392. A Nine-Point Circle

Author(s): W. Gallatly

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391. [K. 8. e. : V. 2. d.] *An extension of a theorem in Pappus.*

There is in Pappus (Bk. IV. Prop. 1) a simple and elegant theorem with which I made acquaintance perhaps more than thirty years ago in a German school book. Some years back, happening to mention it at a meeting of distinguished mathematicians, I was surprised to find none of them had heard of it. By this time it may be fairly well known, for it has been pointed out to me that it is in Casey's *Sequel to Euclid*, p. 18. Ex. 20, though without indication of its origin. I also understand that Pappus is little known to British geometers, a defect which may be remedied by the forthcoming edition to be published by Sir Thomas Heath. Recently an extension of the theorem in question has occurred to me which is of interest, I think, as illustrating the principle of algebraic signs in geometry.

The original proposition finds a parallelogram on a side of a triangle equal to the sum of any two parallelograms on the other two sides. Thus it is far more general (*καθολικώτερον πολλῶν*, as Pappus says) than Euclid I. 47, which is a case of it, and the generalisation of that proposition in Euclid VI.

The extension finds a parallelogram on a side of the triangle which is equal to the difference between any two parallelograms on the other two sides.

The original theorem is as follows:

Let $ABDE$ and $ACKH$ be parallelograms described on the sides AB and AC of the triangle ABC . Let DE and KH , produced if necessary, meet in L . Join AL . From B and C draw BO and CM parallel to AL , meeting DE and KH , or these lines produced, in O and M respectively. Join OM . Then $BCMO$ is the required parallelogram. If LA is produced to meet AB in P , the proof easily follows from Euclid I. 35.

To generalise the theorem we may consider the three parallelograms as three figures forming a system related by the given construction, and seek for generality by varying the position of one of them, while we keep the same connecting construction. If now we move the parallelogram on the base so that one of the angles comes within the corresponding angle of the triangle, P will move to the outside of AB , and the parallelogram on the corresponding side will come from the outside of the triangle and fall partly within it; and we shall now find that the parallelogram on the base is equal to the difference of the parallelograms on the sides. We thus get a construction for the parallelogram on the base equal to the difference between the given ones on the sides.

Further, if we consider the areas of parallelograms described on opposite sides of the same line as of opposite sign, the difference between the parallelograms in the second theorem would be their algebraic sum. Also, in the first figure the proper enunciation would be that the algebraic sum of the parallelograms so described is zero, and this obviously would also be the enunciation of the second theorem, and so the two are embraced in one formula.

It may of course be easily verified that this all holds good if we start with the parallelogram on the base described outside the triangle.

J. COOK WILSON.

392. [K. 2. e.] *A Nine-point Circle.*

Consider any four lines 1, 2, 3, 4.

The circumcentres O_1, O_2, O_3, O_4 of the four triangles formed by them lie on a circle—call it the centre circle.

The circumcircles of these triangles have a common point—call it M —on the centre circle.

The circumcircle of the triangle formed by 1, 2, 3 cuts the centre circle again in a point N_1 , which is the pole of the Simson Line parallel to 4.

We thus have nine concyclic points, $O_1, O_2, O_3, O_4, M, N_1, N_2, N_3, N_4$.

A remarkable circle. Has any geometer tackled it? W. GALLATLY.