frac{r^2}{8} \left(\frac{V_2 - V_1}{D - d} + \frac{V_3 - V_1}{d} \right) \left(\frac{V_1 - V_2}{D - d} - \frac{V_3 - V_1}{d} \right)$$

which depends essentially on d as well as on D .

Let the disk be in the middle, or $2d = D$, and put $X = Mg\Delta N/l$ as above, where M is now the mass of the disk and appurtenances, ΔN the interferometer displacement, l the length of the bifilar pendulum, and g the acceleration of gravity. Then

$$\left(V_2 - V_3 \right) \left(V_1 - \frac{V_2 + V_3}{2} \right) = \frac{MgD^2}{lr^2} \Delta N$$

Let $V_2 = 0$ and $M = \pi r^2 t \rho$, t being the thickness and ρ the density of the disk, the latter being sufficiently large so that the appurtenances may be ignored. Then on commutation and in electrostatic units, ΔN being the mean displacement,

$$V_3 = \frac{MgD^2}{V_1 l r^2} \Delta N = \frac{\pi t \rho g D^2}{V_1 l} \Delta N,$$

which (for a sufficiently large disk) is independent of its area.

To estimate the limiting sensitiveness the following apparently reasonable values may be inserted

$V_1 = \frac{1}{3}$ els., $t = .05$ cm., $\rho = 1$, $D = .1$ cm., $\Delta N = 10^{-4}$ cm., $l = 10^2$ cm.
Whence

$$V_3 = 1.4 \times 10^{-3} \text{ volts.}$$

The limiting sensitiveness is thus of the same order as in the cylindrical case for the same V_1 , as might have been anticipated. It is probable, however, that the present conditions may be more nearly realized in practice, as the apparatus is essentially simpler.

If $V_2 = V_1 = 0$, the idiostatic case is identical with the absolute electrometer.

6. *The closed field electrometer (disk and cylinder combined).* If the two cases of the preceding paragraphs be combined, with the symbols of potential and distance the same as before, the energy of the system is, in electrostatic units,

$$W = \frac{1}{8} \left\{ (V_3 - V_2)^2 \left(\frac{r^2}{d} + L \frac{R+r}{R-r} \right) + (V_1 - V_2)^2 \left(\frac{r^2}{d'} + L' \frac{R+r}{R-r} \right) \right\}$$

and hence the forces become

$$X = \frac{1}{4} \left(\frac{r^2}{d^2} + \frac{R+r}{R-r} \right) (V_3 - V_2) \left(V_1 - \frac{V_2 + V_3}{2} \right) = \frac{Mg}{l} \Delta N$$

If $V_2 = 0$,

$$V_3 = \frac{4 Mg \Delta N}{V_1 (r^2/d^2 + (R+r)/(R-r)) l}$$

after commutation, where l is the length of the cylinder. The sensitiveness of this apparatus depends very largely on d , that is upon the disk, in any practical case; but it should be more nearly absolute. Experiments were not made.

7. *Corrections.* The most important consideration to be made here is the effect attributable to the want of symmetry in the orientation of the disk or movable charge, supposing the latter and the plates are quite parallel. Let k be the axial distance of the uncharged disk from the plane midway between the condenser plates (guard ring). In such a case, if the electric displacement is ΔN , the quantity $\Delta = k + \Delta N$ must be added on one side and subtracted on the other, so that $d = D/2 + \Delta$, $d' = D/2 - \Delta$. Thus the displacing force is, after reduction,

$$X = \frac{r^2}{2} \left(\frac{V_1 - V_2}{D - 2\Delta} + \frac{V_3 - V_1}{D + 2\Delta} \right) \left(\frac{V_1 - V_2}{D - 2\Delta} - \frac{V_3 - V_1}{D + 2\Delta} \right)$$

which may be further reduced.

If nothing is neglected, and under the assumption of a uniform field, for a positive charge, V_3 corresponding to a displacement ΔN^1 with $V_2 = 0$; and for a negative charge, $-V_3$

corresponding to a displacement $-\Delta N''$, the mean of the equations, $\overline{\Delta N} = \frac{1}{2}(\Delta N' + \Delta N'')$, after reduction becomes

$$\frac{Mg \Delta N}{l} \frac{D^2}{r^2} \frac{(1 + 2 \Delta / D)^2}{V_1} = V_3$$

the corrected equation required, reducing to the above case when $\Delta = 0$. The correction factor is thus $1/F = (1 + 2\Delta/D)^2$ and was computed in an auxiliary table.

It is now easy to discuss the conditions of equilibrium, for the forces X are given by the equations (second member referring to the pendulum and third member to the electrical forces)

$$X = \frac{Mg \Delta N}{l} = V_1 V_3 \frac{r^2}{d^2} \frac{1}{(1 + 2 \Delta / D)^2}$$

Hence, if $\Delta N = \Delta$, $k = 0$, the condition under which the disk just moves, without interruption, from the guard ring to the condenser plates, i. e., the limiting value of the potential products, ($V_1 V_3$) for stable positions of the disk, since

$$\frac{dX}{d\Delta} = \frac{Mg}{l} = \left(V_1 V_3 \right) \frac{r^2}{D^2} \frac{-4 \Delta / D}{(1 + 2 \Delta / D)^3}$$

are (in electrostatic units)

$$\left(V_1 V_3 \right) = \frac{Mg}{4l} \frac{D^2}{r^2} \frac{(1 + 2 \Delta / D)^3}{\Delta / D}$$

Assuming $V_1 = 250$ volts, the forces X for the pendulum and the electric field were also computed.

If now, we insert the value of V_3 from the above equation and reduce

$$-6 \Delta / D = 1 \text{ or } -\Delta = D/6$$

Hence the value of V_3 which corresponds to tangency is

$$V_3 = \frac{Mg D^2}{l V_1 r^2} \left(-\frac{1}{6} \right) \left(\frac{2}{3} \right)^2 = \frac{2}{27} \frac{Mg D^2}{l V_1 r^2}$$

or $V_3 = 145$ volts, above which charge the disk passes continuously from guard ring to plate.

If the suspension is provided with a horizontal micrometer by which it can be shifted as a whole from k to k' taking the needle with it, k may be eliminated. But the expression is not simple.

The idiostatic method needs a corresponding correction, and if Δ^2 is neglected in comparison with D ,

$$\frac{2 Mg D^2}{l r^2} \Delta N = V_3^2 \left(1 + 4 \Delta / D \right)$$

As the new factor is practically k or constant, V_3^2 is linear with ΔN . If, as above indicated, the suspension is provided

with a horizontal micrometer by which it can be shifted as a whole from k to k' taking the needle with it, k may be eliminated.

The equation may be written in terms of the two forces X (gravitational and electric)

$$X = \frac{Mg \Delta N}{l} = \frac{4 r^2}{D^2} V_1^2 \frac{\Delta/D}{(1 - 4 \Delta^2/D^2)} = \frac{4 V_1^2 r^2}{D^2} F_1$$

A table was computed for F_1 and the two forces X may be found at once. Again, the condition may be practically stated, inasmuch as the values $dX/d\Delta$ may be easily derived. They are for the pendulum Mg/l and for the electric field

$$\frac{dX}{d\Delta} = \frac{4 V_1^2 r^2}{D^3} \frac{1 + 12 \Delta^2/D^2}{1 - 4 \Delta^2/D^2}$$

Equating them, we derive the voltage of transition to be characterized by (V_1), as

$$(V_1)^2 = \frac{Mg D^3}{4 l r^2}$$

since $\Delta = 0$ at the origin. Hence above (V_1) the disk would move on any slight disturbance without interruption from the guard ring, $\Delta = 0$, to either plate of the condenser; but within (V_1) the apparatus is available for accurate measurement of V_1 , if k is equal to zero by trial.

8. *Experiments. Cylindrical electrometer; movable cylinder within.*—Tests of the apparatus described were begun by using a rather heavy cylinder of aluminum, damped with a vane submerged in a cup of oil. The cylinder itself weighed but 6.546 grams; but air damping was quite inadequate, though the cylinder might easily have been etched to a more appropriate degree of thinness in acid.

It is therefore necessary to make use of paper cylinders, even if the possibility of securing more nearly accurate values of the diameter of cylinder is the advantage of the metallic appurtenance.

No trouble was encountered in using a light plane mirror about 1/4 inch in diameter, with axes for adjustment in a bit of cork. To sharpen the ellipses the (yellow) image at the movable mirror should be screened off, leaving the blue image and but two superposed spectra.

Light gilt paper cylinders, weighing with mirror, etc., only about a gram, were therefore installed. An example (Table 1 and fig. 3 *a* and *b*) of the many results obtained can only be given here, M being the mass and r the radius of the movable cylinder, and R the radius of the fixed cylinder.

TABLE I.

Cylindrical electrometer; movable cylinder within.

I.— $M = 1.105$ g.; $R = 2.07$ cm.; $r = 1.7$ cm.; $l = 21$ cm.;
 $V_2 = V_3 = 0$.

$10^3 \Delta N$	V_3	Voltmeter
0.08	15	15
0.25	28	29
0.60	43	41
1.12	60	61
1.87	77	81

II.— $M = 1.189$ g.; $R = 2.07$ cm.; $r = 1.90$ cm.; $l = 21$ cm.;
 $V_2 = 0$; $V_1 = 250$ volts.

$10^3 \Delta N$	V_3	Voltmeter
1.74	6.1	5
2.17	7.6	7
3.31	11.6	11.5
4.53	16.0	15
5.85	20.6	19

FIG. 3.

FIG. 3. Electrometer data compared with actual voltages.

The observations with the needle charged at 250 volts show the maximum sensitiveness obtained with short suspensions. These results and those by the idiostatic method are quite satisfactory and the differences are more liable to be sought for in the voltmeter with which the comparison was made than in the electrometer. It indicates the sufficiency of the equation assumed, apart from correction for the end of the cylinder, so that the latter cannot be menacing.

In addition to these experiments, a long suspension ($l = 150$ centimeters) in which the sensitiveness would have been increased 7 times or equivalent to 5×10^{-3} centimeter per volt or .006 volt per vanishing ring was installed. The ellipses were easily found but they were continually in motion, owing to the friction of air currents moving across the wires. To make the adjustment available it would have been necessary to build a closed case around the bifilar suspension over 1.5

meters in height. It was not thought worth while to do this, and further experiments were abandoned.

9. *Cylindrical electrometer; movable cylinder within. Summary.*—The above experiments were made in the midst of the turmoil of a large city and near the engineering laboratories of a University. It is rather remarkable, therefore, that the ellipses were so easily found throughout and so easily made use of; but it was quite out of question to use the evanescence of rings by which the sensitiveness could have been increased over twenty fold. Though the work was carefully done, it is intended merely to exhibit the general character of the method, inasmuch as the storage battery which was drawn upon was unavoidably in use elsewhere in the laboratory and the potential may have fluctuated.

The design of the apparatus with a movable inside cylinder is probably the least interesting of those used. Thus it is difficult to keep a paper cylinder quite smooth or to give it a rigorous cylindrical shape and the correction for the ends can scarcely be estimated. For high charges any asymmetry of the movable cylinder is liable to place it in contact with the fixed cylinders. Thus there is a limit of sensitiveness from a purely instrumental point of view, not contemplated in theory. In the above experiments with short suspension (21 centimeters) $\cdot 7 \times 10^{-4}$ centimeter per volt was the largest double displacement practically obtained, which would mean about $\cdot 04$ volt per vanishing ring. With the long suspension $\cdot 006$ volt per ring may be estimated. It is far short of the theoretical datum of §4.

10. *Cylindrical electrometer; movable cylinder without.*—After a few preliminary experiments with the metal cylinder of aluminum, the gilt paper cylinder was tried and was at once successful. The mirror was attached to the paper wing needed for damping by the aid of a bent piece of thin steel wire, cemented on so as to give a horizontal axis on passing through the perforation of the bit of cork holding the mirror. For a vertical axis the whole apparatus is rotated (cf. figs. 1 and 2). The results are given in Table 2 and fig. 4a.

TABLE II.

Cylindrical electrometer; movable cylinder (gilt paper) without.

I.—M = 2.075 g.; *R* = 1.75 cm.; *r* = 1.59 cm.; *l* = 21 cm.; *V*₁ = 250 volts.

Drum	$10^3 \Delta N$	V_2	Voltmeter
11.2	2.79	18.7	18
22.9	5.72	38.2	38
35.3	8.83	59.1	58
27.6	6.90	46.1	48

II.— $M = 1.0930$ grams.

Drum	$10^3 \Delta N$	V_s	Voltmeter
24.9	6.22	21.9	21
47.3	11.82	41.6	41

FIG. 4.

FIG. 4. Electrometer data compared with actual voltages.

The differences in this table are more liable to be of the voltmeter than of the electrometer. The needle easily carried a potential of $V_1 = 250$ volts. With $V_s = 100$ volts, the needle became asymmetric and observations could no longer be taken. The sensitiveness is about .0003 centimeter of double displacement per volt for the short suspension, so that .1 volt per ring is the equivalent datum. The present form is thus easily made as sensitive as the preceding form, besides being much simpler in general design and installation.

The experiment was now pushed a step further in the direction of sensitiveness by using an even lighter paper cylinder with paper wings, the needle having the constants given in the second series of Table 2 and fig. 4b. The sensitiveness is now about .0006 centimeter of double displacement per volt, twice that of the preceding case as it should be. This apparatus did not, however, behave as satisfactorily as the other, there being greater tendency of the movable cylinder to cling to the stable parts, whenever V_1 was high. In case of smaller potentials or of the idiostatic method there would be no difficulty in this respect. No doubt, if greater time and patience were spent on the work (a cylinder of celluloid suggests itself) the final result could be somewhat improved; but the limit found (6×10^{-4} centimeter or 20 rings per volt) will not easily be exceeded in practice.

11. *Disk electrometer. Apparatus and experiments.*—The disk electrometer deserves special attention because it is simpler in design and practically more sensitive than the cylinder types. At the same time, however, it is more treacherous and without special precautions there is danger of short-circuiting the highly charged disk, while the sensitiveness is enormously variable in response to any unsymmetrical position

of the disk. The instrument used is shown in figs. 5 and 6, in sectional side and front elevation. V_1 is the charged disk on the bifilar suspension of very thin copper wire yy' , .007 centimeter in diameter. The disk is made of thin mica, silver-plated, carried by the horizontal axial steel rod dd , and surrounded by the guard ring of very thin copper V_1' . Parallel to the disk and equidistant (d) from it are the plates V_3 and V_2 of the condenser at a distance D apart, so that D is twice d . The plate V_2 is earthed and firmly held on the arm of the interferometer. The plates V_3 and V_2 are spaced at three points

FIG. 5.

FIG. 6.

FIG. 5. Disk electrometer, vertical section. Front view.

FIG. 6. Disk electrometer, vertical section. Side view.

by hard rubber gaskets, cc , through the holes of which hard rubber screws are passed, the plates being ultimately secured by nuts a , a' , and a'' , on the outside of the plates. The nuts a terminate in clamp screws. The guard ring V_1' and the suspension wires yy' are in metallic connection at the tops of the suspension, the latter being the same already shown in figs. 1 and 2. The small mirror, m , is attached to a small plate of cork, n , which is slotted parallel to the rod dd , the latter being clutched by the jaws of cork which make up the sides of the slot. In this way m may be rotated around a horizontal axis, while the apparatus as a whole may be revolved about a vertical axis.

If the grating is capable of being raised or lowered, it is not difficult to adjust the apparatus and find the ellipses. The damping of the disk is naturally good, though it may be improved by surrounding it with the case shown in fig. 1.

12. *Experiments with the disk electrometer.*—The experiments were begun with but a small distance, D , between the plates of the electrometer. In such a case the disk can not carry very high potential before it is drawn across to the condenser plates and short circuited. In fact, though the mica disks may be made very light, the annoyance of short-circuiting is correspondingly increased and the great advantage of sensitiveness cannot for this reason be realized unless the idiostatic method is used.

The disk was charged with the storage battery to 101 volts and a number of small potentials measured at the plates. The computed and uncorrected results were about 3 times as large as the true voltages. The sensitiveness is here surprisingly large, almost 8 drum parts or 2×10^{-3} centimeter per volt, *i.e.*, 70 rings per volt. The distribution of results is, nevertheless, linear and proportional to the true voltages. To account for this result by aid of the equation

$$V_3 = \frac{Mg \Delta N}{l V_1} \frac{D^2}{r^2} \left(1 - 2 \Delta/D \right)^2$$

is not impossible, if the asymmetry is such as to make the terms in Δ/D essentially negative. Yet in the case of a large value of Δ the results are so lacking in probability that a want of uniformity or variation in the field is more liable to be in question.

On spacing the plates further, $D=1.04$ centimeters, and carefully adjusting the disk, it carried a charge equivalent to $V_1=250$ volts easily, so long as V_3 did not exceed about 20 volts.

A new and lighter disk was thereupon introduced, weighing but .607 gram. As it failed to carry a charge quite as large as $V_1=250$ volts, the only available smaller one, $V_1=104.5$ volts, was given to it. The uncorrected results for V_3 are here much nearer the true values, *i.e.*, about 1.2 times too large, than were those of the preceding case, so that the correction for asymmetry is probably applicable. The mean trend of the locus is through zero. Sensitiveness is naturally much lower than above, showing $\Delta N = 5 \times 10^{-4}$ centimeter per volt. The experiment indicates well how markedly dependent the constants of the instrument are on the position of the disk.

The same disk was now weighted with a rider, making the total mass 1.086 grams, with the object of charging it to 250 volts. The uncorrected results, while maintaining proportionality, were about 2 times too large.

The data obtained idiostatically, however, where the deflections are essentially small, fig. 7a, show an excellent agreement with the true voltages. Discrepancies are as liable to be

in the voltmeter as in the electrometer. The values of k given for this table show how nearly a true condition of symmetry was attained, as its mean value from the equation

$$V_s^2 = \frac{2 Mg D^2}{l r^2} \Delta N \left(1 - 2 \frac{\Delta}{D}\right)^2$$

is but $-.0085$ centimeter.

The data, as a whole, illustrate the peculiarities of the disk method very well, giving evidence both of its relatively great

FIG. 7.

FIG. 7. Electrometer data compared with actual voltages.

sensitiveness (the double displacements being even 2×10^{-3} centimeter per volt) on the one hand, and the variation of sensitiveness as the result of a more or less unsymmetric position of the disk on the other. The ratio of observed and actual voltages is fairly constant. The question then arises as to the degree to which the latter difference may be corrected. There is, in many cases, a peculiar shift of the zero of displacement for the uncharged apparatus which is not easily accounted for; though, from another point of view, it is truly astonishing that a suspended disk should adjust itself to a given position with an accuracy comparable with the wave length of light. In the experiments detailed, such difficulties were eliminated by taking mean results, but usually the position in question was actually stable. If the shortcomings can be overcome, the practical limits of the apparatus as here constructed should, for short suspension (23 centimeters), be about $.015$ volt per vanishing ring.

13. *The Same. Further experiments.* The interesting feature of the preceding result is the large range of variability in the sensitiveness of the apparatus. It was therefore thought worth while to throw further light upon the investigation by purposely tipping the apparatus, in order that the disk might lie on one side or the other of the guard ring.

The approximate equation for $k/\Delta N = \frac{MgD^3}{4l r^2 V_1^2} - 1$ applied to the experiments did not give results quite as consistent as was anticipated, inasmuch as the same value of k did not reappear when the apparatus was tipped back apparently to its original position. Moreover, the central position did not clearly correspond to $k=0$. Hence it seems that $k=0$ is determined by the geometry of a non-uniform field. Neither did the values of V_1 correspond very closely to the values of k . The apparatus is more sensitive when the disk V_1 is on the side of the plate V_2 , and about equally so when central or when on the side $V_2 = 0$.

In other experiments the disk was weighted with a rider and a similar series of data was taken, although $V_1 = 250$ volts was carried by the disk only in cases where its position was nearly central. This was even the case on adding heavier riders. The computed values of V_2 were in all cases 2 or 3 times the true values, though the proportionality is maintained. The conditions are thus too complicated for consideration here.

14. *Case of the enclosed disk.* With the object of obtaining a more uniform field, the electrometer was now modified by surrounding the disk with a short cylindric or drum-shaped tube within the guard ring. The plates and the drum guard ring are spaced, as before, by three hard rubber rings, with a hard rubber screw passed through the perforations.

Experiments were begun with the light disk, $M = .607$ gram, which was charged to 110 volts, as it would not carry an appreciably higher voltage. Consequently the rider was thereafter added, making $M = 1.086$ grams, and the disk charged as far as 250 volts. The results are given in fig. 7 *b*. These voltages, though obtained under very different conditions, are practically coincident and the discrepancies are as liable to be in one instrument (voltmeter) as the other. The remaining experiments were of the same nature.

To summarize; the enormous variation of the sensitiveness of the disk electrometers as depending upon the position of the disk, k , has, therefore, failed of interpretation; *i. e.*, the correction for V_2 does not seem to follow the above equations. The field is, therefore, probably far from uniform, possibly consisting of a conical tube of force between plate and disk on the side toward which the disk leans and of a more nearly

cylindric tube on the opposite side, where the tube must pass through the circular perforation in the guard ring. The result is an empiric instrument in which the deflections are proportional to the voltages to be measured, of increased sensitiveness, even 2×10^{-3} centimeter per volt, but not yet sensitive enough to be immediately valuable for refined practical purposes. The idiostatic instrument, for voltages above 20 volts (after the disk has been adjusted for $k = 0$ by comparing the positions of the charged and uncharged disk) may in some cases be useful.

15. *Case of the unsymmetric disk.* The cause of the departure from linearity in the preceding experiments is to be referred to a slight asymmetry in the disk, whereby an effect varying as the square of voltage of the disk is superimposed on an effect varying as its first power. To eliminate the non-linear term, it should be sufficient to obtain no deflection in the case of earthed plates, when the needle is successively charged and uncharged. In fact, the charged non-symmetrical needle between earthed plates introduces an interesting method of electrometry as follows:

Let k be the amount of non-symmetry for a disk whose mean distance from the plates D centimeters apart would be $d = D/2$. Hence for any displacement ΔN the distance of the disk from one condenser plate will be $d = D/2 + (k + \Delta N)$ and from the other plate, $D - d = D/2 - (k + \Delta N)$. Hence the displacement force is (after reduction)

$$X = \frac{V_1^2 r^2}{2} \frac{k + \Delta N}{D/2} \frac{1}{D^2/4 - (k + \Delta N)^2} = \frac{Mg \Delta N}{l}$$

As $(k + \Delta N)^2$ may usually be neglected in comparison with $D^2/4$, the equation becomes

$$V_1^2 = \frac{Mg D^3}{4l r^2} \frac{\Delta N}{k + \Delta N}$$

If k is large as compared with ΔN , which will usually be the case, V_1^2 will vary linearly with ΔN .

To use this method, k must, therefore, be known, and it may be determined with the aid of a given voltage preliminarily. Many experiments were made in this way, an example of results being given in fig. 7 *c*. The asymmetry of the disk was about .5 millimeter, and the plates were over 6 millimeters apart. The equation employed answers the requirements as closely as the observations could be made. At high voltages (100 volts) there is liable to be divergence, while at low voltages the displacement ΔN is too small for accurate work, seeing that the underlying equation is quadratic.

An apparatus like the present, intended for actual measurements, should be provided with a micrometer suspension for

shifting the disk as a whole in the direction of its axis and with more elaborate means for sighting with a view to horizontal and vertical parallelism of the disk with the plates of the condenser than was the case with the improvised apparatus here treated. It has been stated that it is an inversion of the present displacement which is superimposed on the displacement in case of a permanently charged needle, and which thus demands an apparatus tested for absence of non-symmetry, if the voltages are to be proportional to the displacements.

16. *Displacement Interferometry Applied to the Quadrant Electrometer.*—The method of measuring small angles given elsewhere may be made use of in measuring very small voltages or small increments of potential, by attaching a pair of light mirrors, symmetrically and parallel, to the needle of a quadrant electrometer. In the present experiments this was an improvised instrument, constructed by myself, the quadrants being of sheet copper fitted and soldered together and supported on cylinders of hard rubber. The bottom of the stem of the needle was submerged in sulphuric acid as in Kelvin's instrument and the suspension was bifilar. The insulation was throughout excellent. The needle was kept charged to about 150 volts with a Zamboni pile, any variation of charge being indicated by Elster and Geitel's electroscope.

In fig. 8, qq' shows a pair of quadrants in vertical section, E the needle on the stem ss' , the lower end of which is platinum, bent as shown, thus making a clip to hold the light mica vane v (if necessary) submerged in sulphuric acid of the vessel c charged by the Zamboni pile. $R R'$ are the hard rubber supports of the quadrants. At a suitable distance below them the light parallel mirrors m and n (less than 1 centimeter in diameter each) are supported by the light cross piece of hard rubber rr attached to the stem ss' of the needle. The axial line of the needle E is parallel to the line rr between the mirrors and the latter are placed at a horizontal angle of about 45° to rr .

To adjust m and n to adequate parallelism, each is supported by an attached fine needle, fitting snugly in a vertical groove in the ends of rr . The needle, as a whole, is to be clamped at rr in a suitable support and sunlight is to be used. When the horizontal beam reflected from m to n and thence to a distant white screen falls within the direct shadow of n , the needles are fixed in place by resinous cement. The mirrors should be equally high.

The quadrant electrometer with the needle in position was now placed, with the aid of three long foot screws, on a circular platform just below the iron arm holding the fixed interferometer mirror M , and the reflection took place as shown in fig. 9. In effect the arc light from a collimator reflected from

slit being opened wide. The electrometer is now rotated as a whole around the vertical, until the light reflected from n strikes m and a similar screen behind the latter in the line mn is necessary here. Next, the mirror N is adjusted to normality, a white screen behind m in the direction Nm being essential, whereupon the light is reflected from n again toward the grating. As the pencil must pass through the grating G again, this is mounted to be capable of being raised or lowered and rotated around the vertical and horizontal (adjustment screws); but these are the usual adjustments on the interferometer. It follows that the electrometer must also be capable of being raised and lowered on its long foot screws, as already indicated.

Moreover, the spectrum should be so placed by raising and lowering the source of light and including the grating together with N that the higher orders of direct spectra are not in the same plane with the spectra superposed for interference. When the adjustments described are well made, there is no further difficulty in finding the solitary ellipses. As there is compensation owing to the two glass plate windows in the case of the electrometer, the ellipses are liable to be enormous, practically vertical straight lines, which are displaced with correspondingly great rapidity by the micrometer screw, and are therefore hard to find. Hence some counter compensation at the micrometer mirror is desirable, in order that centers may appear, and the displacement may be slower. A compensating plate about 1 centimeter or less in thickness, with the *vertical* focus of the light from the slit on the stationary mirror, produces very clear and sharp ellipses, admirably adapted for measurement.

17. *Observations.*—The stems of the mirrors m and n on rr were about $a = 4.5$ centimeters apart. Hence, since $i = 45^\circ$, $a = 4.5/\sqrt{2}$ and if δ is the path difference $\frac{d\delta}{di} = .0175 \times 2a \sin$

$i = .079$ cm. per degree. Supposing that 1 volt gave a deflection of 45° and that 10^{-4} centimeters are measurable at the micrometer, the micrometer sensitiveness should be $10^{-4}/45 \times .079$, or about 3×10^{-5} volt, *i. e.*, about 10^{-5} volt per vanishing interference ring. It would thus seem probable that with a lighter needle and a more delicate suspension the possibility of measuring 10^{-6} volt would not be out of the question. The following observations show that these surmises are correct so far as the method is concerned.

The great hardship encountered in the present work was the unavoidable agitation of the laboratory, and this unfortunately is insuperable. After the ellipses were found they were always in motion, so that the displacement work was bound to

be rough and the use of interference rings out of the question. As the rings, however, may easily be obtained and used under suitable conditions, and as the purpose of the present paper is merely to test the method, the annoyances in question are of less consequence.

To obtain small potentials, a thin bare German silver wire about 1 meter long was stretched and insulated on a board and the ends supplied with a constant potential difference of one volt. Two points of the wire, one or more centimeters apart, were then used as a source of potential, the wires from these points leading to a Mascart key, which suitably earths and commutates the charged wires leading to the electrometer. A thermo-couple might have been used; but the long wire is preferable because of its simplicity.

In the first experiments made, the suspension was a bifilar about 10 centimeters long (each strand of several silk fibers) and about .5 to 1 millimeter apart. No doubt the torsional stress of the fiber was here of an order commensurate with the bifilar force. The needle, being damped in concentrated sulphuric acid, moved very slowly and about two minutes were allowed for each deflection. It was thus possible to follow the ellipses on commutation, from one to the other extreme elongation, by moving the micrometer screw proportionally to the displacement of ellipses. As a rule, the ellipses were quite clear all the way, showing that the adjustment for parallelism of mirrors on the needle by the aid of sunlight is adequate. The potential of the charge on the needle was of the order of 150 volts.

In the first experiments, $\Delta N/\Delta V = .57$ centimeter was the displacement on the micrometer per volt, so that $\Delta N = 10^{-4}$ centimeter corresponds to .000170 volt, or the sensitiveness is about .000060 volt per vanishing interference ring.

The instrument was now improved by inserting a longer bifilar suspension, consisting of a single fiber of silk. The damper and other details were retained. The potentials were tapped from points of the long wire, respectively 5 and 10 centimeters apart. Owing to the very large deflection, the ellipses were not equally clear throughout the whole displacement from elongation to elongation.

The instrument behaved much better in the present case and the sensitiveness had been much increased, particularly in the later series of observations. Since $\Delta N/\Delta V$ was of the order of 2.9 centimeters per volt, 10^{-4} centimeter corresponds to .000035 volt, *i. e.*, to about .000010 volt per ring, so that under favorable circumstances a few millionths of a volt should be discernible.

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