CRITICISMS AND DISCUSSIONS.

THE WORKS OF WILLIAM OUGHTRED.*

CLAVIS MATHEMATICAE.

William Oughtred (1574 (?)–1660), though by profession a clergyman, was one of the world's great teachers of mathematics and should still be honored as the inventor of that indispensable mechanical instrument, the slide-rule. It is noteworthy that he showed a marked disinclination to give his writings to the press. His first paper on sun-dials was written at the age of twenty-three, but we are not aware that more than one brief mathematical manuscript was printed before his fifty-seventh year. In every instance, publication in printed form seems to have been due to pressure exerted by one or more of his patrons, pupils or friends. Some of his manuscripts were lent out to his pupils who prepared copies for their own use. In some instances they urged upon him the desirability of publication and assisted in preparing copy for the printer. The earliest and best known book of Oughtred was his *Clavis mathematicae*. As he himself informs us, he was employed by the Earl of Arundel about 1628 to instruct the Earl's son, Lord William Howard (afterwards Viscount Stafford), in the mathematics. For the use of this young man Oughtred composed a treatise on algebra which was published in Latin in the year 1631 at the urgent request of a kinsman of the young man, Charles Cavendish, a patron of learning.

The *Clavis mathematicae*,[1] in its first edition of 1631, was a booklet of only 88 small pages. Yet it contained in very condensed form the essentials of arithmetic and algebra as known at that time.

Aside from the addition of four tracts, the 1631 edition underwent some changes in the editions of 1647 and 1648 which two are much alike. The twenty chapters of 1631 are reduced to nineteen in 1647 and in all the later editions. Numerous minute alterations

* For further details see the author's article on "The Life of Oughtred" in *The Open Court*, August, 1915, where fuller references are given to some of the books cited here.

[1] The full title of the *Clavis* of 1631 is as follows: *Arithmeticae in numeris et speciebvs institvtio: Quae tvm logisticae, tvm analyticae, atqve adeo totivs*

from the 1631 edition occur in all parts of the books of 1647 and 1648. The material of the last three chapters of the 1631 edition is re-arranged with some slight additions here and there. The 1648 edition has no preface. In the print of 1652 there are only slight alterations from the 1648 edition; after that the book underwent hardly any changes, except for the number of tracts appended, and brief explanatory notes added at the close of the chapters in the English edition of 1694 and 1702. The 1652 and 1667 editions were seen through the press by John Wallis; the 1698 impression contains on the title-page the words: *Ex Recognitione D. Johannis Wallis, S.T.D. Geometriae Professoris Saviliani.*

The cost of publishing may be a matter of some interest. When arranging for the printing of the 1667 edition of the *Clavis,* Wallis wrote Collins:[2] "I told you in my last what price she [Mrs. Lichfield] expects for it, as I have formerly understood from her, viz., 40 l. for the impression, which is about 9½d. a book."

mathematicae, qvasi clavis est.—Ad nobilissimvm spectatissimumque invenem Dn. Guilelmvm Howard, Ordinis qui dicitur, Balnei Equitem, honoratissimi Dn. Thomae, Comitis Arvndeliae & Surriae, Comitis Mareschalli Angliae, &c filium—Londini, Apud Thomam Harpervm. M.DC.XXXI.

In all there appeared five Latin editions, the second in 1648 at London, the third in 1652 at Oxford, the fourth in 1667 at Oxford, the fifth in 1693 and 1698 at Oxford. There were two independent English editions: the first in 1647 at London, translated in greater part by Robert Wood of Lincoln College, Oxford, as is stated in the preface to the 1652 Latin edition; the second in 1694 and 1702 is a new translation, the preface being written and the book recommended by the astronomer Edmund Halley. The 1694 and 1702 impressions labored under the defect of many sense-disturbing errors due to careless reading of the proofs. All the editions of the *Clavis,* after the first edition, had one or more of the following tracts added on:

Eq. = *De Aequationum affectarum resolutione in numeris.*
Eu. = *Elementi decimi Euclidis declaratio.*
So. = *De Solidis regularibus tractatus.*
An. = *De Anatocismo, sive usura composita.*
Fa. = *Regula falsae positionis.*
Ar. = *Theorematum in libris Archimedis de Sphaera & cylindro declaratio.*
Ho. = *Horologia scioterica in plano, geometricè delineandi modus.*

The abbreviated titles given here are, of course, our own. The lists of tracts added to the *Clavis mathematicae* of 1631 in its later editions, given in the order in which the tracts appear in each edition, are as follows: *Clavis* of 1647, *Eq., An., Fa., Ho.*; *Clavis* of 1648, *Eq., An., Fa., Eu., So.*; *Clavis* of 1652, *Eq., Eu., So., An., Fa., Ar., Ho.*; *Clavis* of 1667, *Eq., Eu., So., An., Fa., Ar., Ho.*; *Clavis* of 1693 and 1698, *Eq., Eu., So., An., Fa., Ar., Ho.*; *Clavis* of 1694 and 1702, *Eq.*

The title-page of the *Clavis* was considerably modified after the first edition. Thus, the 1652 Latin edition has this title-page: *Guilelmi Oughtred Aetonensis, quondam Collegii Regalis in Cantabrigia Socii, Clavis mathematicae denvo limata, sive potius fabricata. Cum aliis quibusdam ejusdem commentationibus, quae in sequenti pagina recensentur. Editio tertia auctior & emendatior Oxoniae, Excudebat Leon. Lichfield, Veneunt apud Tho. Robinson. 1652.*

[2] Rigaud, *op. cit.,* Vol. II, p. 470.

As compared with other contemporary works on algebra, Oughtred's distinguishes itself for the amount of symbolism used, particularly in the treatment of geometric problems. Extraordinary emphasis was placed upon what he called in the *Clavis* the "analytical art."[3] By that term he did not mean our modern analysis or analytical geometry, but the art "in which by taking the thing sought as knowne, we finde out that we seeke."[4] He meant to express by it condensed processes of rigid, logical deduction expressed by appropriate symbols, as contrasted with mere description or elucidation by passages fraught with verbosity In the preface to the first edition (1631) he says:

"In this little book I make known...the rules relating to fundamentals, collected together, just like a bundle, and adapted to the explanation of as many problems as possible."

As stated in this preface, one of his reasons for publishing the book, is "...that like Ariadne I might offer a thread to mathematical study by which the mysteries of this science might be revealed, and direction given to the best authors of antiquity, Euclid, Archimedes, the great geometrician Apollonius of Perga, and others, so as to be easily and thoroughly understood, their theorems being added, not only because to many they are the height and depth of mathematical science (I ignore the would-be mathematicians who occupy themselves only with the so-called practice, which is in reality mere juggler's tricks with instruments, the surface so to speak, pursued with a disregard of the great art, a contemptible picture), but also to show with what keenness they have penetrated, with what mass of equations, comparisons, reductions, conversions and disquisitions these heroes have ornamented, increased and invented this most beautiful science."

The *Clavis* opens with an explanation of the Hindu-Arabic notation of decimal fractions. Noteworthy is the absence of the words "million," "billion," etc. Although used on the continent by certain mathematical writers long before this, these words did not become current in English mathematical books until the eighteenth century. The author was a great admirer of decimal fractions, but failed to introduce the notation which in later centuries came to be

[3] See, for instance, the *Clavis mathematicae* of 1652, where he expresses himself thus (p. 11): "*Speciosa haec Arithmetica arti Analyticae (per quam ex sumptione quaesiti, tanquam noti, investigatur quaesitum) multo accommodatior est, quam illa numerosa.*"

[4] Oughtred, *The Key of the Mathematicks,* London, 1647, p. 4.

universally adopted. Oughtred wrote 0.56 in this manner 0|56; the point he used to designate ratio. Thus 3:4 was written by him 3·4. The decimal point (or comma) was first used by the inventor of logarithms, John Napier, as early as 1616 and 1617. Although Oughtred had mastered the theory of logarithms soon after their publication in 1614 and was a great admirer of Napier, he preferred to use the dot for the designation of *ratio*. This notation of ratio is used in all his mathematical books, except in two instances. The two dots (:) occur as symbols of ratio in some parts of Oughtred's posthumous work, *Opuscula mathematica hactenus inedita*, Oxford, 1677, but may have been due to the editors and not to Oughtred himself. Then again the two dots (:) are used to designate ratio on the last two pages of the tables of the Latin edition of Oughtred's *Trigonometria* of 1657. In all other parts of that book the dot (.) is used. Probably some one who supervised the printing of the tables introduced the (:) on the last two pages, following the logarithmic tables, where methods of interpolation are explained. The probability of this conjecture is the stronger, because in the English edition of the *Trigonometrie*, brought out the same year (1657) but *after* the Latin edition, the notation (:) at the end of the book is replaced by the usual (.), except that in some copies of the English edition the explanations at the end are omitted altogether.

Oughtred introduces an interesting, and at the same time new, feature of an abbreviated multiplication and an abbreviated division of decimal fractions. On this point he took a position far in advance of his time. The part on abbreviated multiplication was re-written in slightly enlarged form and with some unimportant alterations in the later editions of the Clavis. We give it as it occurs in the revision. Four cases are given. In finding the product of 246|914 and 35|27, "if you would have the Product without any Parts" (without any decimal part), "set the place of Unity of the lesser under the place of Unity in the greater: as in the Example," writing the figures of the lesser number in *inverse order*. From the example it will be seen that he begins by multiplying by 3, the right-hand digit of the multiplier. In the first edition of the *Clavis* he began with 7, the left digit. Observe also that he "carries" the nearest tens in the product of each lower digit and the upper digit one place to its

```
2 4 6|9 1 4
7 2|5 3
-------
7 4 0 7
1 2 3 5
    4 9
    1 7
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8 7 0 8
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right. For instance, he takes $7\times4=28$ and carries 3, then he finds $7\times2+3=17$ and writes down 17.

The second case supposes that "you would have the Product with some places of parts" (decimals), say 4; "Set the place of Unity of the lesser Number under the Fourth place of the Parts of the greater." The multiplication of 246|914 by 35|27 is now performed thus:

```
2 4 6|9 1 4
        7 2|5 3
---------------
7 4 0 7 4 2 0 0
1 2 3 4 5 7 0 0
    4 9 3 8 2 8
    1 7 2 8 4 0
---------------
8 7 0 8|6 5 6 8
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In the third and fourth cases are considered factors which appear as integers, but are in reality decimals; for instance, the sine of 54° is given in the tables as 80902 when in reality it is .80902.

Of interest as regards the use of the word "parabola" is the following (*Clavis,* 1694, p. 19 and the *Clavis* of 1631, p. 8): "The Number found by Division is called the *Quotient,* or also *Parabola,* because it arises out of the Application of a plain Number to a given Longitude, that a congruous Latitude may be found." This is in harmony with etymological dictionaries which speak of a parabola as the application of a given area to a given straight line. The dividend or product is the area; the divisor or factor is the line.

Oughtred gives two processes of long division. The first is identical with the modern process, except that the divisor is written below every remainder, each digit of the divisor being crossed out as soon as it has been used in the partial multiplication. The second method of long division is one of the several types of the old "scratch method." This antiquated process held its place by the side of the modern method in all editions of the *Clavis.* The author divides 467023 by 357|0926425, giving the following instructions: "Take as many of the first Figures of the Divisor as are necessary, for the first Divisor, and then in every following particular Division drop one of the Figures of the Divisor towards the Left Hand, till you have got a competent Quotient." He does not explain abbreviated division as thoroughly as abbreviated multiplication.

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                        17
                       303
                      2803
                    109930
357 |0926425)       467023   (1307 |80
                    357093
                    107127
                      2500
                       286
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Oughtred does not examine the degree of reliability or accuracy of his processes of abbreviated multiplication and division. Here as in other places he gives in cendensed statement the mode of procedure, without further discussion.

He does not attempt to establish the rules for the addition, subtraction, multiplication and division of positive and negative numbers. "If the Signs are both alike, the Product will be affirmative, if unlike, negative"; then he proceeds to applications. This attitude is superior to that of many writers of the eighteenth and nineteenth centuries, on pedagogical as well as logical grounds: Pedagogically, because the beginner in the study of algebra is not in a position to appreciate an abstract train of thought, as every teacher well knows, and derives better intellectual exercise from the applications of the rules to problems; logically, because the rule of signs in multiplication does not admit of rigorous proof, unless some other assumption is first made which is no less arbitrary than the rule itself. It is well known that the proofs of the rule of signs given by eighteenth-century writers are invalid. Somewhere they involve some surreptitious assumption. This criticism applies even to the proof given by Laplace, which tacitly assumes the distributive law in multiplication.

A word should be said on Oughtred's definition of + and –. He recognizes their double function in algebra by saying (*Clavis*, 1631, p. 2): "*Signum additionis, sive affirmationis, est + plus*" and "*Signum subductionis, sive negationis est – minus.*" They are symbols which indicate the *quality* of numbers in some instances and *operations* of addition or subtraction in other instances. In the 1694 edition of the *Clavis*, thirty-four years after the death of Oughtred, these symbols are defined as signifying operations only, but are actually used to signify the quality of numbers as well. In this respect the 1694 edition marks a recrudescence.

The characteristic in the *Clavis* that is most striking to a modern reader is the total absence of indices or exponents. There is much discussion in the leading treatises of the latter part of the sixteenth and the early part of the seventeenth centuries on the theory of indices, but modern exponential notation, a^n, is of later date. The modern notation, for positive integral exponents, first appears in Descartes's *Géométrie*, 1637; fractional and negative exponents were first used in the modern form by Sir Isaac Newton, in his announcement of the binomial formula, in a letter written in 1676. This total absence of our modern exponential notation in Oughtred's *Clavis* gives it a strange aspect. Like Vieta, Oughtred uses ordinarily the capital letters, A, B, C,... to designate given numbers; A^2 is written Aq, A^3 is written Ac; for A^4, A^5, A^6 he has, respectively, Aqq, Aqc, Acc. Only on rare occasions, usually when some parallelism in notation is aimed at, does he use small letters[5] to represent numbers or magnitudes. Powers of binomials or polynomials are marked by prefixing the capital letters Q (for square), C (for cube), QQ (for the fourth power), QC (for the fifth power), etc.

Oughtred does not express aggregation by ( ). Parentheses had been used by Girard, and by Clavius as early as 1609,[6] but did not come into general use in mathematical language until the time of Leibniz and by the Bernoullis. Oughtred indicates aggregation by writing a colon (:) at both ends. Thus, Q:A−E: means with his $(A-E)^2$. Similarly, $\sqrt{}$q:A+E: means $\sqrt{(A+E)}$. The two dots at the end are frequently omitted when the part affected includes all the terms of the polynomial to the end. Thus, C:A+B −E=.. means $(a+B-E)^3=$.. There are still further departures from this notation, but they occur so seldom that we incline to the interpretation that they are simply printer's errors. For proportion Oughtred uses the symbol (: :). The proportion $a:b=c:d$ appears in his notation $a\cdot b::c\cdot d$. Apparently, a proportion was not fully recognized in his day as being the expression of an equality of ratios. That probably explains why he did not use = here as in the notation of ordinary equations. Yet Oughtred must have been very close to the interpretation of a proportion as an equality; for he says in his *Elementi decimi Euclidis declaratio, "proportio, sive ratio aequalis* : :" That he introduced this extra symbol, when

[5] See, for instance, Oughtred's *Elementi decimi Euclidis declaratio*, 1652, p. 1, where he uses A and E, and also *a* and *e*.

[6] See *Christophori Clavii Bambergensis Operum mathematicorum, tomus secundus*, Moguntiae, M.DC.XI, Algebra, p. 39.

the one for equality was sufficient, is a misfortune. Simplicity demands that no unnecessary symbols be introduced. However, Oughtred's symbolism is certainly superior to those which preceded. Consider the notation of Clavius.[7] He wrote $20:60=4:x$, $x=12$, thus: "20.60.4? *fiunt* 12." The insufficiency of such a notation in the more involved expressions frequently arising in algebra is readily seen. Hence Oughtred's notation (: :) was early adopted by English mathematicians. It was used by John Wallis at Oxford, by Samuel Foster at Gresham College, by James Gregory of Edinburgh, by the translators into English of Rahn's algebra and by many other early writers. Oughtred has been credited generally with the introduction of St. Andrews's cross × as the symbol for multiplication in the *Clavis* of 1631. We have discovered that this symbol, or rather the letter x which closely resembles it, occurs as the sign of multiplication thirteen years earlier in an anonymous "Appendix to the Logarithmes, shewing the practise of the Calculation of Triangles etc." to Edward Wright's translation of John Napier's *Descriptio*, published in 1618. Later we shall give our reasons for believing that Oughtred is the author of that "Appendix." The × has survived as a symbol of multiplication.

Another symbol introduced by Oughtred and found in modern book is $\sim$, expressing difference; thus $C \sim D$ signifies the difference between C and D, even when D is the larger number.[8] This symbol was used by John Wallis in 1657.[9]

Oughtred represented in symbols also certain composite expressions, as for instance $A+E=Z$, $A-E=X$, where A is greater than E. He represented by a symbol also each of the following: A^2+E^2, A^3+E^3, A^2-E^2, A^3-E^3.

Oughtred practically translated the 10th book of Euclid from its ponderous rhetorical form into that of brief symbolism. An appeal to the eye was a passion with Oughtred. The present writer has collected the different mathematical symbols used by Oughtred and has found more than one hundred and fifty of them.

The differences between the seven different editions of the *Clavis* lie mainly in the special parts appended to some editions and dropped in the latest editions. The part which originally con-

[7] *Christophori Clavii operum mathematicorum Tomus Secundus*, Moguntiae, M.DC.XI., *Epitome arithmeticae*, p. 36.

[8] See *Elementi decimi Euclidis declaratio*, 1652, p. 2.

[9] See Johannis Wallisii *Operum mathematicorum pars prima*, Oxonii, 1657, p. 247.

stituted the *Clavis* was not materially altered, except in two or three of the original twenty chapters. These changes were made in the editions of 1647 and 1648. After the first edition, great stress was laid upon the theory of indices upon the very first page as also in passages further on. Of course, Oughtred did not have our modern notation of indices or exponents, but their theory had been a part of algebra and arithmetic for some time. Oughtred incorporated this theory in his brief exposition of the Hindu-Arabic notation and in his explanation of logarithms. As previously pointed out, the last three chapters of the 1631 edition were considerably rearranged in the later editions and combined into two chapters, so that the *Clavis* proper had nineteen chapters instead of twenty in the editions after the first. These chapters consisted of applications of algebra to geometry and were so framed as to constitute a severe test of the student's grip of the subject. The very last problem deals with the division of angles into equal parts. He derives the cubic equation upon which the trisection depends algebraically, also the equations of the fifth degree and seventh degree upon which the divisions of the angle into 5 and 7 equal parts depend, respectively. The exposition was severely brief, yet accurate. He did not believe in conducting the reader along level paths or along slight inclines. He was a guide for mountain climbers and woe unto him who lacked nerve.

Oughtred lays great stress upon expansions of powers of a binomial. He makes use of these expansions in the solution of numerical equations. To one who does not specialize in the history of mathematics such expansions may create surprise, for did not Newton invent the binomial theorem after the death of Oughtred? As a matter of fact, the expansions of positive integral powers of a binomial were known long before Newton, not only to seventeenth-century but even sixteenth-century mathematicians. Oughtred's *Clavis* of 1631 gave the binomial coefficients for all powers up to and including the tenth. What Newton really accomplished was the generalization of the binomial expansion which makes it applicable to negative and fractional exponents and converts it into an infinite series.

As a specimen of Oughtred's style of writing we quote his solution of quadratic equations, accompanied by a translation into English and into modern mathematical symbols.

As a preliminary step he lets[10]

[10] *Clavis* of 1631, Chap. XIX, sect. 5, p. 50.

$$Z = A + E \quad \text{and} \quad A > E;$$

he lets also $X = A - E$. From these relations he obtains identities which, in modern notation, are $\frac{1}{4}Z^2 - AE = \frac{1}{4}(\frac{1}{2}Z - E)^2 = \frac{1}{4}X^2$. Now, if we know Z and AE, we can find $\frac{1}{2}X$. Then $\frac{1}{2}(Z + X) = A$, and $\frac{1}{2}(Z - X) = E$, and

$$A = \tfrac{1}{2}Z + \sqrt{\tfrac{1}{4}Z^2 - AE}.$$

Having established these preliminaries, he then proceeds. (We translate the Latin passage, using the modern exponential notation and parentheses.)

"Given therefore an unequally divided line Z (10), and a rectangle beneath the segments AE (21) which is a gnomon. Half the difference of the segments $\frac{1}{2}X$ is given, and consequently the segment itself. For, if one of the two segments is placed equal to A, the other will be $Z - A$. Moreover, the rectangle is $ZA - A^2 = AE$. And because Z and AE are given, and there is $\frac{1}{4}Z^2 - AE = \frac{1}{4}X^2$, and by 5c.18, $\frac{1}{2}Z + \frac{1}{2}X = A$, and $\frac{1}{2}Z - \frac{1}{2}X = E$, the equation will be solved thus: $\frac{1}{2}Z \pm \sqrt{(\frac{1}{4}Z^2 - AE)} = A \begin{cases} \text{major segment} \\ \text{minor segment.} \end{cases}$

"And so an equation having been proposed in which three species (terms) are in equally ascending powers, the highest species, moreover, being negative, the given magnitude which constitutes the middle species is the line to be bisected. And the given absolute magnitude to which it is equal is the rectangle beneath the unequal segments, without gnomon. As $ZA - A^2 = AE$, or in numbers, $10x - x^2 = 21$. And A or x is one of the two unequal segments. It may be found thus:

"The half of the middle species is $Z^2/2$ (5). its square is $Z^2/4$ (25). From it subtract the absolute term AE (21), and $(Z^2/4) - A^2$ (4) will be the square of half the difference of the segments. The square root of this, $\sqrt{[(Z^2/2)^2 - AE]}$ (2) is half the difference. If you add it to half the coefficient $Z/2$ (5), the longer segment is obtained, if you subtract it, the smaller segment is obtained. I say:

$$(Z/2) + [\sqrt{(Z^2/4 - AE)}] = A \begin{cases} \text{major segment} \\ \text{minor segment.''} \end{cases}$$

The quadratic equation $Aq + ZA = AE$ receives similar treatment. This and the preceding equation, $ZA - Aq = AE$, constitute together a solution of the general quadratic equation, $x^2 + ax = b$, provided that E or Z are not restricted to positive values, but admit of being either positive or negative, a case not adequately treated by Oughtred. Imaginary numbers and imaginary roots receive no consideration whatever.

A notation suggested by Vieta and favored by Girard made vowels stand for unknowns and consonants for knowns. This conventionality was adopted by Oughtred in parts of his algebra, but not throughout. Near the beginning he used Q to designate the unknown, though usually this letter stood with him for the "square" of the expression after it.[11]

We quote the description of the *Clavis* that was given by Oughtred's greatest pupil, John Wallis. It contains additional information of interest to us. Wallis devotes Chapter XV of his *Treatise of Algebra*, London, 1685, pp. 67-69, to Mr. Oughtred and his *Clavis*, saying:

"Mr. William Oughtred (our Country-man) in his *Clavis Mathematicae*, (or Key of Mathematicks,) first published in the Year 1631, follows Vieta (as he did Diophantus) in the use of the Cossick Denominations; omitting (as he had done) the names of *Sursolids*, and contenting himself with those of *Square* and *Cube*, and the Compounds of these.

"But he doth abridge Vieta's Characters or Species, using only the letters q, c, &c. which in Vieta are expressed (at length) by *Quadrate, Cube*, &c. For though when Vieta first introduced this way of Specious Arithmetick, it was more necessary (the thing being new,) to express it in words at length: Yet when the thing was once received in practise, Mr. Oughtred (who affected brevity, and to deliver what he taught as briefly as might be, and reduce all to a short view,) contented himself with single Letters instead of Those words.

"Thus what Vieta would have written

$$\frac{\text{A } \textit{Quadrate}, \text{ into B } \textit{Cube},}{\text{CDE } \textit{Solid},} \ \textit{Equal to} \text{ F. G. } \textit{Plane},$$

would with him be thus expressed

$$\frac{A_q\, B_c}{CDE} = FG.$$

"And the better to distinguish upon the first view, what quantities were Known, and what Unknown, he doth (usually) denote the Known by *Consonants*, and the Unknown by *Vowels*; as Vieta (for the same reason) had done before him.

[11] We have noticed the representation of known quantities by consonants and the unknown by vowels in Wingate's *Arithmetick made easie*, edited by

"He doth also (to very great advantage) make use of several Ligatures, or Compendious Notes, to signify *Summs, Differences,* and *Rectangles* of several Quantities. As for instance, Of two Quantities A (the Greater), and E (the Lesser), the Sum he calls Z, the Difference X, the Rectangle AE...."

"Which being of (almost) a constant signification with him throughout, do save a great circumlocution of words, (each Letter serving instead of a Definition;) and are also made use of (with very great advantage) to discover the true nature of divers intricate Operations, arising from the various compositions of such Parts, Sums, Differences, and Rectangles; (of which there is great plenty in his *Clavis,* Cap. 11, 16, 18, 19. and elsewhere,) which without such Ligatures, or CompendiousNotes, would not be easily discovered or apprehended..

"In know there are who find fault with his *Clavis,* as too obscure, because so short, but without cause; for his words be always full, but not Redundant, and need only a little attention in the Reader to weight the force of every word, and the Syntax of it;... And this, when once apprehended, is much more easily retained, than if it were expressed with the prolixity of some other Writers; where a Reader must first be at the pains to weed out a great deal of superfluous Language, that he may have a short prospect of what is material; which is here contracted for him in short Synopsis"...

"Mr. Oughtred in his *Clavis,* contents himself (for the most part) with the solution of Quadratick Equations, without proceeding (or very sparingly) to Cubick Equations, and those of Higher Powers; having designed that Work for an *Introduction* into *Algebra* so far, leaving the Discussion of Superior Equations for another work.... He contents himself likewise in Resolving Equations, to take notice of the *Affirmative* or *Positive Roots*; omitting the *Negative* or *Ablative* Roots, and such as are called *Imaginary* or *Impossible Roots.* And of those which he calls *Ambiguous* Equations, (as having more Affirmative Roots than one,) he doth not (that I remember) any where take notice of more than *Two* Affirmative Roots: (Because in Quadratick Equations, which are those he handleth, there are indeed no more.) Whereas yet in *Cubick* Equations, there may be *Three,* and in those of Higher Powers, yet more. Which Vieta was well aware of, and men-

John Kersey, London, 1650, algebra, p. 382; and in the second part, section 19, of Jonas Moore's *Arithmetick in two parts,* London, 1660, second part; Moore suggests as an alternative the use of *z, y, x,* etc. for the unknowns. The practice of representing unknowns by vowels did not spread widely in England.

tioneth in some of his Writings; and of which Mr. Oughtred could not be ignorant."

OUGHTRED'S CIRCLES OF PROPORTION AND TRIGONOMETRY.

Oughtred wrote and had published three important mathematical books, the *Clavis*, the *Circles of Proportion*[12] and a *Trigonometry*.[13] This last appeared in the year 1657 at London, in both Latin and English.

It is claimed that the trigonometry was "neither finished nor published by himself, but collected out of his scattered papers; and though he connived at the printing it, yet imperfectly done, as appears by his MSS.; and one of the printed Books, corrected by his own Hand."[14] Doubtless more accurate on this point is a letter of Richard Stokes who saw the book through the press:[15]

"I have procured your Trigonometry to be written over in a fair hand, which when finished I will send to you, to know if it be according to your mind; for I intend (since you were pleased to give your assent) to endeavour to print it with Mr. Briggs his Tables, and so soon as I can get the Prutenic Tables I will turn those of the sun and moon, and send them to you."

In the preface to the Latin edition Stokes writes:

"Since this trigonometry was written for private use without the intention of having it published, it pleased the Reverend Author, before allowing it to go to press, to expunge some things, to change

[12] There are two title-pages to the edition of 1632. The first title-page is as follows: *The Circles of Proportion and The Horizontall Instrument. Both invented, and the uses of both Written in Latine by Mr. W. O. Translated into English: and set forth for the publique benefit by William Forster. London. Printed for Elias Allen maker of these and all other mathematical Instruments, and are to be sold at his shop over against St. Clements church with out Temple-barr. 1632. T. Cecill Sculp.*

In 1633 there was added the following, with a separate title-page: *An addition unto the Use of the Instrument called the Circles of Proportion. London, 1633,* this being followed by Oughtred's *To the English Gentrie etc.* In the British Museum there is a copy of another impression, dated 1639, with the *Addition unto the use of the Instrument etc.,* bearing the original date, 1633, and with the epistle, *To the English Gentrie* etc., inserted immediately after Forster's dedication, instead of at the end of the volume.

[13] The complete title of the English edition is as follows: *Trigonometrie, or, The manner of calculating the Sides and Angles of Triangles, by the Mathematical Canon, demonstrated. By William Oughtred Etoneus. And published by Richard Stokes Fellow of Kings Colledge in Cambridge, and Arthur Haughton Gentleman. London, Printed by R. and W. Leybourn, for Thomas Johnson at the Golden Key in St. Paul's Church-yard.* M.DC.LXII.

[14] Jer. Collier, *The Great Historical, Geographical, Genealogical and Poetical Dictionary*, Vol. II, London, 1701, art. "Oughtred."

[15] Rigaud, *op. cit.*, Vol. I, p. 82.

other things and even to make some additions and insert more lucid methods of exposition."

This much is certain, the *Trigonometry* bears the impress characteristic of Oughtred. Like all his mathematical writings, the book was very condensed. Aside from the tables, the text covered only 36 pages. Plane and spherical triangles were taken up together. The treatise is known in the history of trigonometry as among the very earliest works to adopt a condensed symbolism so that equations involving trigonometric functions could be easily taken in by the eye. In the work of 1657 contractions are given as follows. *s* = sine, *t* = tangent, *se* = secant, *s co* = cosine (sine complement), *t co* = cotangent, *se co* = cosecant, *log* = logarithm, *Z cru* = sum of the sides of a rectangle or right angle, *X cru* = difference of these sides. It has been generally overlooked by historians that Oughtred used the abbreviations of trigonometric functions named above, a quarter of a century earlier, in his *Circles of Proportion,* 1632, 1633. Moreover, he used sometimes also the abbreviations which are current at the present time, namely sin = sine, tan = tangent, sec = secant. We know that the *Circles of Proportion* existed in manuscript many years before they were published. The symbol *sv* for *sinus versus* occurs in the *Clavis* of 1631. The great importance of well-chosen symbols needs no emphasis to readers of the present day. With reference to Oughtred's trigonometric symbols, Augustus De Morgan said:[16] "This is so very important a step, simple as it is, that Euler is justly held to have greatly advanced trigonometry by its introduction. Nobody that we know of has noticed that Oughtred was master of the improvement, and willing to have taught it, if people would have learnt." We find, however, that even Oughtred cannot be given the whole credit in this matter. As early as 1624, the contractions *sin* for sine and *tan* for tangent appear on the drawing representing Gunter's scale, but Gunter did not use them in his books, except in the drawing of his scale.[17] A closer competitor for the honor of first using these trigonometric abbreviations is Richard Norwood in his *Trigonometrie,* London, 1631, where *s* stands for sine, *t* for tangent, *sc* for sine complement (cosine), *tc* for tangent complement (cotangent), and *sec* for secant. Norwood was a teacher of mathematics in London and a well-known

[16] A. De Morgan, *Budget of Paradoxes,* London, 1872, p. 451; 2d edition, Chicago, 1915, Vol. II, p. 303.

[17] E. Gunter, *Description and Use of the Sector, the Crosse-staffe and other Instruments,* London, 1624. The second book, p. 31.

writer of books on navigation. Aside from the abbreviations just cited, Norwood did not use nearly as much symbolism in his mathematics as did Oughtred. The innovation of designating the sides and angles of a triangle by A, B, C and *a*, *b*, *c*, so that A was opposite *a*, B opposite *b*, and C opposite *c*, is attributed to Leonard Euler (1753), but was first used by Richard Rawlinson of Queen's College, Oxford, sometime after 1655 and before 1668. Oughtred did not use Rawlinson's notation.[18]

Mention should be made of trigonometric symbols used even earlier than any of the preceding, in "An Appendix to the Logarithmes, shewing the practise of the Calculation of Triangles, etc." printed in Edward Wright's edition of Napier's *A Description of the Admirable Table of Logarithmes*, London, 1618. We referred to this "Appendix" in tracing the origin of the sign ×. It contains, on page 4, the following passage: "For the Logarithme of an arch or an angle I set before (s), for the antilogarithme or compliment thereof (s*) and for the Differential (t)." In further explanation of this rather unsatisfactory passage, the author (Oughtred?) says, "As for example: sB+BC=CA. that is, the Logarithme of an angle B. at the Base of a plane right-angled triangle, increased by the addition of the Logarithm of BC, the hypothenuse thereof, is equall to the Logarithme of CA the cathetus."

Here "logarithme of an angle B" evidently means "log sin B," just as with Napier, "Logarithms of the arcs" signifies really "Logarithms of the sines of the angles." In Napier's table, the numbers in the column marked "Differentiae" signify log. sine minus log. cosine of an angle; that is, the logarithms of the tangents. This explains the contraction (t) in the "Appendix." The conclusion of all this is that as early as 1618 the signs *s*, *s**, *t* were used for *sine*, *cosine*, and *tangent*, respectively.

In trigonometry English writers of the first half of the seventeenth century used contractions more freely than their continental contemporaries, yea even more freely than English writers of a later period. Von Braunmühl, the great historian of trigonometry, gives Oughtred much praise for his trigonometry, and points out that half a century later the army of writers on trigonometry had hardly yet reached the standard set by Oughtred's[19] analysis. Oughtred must be credited also with the first complete proof that

[18] F. Cajori, "On the History of a Notation in Trigonometry" in *Nature*, Vol. 94, 1915, pp. 642, 643.

[19] A. v. Braunmühl, *Geschichte der Trigonometrie*, 2. Teil, Leipsic, 1903, pp. 42, 91.

was given to the first two of "Napier's analogies." His trigonometry contains seven-place tables of sines, tangents and secants, and six-place tables of logarithmic sines and tangents; also seven-place logarithmic tables of numbers. At the time of Oughtred there was some agitation in favor of a wider introduction of decimal systems. This movement is reflected in these tables which contain the centesimal division of the degree, a practice which is urged for general adoption in our own day, particularly by the French.

SOLUTION OF NUMERICAL EQUATIONS.

In the solution of numerical equations Oughtred does not mention the sources from which he drew, but the method is substantially that of the great French algebraist Vieta, as explained in a publication which appeared in 1600 in Paris under the title, *De numerosa potestatum purarum atque adfectarum ad exegesin resolutione tractatus*. In view of the fact that Vieta's process has been described inaccurately by leading modern historians including H. Hankel[20] and M. Cantor,[21] it may be worth while to go into some detail.[22] By them it is made to appear as identical with the procedure given later by Newton. The two are not the same. The difference lies in the divisor used. What is now called "Newton's method" is Newton's method as modified by Joseph Raphson.[23] The Newton-Raphson method of approximation to the roots of an equation $f(x)=0$ is usually given the form $a-[f(a)/f'(a)]$, where a is an approximate value of the required root. It will be seen that the divisor is $f'(a)$. Vieta's divisor is different; it is

$$|f(a+s_1)-f(a)|-s^n,$$

where $f(x)$ is the left of the equation $f(x)=k$, n is the degree of equation and s_1 is a unit of the denomination of the digit next to be found. Thus in $x^3+420000x=247651713$, it can be shown that 417 is approximately a root; suppose that a has been taken to be 400, then $s_1=10$; but if, at the next step of approximation, a is taken

[20] H. Hankel, *Geschichte der Mathematik im Alterthum und Mittelalter*, Leipsic, 1874, pp. 369, 370.

[21] M. Cantor, *Vorlesungen über die Geschichte der Mathematik*, II, 1900, pp. 640, 641.

[22] This matter has been discussed in a paper "A History of the Arithmetical Methods of Approximation etc." by F. Cajori, in the *Colorado College Publication*, General Series No. 51, 1910, pp. 182-184. Later this subject was again treated by G. Eneström in *Bibliotheca mathematica*, 3. Folge, Vol. 11, 1911, pp. 234, 235.

[23] See F. Cajori, *loc. cit.*, p. 193.

to be 410, then $s_1 = 1$. In this example, taking $a = 400$, Vieta's divisor would have been 9120000; Newton's divisor would have been 900000.

A comparison of Vieta's method with the Newton-Raphson method reveals the fact that Vieta's divisor is more reliable, but labors under the very great disadvantage of requiring a much larger amount of computation. The latter divisor is accurate enough and easier to compute. Altogether the Newton-Raphson process marks a decided advance over that of Vieta.

As already stated, it is the method of Vieta that Oughtred explains. The Englishman's exposition is an improvement on that of Vieta, printed forty years earlier. Nevertheless, Oughtred's explanation is far from easy to follow. The theory of equations was at that time still in its primitive stage of development. Algebraic notation was not sufficiently developed to enable the argument to be condensed into a form easily surveyed. So complicated does Vieta's process of approximation appear, that M. Cantor failed to recognize that Vieta possessed a uniform mode of procedure. But when one has in mind the general expression for Vieta's divisor which we gave above, one will recognize that there was marked uniformity in Vieta's approximations.

Oughtred allows himself twenty-eight sections in which to explain the process and at the close cannot forbear remarking that 28 is a "perfect" number (being equal to the sum of its divisors, 1, 2, 4, 7, 14).

The early part of his exposition shows how an equation may be transformed so as to make its roots 10, 100, 1000 or 10^m times smaller. This simplifies the task of "locating a root"; that is, of finding between what integers the root lies.

Taking one of Oughtred's equations, $x^4 - 72x^3 + 238600x = 8725815$, upon dividing $72x^3$ by 10, $238600x$ by 1000, and 8725815 by 10,000, we obtain $x^4 - 7\cdot 2x^3 + 238\cdot 6x = 872\cdot 5$. Dividing both sides by x, we obtain $x^3 + 238\cdot 6 - 7\cdot 2x^2 = x)872\cdot 5$. Letting $x = 4$, we have $64 + 238\cdot 6 - 115\cdot 2 = 187\cdot 4$.

But $4)872\cdot 5(218\cdot 1$; 4 is too small. Next let $x = 5$, we have $125 + 238\cdot 6 - 180 = 183\cdot 6$.

But $5)872\cdot 5(174\cdot 5$; 5 is too large. We take the lesser value, $x = 4$, or in the original equation, $x = 40$. This method may be used to find the second digit in the root. Oughtred divides both sides of the equation by x^2, and obtains $x^2 + x)238600 - 72x = x^2)8725815$. He tries $x = 47$ and $x = 48$, and finds that $x = 47$.

He explains also how the last computation may be done by logarithms. Thereby he established for himself the record of being the first to use logarithms in the solution of affected equations.

"Exemplus II.

$$1 + 420000l = 247651713$$

Hoc est, $L_c + C_q\, L = D_c$.

	247	651	713	(417
	42	000	0	C_q
	64			A_c
	168	000	0	C_q A
	232	000	0	Ablatit.
R	15	651	713	
	4	8		3 A_q
		12		3 A
	4	200	00	C_q
	9	120	00	Divisor.
	4	8		3 A_q E
		12		3 A E_q
		1		E_c
	4	200	00	C_q E
	9	121	00	Ablatit.
R	6	530	713	
		504	3	3 A_q
		1	23	3 A
		420	000	C_q
		925	530	Divisor.
	3	530	1	3 A_q E
		60	27	3 A E_q
			343	E_c
	2	940	000	C_q E
	6	530	713	Ablatit."

4	1	
16	8	
		1
16	8	1

As an illustration of Oughtred's method of approximation, after the root sought has been located, we choose for brevity a cubic in preference to a quartic. We select the equation $x^3 +$

$420000x = 247651713$. By the process explained above a root is found to lie between $x = 400$ and $x = 500$. From this point on, the approximation as given by Oughtred is as shown on previous page.

In further explanation of this process, observe that the given equation is of the form $L_c = C_q L = D_c$, where L is our x, $C_q = 420000$, $D_c = 247651713$. In the first step of approximation, let $L = A + E$, where $A = 400$ and E is, as yet, undetermined. We have $L_c = (A+E)^3 = A^3 + 3A^2E + 3AE^2 + E^3$

and $C_q L = 420000\,(A+E)$.

Subtract from 247651713 the sum of the known terms A^3 (his A_c) and $420000\,A$ (his $C_q A$). This sum is 232000000; the remainder is 15651713.

Next, he evaluates the coefficients of E in $3\,A^2E$ and $420000\,E$, also 3A, the coefficient of E^2. He obtains $3A^2 = 480000$, $3A = 1200$, $C_q = 420000$. He interprets $3A^2$ and C_q as tens, 3A as hundreds. Accordingly, he obtains as their sum 9120000, which is the *Divisor* for finding the second digit in the approximation. Observe that this divisor is the value of $|f(a+s_1) - f(a)| - s_1^n$ in our general expression, where $a = 400$, $s_1 = 10$, $n = 3$, $f(x) = x^3 + 420000x$.

Dividing the remainder 15651713 by 9120000, he obtains the integer 1 in tens place; thus $E = 10$, approximately. He now computes the terms $3A^2E$, $3AE^2$ and E^3 to be respectively, 4800000, 120000, 1000. Their sum is 9121000. Subtracting it from the previous remainder 15651713, leaves the new remainder, 6530713.

From here on each step is a repetition of the preceding step. The new A is 410, the new E is to be determined. We have now in closer approximation, $L = A + E$. This time we do not subtract A^3 and $C_q A$, because this subtraction is already affected by the preceding work.

We find the second trial divisor by computing the sum of $3A^2$, 3A and C_q; that is, the sum of 504300, 1230, 420000, which is 925530. Again, this divisor can be computed by our general expression for divisors, by taking $a = 410$, $s_1 = 1$, $n = 3$.

Dividing 6530713 by 925530 yields the integer 7. Thus $E = 7$. Computing $3A^2E$, $3AE^2$, E^2 and subtracting their sum, the remainder is 0. Hence 417 is an exact root of the given equation.

Since the extraction of a cube root is merely the solution of a pure cubic equation, $x^3 = n$, the process given above may be utilized in finding cube roots. This is precisely what Oughtred does in Chapter XIV of his *Clavis*. If the above computation is modified by putting $C_q = 0$, the process will yield the approximate cube root of 247651713.

Oughtred solves 16 examples by the process of approximation here explained. Of these, 9 are cubics, 5 are quartics, and 2 are quintics. In all cases he finds only one or two real roots. Of the roots sought, five are irrational, the remaining are rational and are computed to their exact values. Three of the computed roots have 2 figures each, 9 roots have 3 figures each, 4 roots have 4 figures each. While no attempt is made to secure all the roots—methods of computing complex roots were invented much later—he computes roots of equations which involve large coefficients and some of them are of a degree as high as the fifth. In view of the fact that many editions of the *Clavis* were issued, one impression as late as 1702, it contributed probably more than any other book to the popularization of Vieta's method in England.

Before Oughtred, Thomas Harriot and William Milbourn are the only Englishmen known to have solved numerical equations of higher degrees. Milbourn published nothing. Harriot slightly modified Vieta's process by simplifying somewhat the formation of the trial divisor. This method of approximation was the best in existence until the publication by Wallis in 1685 of Newton's method of approximation.

LOGARITHMS.

Oughtred's treatment of logarithms is quite in accordance with the more recent practice.[24] He explains the finding of the *index* (our *characteristic*); he states that "the sum of two Logarithms is the Logarithm of the Product of their Valors; and their difference is the Logarithm of the Quotient," that "the Logarithm of the side [436] drawn upon the Index number [2] of dimensions of any Potestas is the logarithm of the same Potestas" [436^2], that "the logarithm of any Potestas [436^2] divided by the number of its dimensions [2] affordeth the Logarithm of its Root [436]." These statements of Oughtred occur for the first time in the *Key of the Mathematicks* of 1647; the *Clavis* of 1631 contains no treatment of logarithms.

If the characteristic of a logarithm is negative, Oughtred indicates this fact by placing the = *above* the characteristic. He separates the characteristic and mantissa by a *comma*, but still uses

[24] See William Oughtred's *Key of the Mathematicks*, London, 1494, pp. 173-175, tract, "Of the Resolution of the Affected Equations," or any edition of the *Clavis* after the first.

the sign L to indicate decimal fractions. He uses the contraction "log."

INVENTION OF THE SLIDE RULE; CONTROVERSY ON PRIORITY OF INVENTION.

Oughtred's most original line of scientific activity is the one least known to the present generation. Augustus De Morgan, in speaking of Oughtred who was sometimes called "Oughtred Aetonensis," remarks: "He is an animal of extinct race, an Eton mathematician. Few Eton men, even of the minority which knows what a sliding rule is, are aware that the inventor was of their own school and college."[25] The invention of the slide rule has, until recently,[26] been a matter of dispute; it has been erroneously ascribed to Edmund Gunter, Edmund Wingate, Seth Partridge and others. We have been able to establish that William Oughtred was the first inventor of slide rules, though not the first to publish thereon. We shall see that Oughtred invented slide rules about 1622, but the descriptions of his instruments were not put into print before 1632 and 1633. Meanwhile one of his own pupils, Richard Delamain, who probably invented the circular slide rule independently, published a description in 1630, at London, in a pamphlet of 32 pages entitled *Grammelogia; or the Mathematicall Ring.* In editions of this pamphlet which appeared during the following three or four years, various parts were added on, and some parts of the first and second editions eliminated. Thus Delamain antedates Oughtred two years in the publication of a description of a circular slide rule. But Oughtred had invented also a rectilinear slide rule, a description of which appeared in 1633. To the invention of this Oughtred has a clear title. A bitter controversy sprang up between Delamain on one hand, and Oughtred and some of his pupils on the other, on the priority and independence of invention of the circular slide rule. Few inventors and scientific men are so fortunate as to escape contests. The reader needs only to recall the disputes which have arisen, involving the researches of Sir Isaac Newton and Leibniz on the differential and integral calculus, of Thomas Harriot and René Descartes relating to the theory of equations, of Robert Mayer, Hermann v. Helmholtz and Joule on the principle of the conservation of energy, or of Robert Morse, Joseph Henry, Gauss and Weber, and others on the telegraph, to see that questions of

[25] A. De Morgan, *op. cit.*, p. 451; 2d ed., II, p. 303.

[26] See F. Cajori, *History of the Logarithmic Slide Rule*, New York, 1909, pp. 7-14, Addenda, p. ii.

priority and independence are not uncommon. The controversy between Oughtred and Delamain embittered Oughtred's life for many years. He refers to it in print on more than one occasion. We are preparing a separate article giving the details of this controversy and shall confine ourselves at present to the statement that it is by no means clear that Delamain stole the invention from Oughtred; Delamain was probably an independent inventor. Moreover, it is highly probable that the controversy would never have arisen, had not some of Oughtred's pupils urged and forced him into it. William Forster stated in the preface to the *Circles of Proportion* of 1632 that while he had been carefully preparing the manuscript for the press, "another to whom the Author [Oughtred] in a louing confidence discovered this intent, using more hast then good speed, went about to preocupate." It was this passage which started the conflagration. Another pupil, W. Robinson, wrote to Oughtred, when the latter was preparing his *Apologeticall Epistle* as a reply to Delamain's counter-charges:[27] "Good sir, let me be beholden to you for your Apology whensoever it comes forth, and (if I speak not too late) let me entreat you, whip ignorance well on the blind side, and we may turn him round, and see what part of him is free." As stated previously, Oughtred's circular slide rule was described by him in his *Circles of Proportion*, London, 1632, which was translated from Oughtred's Latin manuscript and then seen through the press by his pupil, William Forster. In 1633 appeared *An Addition unto the Use of the Instrument called the Circles of Proportion* which contained at the end "The Declaration of the two Rulers for Calculation," giving a description of Oughtred's rectilinear slide rule. This *Addition* was bound with the *Circles of Proportion* as one volume. About the same time Oughtred described a modified form of the rectilinear slide rule, to be used in London for gauging.[28]

MINOR WORKS.

Among the minor works of Oughtred must be ranked his booklet of forty pages to which reference has already been made, entitled, *The New Artificial Gauging Line or Rod*, London, 1633. His different designs of slide rules and his inventions of sun-dials as well as his exposition of the making of watches show that he displayed unusual interest and talent in the various mathematical

[27] Rigaud, *op. cit.*, Vol. I, p. 12.

[28] *The New Artificial Gauging Line or Rod: together with rules concerning the use thereof: Invented and written by William Oughtred.* London, 1633.

instruments. A short tract on watch-making was brought out in London as an appendix to the *Horological Dialogues* of a clock and watch maker who signed himself "J. S." (John Smith?). Oughtred's tract appeared with its own title-page, but with pagination continued from the preceding part, as *An Appendix wherein is contained a Method of Calculating all Numbers for Watches. Written originally by that famous Mathematician Mr. William Oughtred, and now made Publick. By J. S. of London, Clock-maker.* London, 1675.

"J. S." says in his preface:

"The method following was many years since Compiled by Mr. Oughtred for the use of some Ingenious Gentlemen his friends, who for recreation at the University, studied to find out the reason and Knowledge of Watch-work, which seemed also to be a thing with which Mr. Oughtred himself was much affected, as may in part appear by his putting out of his own Son to the same Trade, for whose use (as I am informed) he did compile a larger tract, but what became of it cannot be known."

Notwithstanding Oughtred's marked activity in the design of mathematical instruments, and his use of surveying instruments, he always spoke in depreciating terms of their importance and their educational value. In his epistle against Delamain he says:[29]

"The Instruments I doe not value or weigh one single penny. If I had been ambitious of praise, or had thought them (or better then they) worthy, at which to have taken my rise, out of my secure and quiet obscuritie, to mount up into glory, and the knowledge of men: I could have done it many yeares before...

"Long agoe, when I was a young student of the Mathematicall Sciences, I tryed many wayes and devices to fit my selve with some good Diall or Instrument portable for my pocket, to finde the houre, and try other conclusions by, and accordingly framed for that my purpose both Quadrants, and Rings, and Cylinders, and many other composures. Yet not to my full content and satisfaction; for either they performed but little, or els were patched up with a diversity of lines by an unnaturall and forced contexture. At last I.... found what I had before with much studie and paines in vaine sought for."

Mention has been made on the previous pages of two of his papers on sun-dials, and prepared (as he says) when he was in his twenty-third year, and was first published in the *Clavis* of 1647. The second paper appeared in his *Circles of Proportion.*

[29] W. Oughtred, *Apologeticall Epistle*, p. 13

Both before and after the time of Oughtred much was written on sun-dials. Such instruments were set up against the walls of prominent buildings, much as the faces of clocks in our time. The inscriptions that were put upon sun-dials are often very clever: "I count only the hours of sunshine," "Alas, how fleeting." A sun-dial on the grounds of Merchiston Castle, in Edinburgh, where the inventor of logarithms, John Napier, lived for many years, bears the inscription, "Ere time be tint, tak tent of time" (Ere time be lost, take heed of time).

Portable sun-dials were sometimes carried in pockets, as we carry watches. Thus Shakespeare, in *As You Like It*, Act II, Sc. 7:

"And he drew a diall from his poke."

Watches were first made for carrying in the pocket about 1658.

Because of this literary, scientific and practical interest in methods of indicating time it is not surprising that Oughtred devoted himself to the mastery and the advancement of methods of time-measurement.

Besides the accounts previously noted, there came from his pen: *The Description and Use of the double Horizontall Dyall: Whereby not onely the hower of the day is shewne; but also the Meridian Line is found: And most Astronomical Questions, which may be done by the Globe, are resolved. Invented and written by W. O.*, London, 1636.

The "Horizontall Dyall" and "Horologicall Ring" appeared again as appendices to Oughtred's translation from the French of a book on mathematical recreations.

The fourth French edition of that work appeared in 1627 at Paris, under the title of *Recreations mathematiqve,* written by "Henry van Etten," a pseudonym for the French Jesuit Jean Leurechon (1591-1690). English editions appeared in 1633, 1653 and 1674. The full title of the 1653 edition conveys an idea of the contents of the text:

Mathematicall Recreations, or, A Collection of many Problemes, extracted out of the Ancient and Modern Philosophers, as Secrets and Experiments in Arithmetick, Geometry, Cosmographie, Musick, Opticks, Architecture, Statick, Mechanicks, Chemistry, Waterworks, Fire-works, &c. Not vulgarly manifest till now. Written first in Greek and Latin, lately compil'd in French, by Henry Van Etten, and now in English, with the Examinations and Augmenta-

tions of divers Modern Mathematicians. Whereunto is added the Description and Use of the Generall Horologicall Ring. And The Double Horizontall Diall. Invented and written by William Oughtred. London, Printed for William Leake, at the Signe of the Crown in Fleet-street, between the two Temple-Gates. MDCLIII.

The graphic solution of spherical triangles by the accurate drawing of the triangles on a sphere and the measurement of the unknown parts in the drawing, was explained by Oughtred in a short tract which was published by his son-in-law, Christopher Brookes, under the following title:

The Solution of all Sphaerical Triangles both right and oblique By the Planisphaere: Whereby two of the Sphaerical partes sought, are at one position most easily found out. Published with consent of the Author, By Christopher Brookes, Mathematique Instrument-maker, and Manciple of Wadham Colledge, in Oxford.

Brookes says in the preface: "I have oftentimes seen my Reverend friend Mr. W. O. in his resolution of all sphaericall triangles both right and oblique, to use a planisphaere, without the tedious labour of Trigonometry by the ordinary Canons: which planisphaere he had delineated with his own hands, and used in his calculations more than Forty years before."

Interesting as one of the sources from which Oughtred obtained his knowledge of the conic sections is his study of Mydorge. A tract which he wrote thereon was published by Jonas Moore, in his *Arithmetick in two books....* [containing also] *the two first books of Mydorgius his conical sections analyzed by that reverend devine Mr. W. Oughtred, Englished and completed with cuts.* London, 1660. Another edition bears the date 1688.

To be noted among the minor works of Oughtred are his posthumous papers. He left a considerable number of mathematical papers which his friend Sir Charles Scarborough had revised under his direction and published at Oxford in 1676 in one volume under the title, *Gulielmi Oughtredi, Etonensis, quondam Collegii Regalis in Cantabrigia Socii, Opuscula Mathematica hactenus inedita.* Its nine tracts are of little interest to a modern reader.

Here we wish to give our reasons for our belief that Oughtred is the author of an anonymous tract on the use of logarithms and on a method of logarithmic interpolation which, as previously noted, appeared as an "Appendix" to Edward Wright's translation into English of John Napier's *Descriptio*, under the title, *A Description of the Admirable Table of Logarithmes,* London, 1618.

The "Appendix" bears the title, "An Appendix to the Logarithmes, showing the practise of the Calculation of Triangles, and also a new and ready way for the exact finding out of such lines and Logarithmes as are not precisely to be found in the Canons." It is an able tract. A natural guess is that the editor of the book, Samuel Wright, a son of Edward Wright, composed this "Appendix." More probable is the conjecture which (Dr. J. W. L. Glaisher informs me) was made by Augustus De Morgan, attributing the authorship to Oughtred. Two reasons in support of this are advanced by Dr. Glaisher, the use of *x* in the "Appendix" as the sign of multiplication (to Oughtred is generally attributed the introduction of the cross × for multiplication in 1631), and the then unusual designation "cathetus" for the vertical leg of a right triangle, a term appearing in Oughtred's books. We are able to advance a third argument, namely the occurrence in the "Appendix" of (S*) as the notation for sine complement (cosine), while Seth Ward, an early pupil of Oughtred, in his *Idea trigonometriae demonstratae,* Oxford, 1654, used a similar notation (S'). It has been stated elsewhere that Oughtred claimed Seth Ward's exposition of trigonometry as virtually his own. Attention should be called also to the fact that, in his *Trigonometria,* page 2, Oughtred uses (') to designate 180°-angle.

COLORADO COLLEGE. FLORIAN CAJORI.

BERGSON'S THEORY OF INTUITION.

Probably the best example of Bergson's application of the intuitive method is to be found in his account of the ideal genesis of the intelligence in the third chapter of *Creative Evolution.* This gives us the gist of his whole philosophy, and serves to illustrate the difficulties of Bergson's view not only of the nature of intellect, but also of intuition itself. What Bergson proposes to do is "to engender intelligence, by setting out from the consciousness which envelopes it"; that is to say, he proposes that we should actually experience in our own selves the process by which duration, which is pure heterogeneity and pure activity, is degraded into the spatializing intellect and spatialized matter. The intellect left to itself, Bergson argues, naturally tends to the homogeneous and the extended and the static. That is to say, the impression we get of the intellect is as of something unmaking itself. "Extension appears only as a tension which is interrupted." But this suggests to us a