as yet absolutely certain of the developmental origin of many structures in the body, and further research may clear up some of the apparent discrepancies now incidental to the embryogenetic classification. The histogenetic classification itself is not altogether free from the reproach levelled at the embryogenetic one by Prof. Adami.

As regards the genesis of new growths, the various hypotheses are discussed at considerable length, and an admirable survey of this vexed and complex question is presented to the reader. Prof. Adami considers that no parasitic hypothesis suffices, that Beard's hypothesis of aberrant and misplaced germs and trophoblastic cells (so much in evidence lately in connection with a certain form of treatment) is inadequate, seeks for an explanation in the hypothesis of a change (? a mutation) in the biological properties of the cells giving origin to tumours, and considers that there is no one specific cause; with all these we cordially agree.

The concluding portion of the book deals with the regressive changes, the degenerations and infiltrations, calcification, pigmentation, &c. The book altogether is an inspiring one, and the careful reader will not only gather what is already known, but will be led to infer in what directions further progress lies. A notable feature of it is the attempt made, usually successfully, to ensure a basis on a sound foundation of general biology. It is carefully and adequately illustrated, and the numerous diagrams and schemata serve to render many of the more abstruse conceptions clear and intelligible.

A NEW WAY IN ARITHMETIC.

Theorie der algebraischen Zahlen. By Dr. Kurt Hensel. Erster Band. Pp. xii+350. (Berlin and Leipzig: Teubner, 1908.) Price 14 marks.

IN this volume Dr. Hensel gives the first instalment of a treatise on algebraic numbers, embodying an independent method on which he has been engaged for the last eighteen years. Its leading idea may be illustrated by the following example. Let us take the solvable congruence, $x^2 \equiv 2$ (mod. 7), the roots of which are $x \equiv 3$ and $x \equiv 4$. The same congruence can be solved with respect to the moduli 7^2, 7^3, 7^4, &c., and we obtain the solutions, in least positive residues, (3, 4), (10, 39), (108, 235), (2116, 285), and so on. Taking the first number in each bracket and expressing it in the septenary scale, only writing the digits in the reverse of the usual order, we obtain the associated solutions, 3, 31, 312, 3126; and it is clear that if $x = a_1 a_2 \ldots a_n$ is a solution of $x^2 \equiv 2$ (mod. 7^n), we can find a number $a_1 a_2 \ldots a_n a_{n+1}$, which is a solution of $x^2 \equiv 2$ (mod. 7^{n+1}). There is thus a definite sequence of digits, 3, 1, 2, 6, ... a_n, ... such that each is a least positive residue of 7 (or zero), and such that $3126 \ldots a_n$ is a solution of $x^2 \equiv 2$ (mod. 7^n). This sequence may be said to be the symbolical septenary representation of $\sqrt{2}$. But conversely we may take any such sequence, $a_1 a_2 \ldots a_n \ldots$ and define it as a septenary number, in an extended sense. All such numbers form a corpus, provided we introduce septenary fractions of the same type. Since $-1 \equiv (7^n - 1)$ (mod. 7^n), the symbolical form of -1 is $666 \ldots$ or $\dot{6}$; hence every ordinary positive or negative integer or fraction has a symbolic expression which is wholly or partly periodic, *e.g.* $2/3 = (3+\dot{6})/3 = 3\dot{2}$, and so on. Similar results hold for any prime modulus; when the modulus is composite, some curious anomalies occur.

Now let $w_1, w_2, \ldots w_n$ be a basis of an algebraic corpus; we may form symbols of the type $A_1 w_1 + A_2 w_2 + \ldots + A_n w_n$, where $A_1, A_2, \ldots A_n$ are numbers of the kind just described. These new symbols may be called "numbers," and by making use of them Dr. Hensel obtains all the most important known properties of algebraic numbers with surprising facility; he also adds results of his own which are of great interest and beauty. Calling a symbol such as A_1 a p-adic number, we may call $F(x) = A_1 x^n + \ldots + A^n$, a p-adic function; it is shown how to determine, by a finite process, the irreducible p-adic factors of $F(x)$, and by a series of propositions we are led up to the remarkable theorem (p. 159) that if $f(x) = x^\lambda + a_1 x^{\lambda-1} + \ldots + a_\lambda$, the coefficients being integral p-adic numbers, and $f(x)$ irreducible, then if p^δ is the highest power of p which divides the discriminant of $f(x)$, and if α_1 is a root of the ordinary equation

$$\phi(x) = 0$$

obtained from $f(x)$ by omitting all the digits of $a_1, a_2, \ldots a_\lambda$ beyond the $(\delta - 1)$th place, the equation $f(x) = 0$ will have precisely λ conjugate roots $\xi_1, \xi_2, \ldots \xi_\lambda$ expressible as conjugate p-adic numbers in the corpora $(\alpha_1), (\alpha_2) \ldots (\alpha_\lambda)$. This fundamental fact leads to a host of consequences, among them a comparatively simple treatment of a well-known problem, namely, the resolution into their prime ideal factors of the real primes which divide the discriminant of a given corpus. It also leads to a complete theory of congruential roots of unity; the theory of units in a given corpus is not discussed in this volume.

On pp. 308 and following will be found a complete solution of the problem of resolving a given real prime into its ideal factors within a given corpus; this involves the Kronecker method, in which *umbrae* are used, and probably there is no certain practical way which can dispense with them. As an illustration, it is shown that in the corpus defined by the equation $\alpha^3 - \alpha^2 - 2\alpha - 8 = 0$, the number 2 is the product of three ideal primes, which are actually determined.

One of the last theorems proved in this volume may be stated in the following terms:—If a corpus is defined by an equation $f(x) = 0$, which is not Galoisian in the field of ordinary numbers, we cannot make the field Galoisian by the introduction of p-adic numbers.

The value of the treatise can hardly be overrated, and its completion will be anxiously expected. It is interesting to compare it with Hensel and Landsberg's treatise on algebraic functions, and observe the points of contact. A special feature is that in the arithmetical work, like the other, there are expansions in

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fractional powers; in the algebraic theory it is almost impossible to avoid this, except by tedious divagations, but in the theory of numbers such symbols ought to be avoided if possible, and their occurrence here may cause some readers a shade of regret. G. B. M.

LISSAJOUS'S FIGURES.

Harmonic Vibrations and Vibration Figures. By J. Goold, C. E. Benham, R. Kerr, and Prof. L. R. Wilberforce. Edited by H. C. Newton. (London: Newton and Co., n.d.) Price 6*s*. net.

THE four authors of this book have each contributed an account of the construction and use of apparatus which they have invented or brought to perfection, the several parts of the book being independent of one another, but related by the similarity of the subject-matter. Lissajous's figures were originally introduced as a convenient method of illustrating optically or mechanically acoustic phenomena, but the beauty and perfection of the results obtained by the compound pendulum of Tisley, and later by the twin elliptic pendulum of Goold, have made the subject sufficiently attractive to be pursued for its own sake. As two leading scientific publishers declined to take the book on the ground that it could not pay, we are indebted to Messrs. Newton and Co. for rescuing and producing a book which will be valued in many quarters.

Mr. Benham writes the history of the harmonograph, and describes his own triple pendulum and his own modification of Goold's twin elliptic pendulum. He also gives valuable information to anyone who would construct his apparatus as to the details which are necessary for success. The construction of the ruling pen, choice of inks or dyes, the selection of suitable paper, interesting dodges with photographic plates or with successive chemicals, are a few only of the tips or dodges described. The extremely beautiful stereoscopic effects obtained by viewing two nearly identical harmonograph figures with a stereoscope are described and illustrated, as is the curious change which occurs when such a pair of figures are slowly turned round at the same time, so as to change their relative aspect, the series of lines all appearing on the surface of a cylinder in the one position, and gradually merging into a series, each of which lies between the last one and the axis in the other position. In the case of figures drawn by the twin elliptic pendulum, where it would be next to impossible to draw two successive figures which should be sufficiently alike, the ingenious plan is adopted of selecting those which have a two-fold symmetry, but in which the two halves on opposite sides of the centre are not quite identical, and then simply turning one upside down, in order to obtain stereoscopic shell-like structures of wonderful beauty. Several examples of the marvellous beauty of the twin elliptic pendulum's work are given, in which it is difficult to know whether the forms of the curves or the water-mark patterns are the more to be admired.

Visitors at soirées of the Royal Society will remember the curves drawn by Mr. Goold's big twin elliptic pendulum, as also that queer vibrating and droning steel plate, which gave rise to so many curious phenomena. One passage from Mr. Goold's description may here be quoted.

"If . . . a small chain be thrown on the vibrating plate, it will immediately settle itself on the curved line between the vortices and . . . will crawl away to the nearest vortex, and there coil itself up like a serpent, continuing to rotate as long as the plate remains sufficiently excited."

This is one only of a number of curious results obtained by Mr. Goold.

Mr. Richard Kerr describes a form of geometric pen, capable of producing very beautiful patterns. This is followed by an account of Mr. Lewis Wright's method of projecting Lissajous's figures on a screen, using reeds in the place of tuning forks, and Prof. Wilberforce describes his well-known sympathetic vibrations obtained by the aid of one or two torsion springs.

This is an excellent book for the Christmas holidays.

C. V. BOYS.

OUR BOOK SHELF.

Cattle of Southern India. By Lieut.-Col. W. D. Gunn. Department of Agriculture, Vol. III., Bulletin No. 60. Pp. 65; plates. (Madras: 1909.) Price 3*s*.

ALTHOUGH the existence of a number of local breeds and sub-breeds of Indian humped cattle (*Bos indicus*) is familiar to Anglo-Indians, comparatively little is known about them in this country, and it is, therefore, highly satisfactory that Col. Gunn, Superintendent of the Indian Civil Veterinary Department at Madras, has furnished us with this elaborately illustrated account of the various types to be met with in southern India. It is, however, a matter for regret that the author did not see his way to make his work complete by including the breeds found in other parts of India. As to the origin of humped cattle, the author is silent, and perhaps wisely so, since, so far as we are aware, nothing definite has hitherto been ascertained with regard to this subject.

If we rightly understand him—and his classification is by no means so clear and unmistakable as it might be—the author considers that there are two main types of large-humped cattle in southern India, namely, the Mysore and the Ongole, or Nellore. The former, which are characterised by the long, more or less upwardly directed, slightly tapering horns, and generally iron-grey or bluish colour, are, however, divisible into a number of sub-breeds, such as the Amrat Mahal, Hallikar, Alumbadi, &c., all of which come under the native designation of Doddadana, or large cattle, in contradistinction to the Nadudana, or ordinary small village cattle. The finest of all are the cattle of the Amrat Mahal breed, which were formerly owned by Tippu Sultan, but became the property of the British Government after the fall of Seringapatam, although the management of the herds remained for a time under the control of the Maharaja of Mysore, on condition of his supplying a specified number of bullocks. In the old days of Indian warfare these cattle were of the greatest value for transport-purposes on account of their rapid pace.

The Nellore, or Ongole, cattle, on the other hand, carry short and somewhat stumpy horns, which are, however, longer in cows than in bulls, and have an outward and slightly backward direction. Formerly black-and-white was in fashion, but white is

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