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THE STRUCTURE OF γ RAYS ON THE BASIS OF THE ELECTRO-MAGNETIC THEORY OF LIGHT.

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THE difficulties in the theories of radiation have become so great in the last few years that only a very fundamental new idea will be able to bring harmony into the present chaos of facts and theories. Before that solving idea appears, we can only draw all the logical conclusions which follow from a given hypothesis and compare these conclusions with experiments. This procedure leads to the result that neither one of the present hypotheses is able to coördinate satisfactorily all the facts. The electromagnetic undulatory theory of light explains the phenomena of reflection, refraction, interference, diffraction, etc., but it fails to explain the phenomena of radiation, the photoelectric and the related effects. The corpuscular theories and the quantum theories on the other hand are especially invented for the explanation of the latter group of phenomena, failing to account for the first group. Not only are the fundamental assumptions of the two theories different, but the older electromagnetic undulatory theory gives not only mathematical relations between the different quantities involved, but it visualizes at the same time, the phenomena, which are explained, so that we can see the mechanism of the processes. The recent quantum theories on the other hand give us only algebraic relations between different quantities such as the law of the photoelectric effect $\frac{1}{2}mv^2 = hn - V_0$, without giving us the least idea as to the mechanism of the phenomenon. It seems to be advantageous if not necessary for a theory to visualize the phenomena. Led by this idea, I shall show by the following figures and calculations that the electromagnetic theory of very hard Roentgen and γ rays leads to conclusions which remind us of an emission or corpuscular theory of rays. The problem has already been solved by A. Sommerfeld,¹ "Über

¹ Sitzungsberichte der K. B. Akademie der Wissenschaften, 41, p. 1, 1911.

die Structur der γ Strahlen." I shall use, however, a different method which allows the visualizing of the remarkable conclusion of the electromagnetic theory.

An electron moves with the velocity v in the x direction; the electric force E at a distance r from the charge making an angle φ with the direction of motion is equal to

$$E = \frac{e \left(1 - \frac{v^2}{c^2}\right) \sqrt{x^2 + y^2 + z^2}}{\left[x^2 + (y^2 + z^2) \left(1 - \frac{v^2}{c^2}\right)\right]^{\frac{3}{2}}}.$$

x, y, z are the coördinates of the point in which the electric force E is measured, the zero of the system of the coördinates lies in the electron. We can express E also in the following way

$$E = \frac{e \left(1 - \frac{v^2}{c^2}\right)}{r^2 \left(1 - \left(\frac{v^2}{c^2}\right) \sin^2 \varphi\right)^{\frac{3}{2}}}.$$

The magnetic force H is given by

$$H = \frac{Ev \sin \varphi}{c}.$$

These equations tell us that with increasing velocity v the electromagnetic field following the electron, is more and more compressed as it were toward the equatorial plane, being perpendicular to the direction of motion.

Fig. 1.

If now the electron comes into collision with a metallic plate it will come to rest during an interval of time t and move meanwhile over the distance l . If we draw the line of force t seconds after the collision began, we find the well-known disturbance within the Roentgen pulse, whose thickness

δ is a function of ϑ given by the equation

$$\delta = \frac{l}{v} (2c - v \cos \vartheta).$$

Within the sphere with radius ρ' drawn round about B of Fig. 1 the final position of rest of the electron, we find the ordinary electrostatic field. Without the sphere with radius ρ , drawn about o , where the collision at

the moment $t = 0$ began, we find the electromagnetic field accompanying the electron in motion. The electric force E , pointing toward o' where the electron at the moment t would be, if it had not been stopped, is equal to

$$E = \frac{e(1 - \beta^2)}{r^2(1 - \beta^2 \sin^2 \varphi)^{\frac{3}{2}}}; \quad \beta = \frac{v}{c}.$$

The component E_n of E in the direction ρ is equal to

$$\begin{aligned} E_n &= \frac{e(1 - \beta^2)r \cos(\varphi - \vartheta)}{r^3(1 - \beta^2 \sin^2 \varphi)^{\frac{3}{2}}}, \\ r \cos(\varphi - \vartheta) &= \rho(1 - \beta \cos \vartheta), \\ r^3(1 - \beta^2 \sin^2 \varphi)^{\frac{3}{2}} &= \rho^3(1 - \beta \cos \vartheta)^3, \\ E_n &= \frac{e(1 - \beta^2)}{\rho^2(1 - \beta \cos \vartheta)^2}. \end{aligned}$$

We shall now calculate the tangential electric force E_t in the shell. In Fig. 1 we consider the volume cut out of the shell by a cone with an angle ϑ . As there are no charges within this cap, the resultant flux of the electric force flowing out of the volume must be equal to zero. The inflowing flux ϕ_1 is due to the component E_t' and to the force e/ρ'^2 or approximately e/ρ^2

$$\begin{aligned} \phi_1 &= E_t \delta 2\pi\rho \sin \vartheta + \frac{e}{\rho^2} 2\pi\rho(\rho - \rho \cos \vartheta), \\ &= E_t \delta 2\pi\rho \sin \vartheta + e 2\pi(1 - \cos \vartheta). \end{aligned}$$

The outflowing flux ϕ_2 is due to the force E_n and is equal to

$$\begin{aligned} \phi_2 &= \int_0^\vartheta E_n 2\pi\rho \sin \vartheta d\vartheta \cdot \rho \\ &= e(1 - \beta^2) 2\pi \int_0^\vartheta \frac{\sin \vartheta d\vartheta}{(1 - \beta \cos \vartheta)^2} \\ &= e(1 + \beta) 2\pi \frac{1 - \cos \vartheta}{1 - \beta \cos \vartheta} \\ \phi_1 &= \phi_2, \end{aligned}$$

hence

$$\begin{aligned} E_t \delta 2\pi\rho \sin \vartheta + e 2\pi(1 - \cos \vartheta) &= e(1 + \beta) 2\pi \frac{1 - \cos \vartheta}{1 - \beta \cos \vartheta} \\ E_t &= \frac{e\beta \sin \vartheta}{\delta\rho(1 - \beta \cos \vartheta)} = \frac{ev \sin \vartheta}{\delta\rho(c - v \cos \vartheta)}. \end{aligned}$$

In every electromagnetic disturbance the electrostatic energy is equal to the magnetic energy. This is the case if we assume for the magnetic

force H in the pulse the expression

$$H = cE_t.$$

The distribution of the lines of force is shown in Fig. 2, and in Fig. 3 for the limiting case where the velocity v becomes equal to the velocity

Fig. 2.

Fig. 3.

c of light. We see from these figures that with increasing velocity v the electric force E_t becomes a maximum in a certain direction ϑ_m where most of the energy flows away from the electron, and that in the limit for $v = c$ the energy flows out in the direction $\vartheta_m = 0$, that is in the direction of motion of the electron. When the velocity v is so small that it can be neglected with respect to c , then on the contrary, the maximum of the energy radiates away from the electron in the equatorial plane.

$$E_t = \frac{ev^2 \sin \vartheta}{l\rho(c - v \cos \vartheta)(2c - v \cos \vartheta)}.$$

If we neglect at first the change in the factor $1/(2c - v \cos \vartheta)$, we get

$$E_t = \frac{ev^2 \sin \vartheta}{l\rho(c - v \cos \vartheta)}$$

and

$$\frac{\partial E_t}{\partial \vartheta} = \frac{e v^2 (c \cos \vartheta - v)}{l\rho (c - v \cos \vartheta)^2},$$

which vanishes when $\cos \vartheta = v/c$; for this angle the electric force is a maximum and the energy radiating away from the electron becomes approximately a maximum also.

When $\frac{v}{c} = 0.9$ then $\vartheta = 25^\circ 45'$,

when $\frac{v}{c} = 0.99$ then $\vartheta = 8^\circ$ and when

$\frac{v}{c} = 1$ then $\vartheta = 0^\circ$.

The effect becomes marked only when the velocity v approaches that of light. Returning to the complete expression E_t , we find that it becomes a maximum for the angle ϑ determined by

$$\cos^3 \vartheta - 2 \left(1 + \frac{c^2}{v^2} \right) \cos \vartheta + 3 \frac{c}{v} = 0.$$

For

$$\frac{c}{v} = \frac{10}{9},$$

the three roots are -2.42 , 1.497 and 0.9210 , hence the angle $\vartheta = 22^\circ 55'$. But as the shell has not everywhere the same thickness, this angle does not accurately indicate the direction in which most of the energy is radiated away.

The energy of the electric force E_t within the elementary volume $2\pi\rho \sin \vartheta \rho d\vartheta d\delta$ is equal to

$$\begin{aligned} dE_{es} &= \frac{1}{8\pi} E_t^2 2\pi\rho \sin \vartheta \rho d\vartheta d\delta \\ &= \frac{1}{4} e^2 \frac{v^3 \sin^3 \vartheta d\vartheta d\delta}{l(2c - v \cos \vartheta)(c - v \cos \vartheta)^2}. \end{aligned}$$

The electrostatic energy in the cap under the angle ϑ is given by the integral

$$\begin{aligned} E_{es} &= -\frac{1}{4} \frac{e^2 v^3}{l} \int_0^\vartheta \frac{\sin^2 \vartheta d\vartheta \cos \vartheta}{(2c - v \cos \vartheta)(c - v \cos \vartheta)^2} \\ &= \frac{1}{4} \frac{e^2}{l} \left\{ \frac{3c^2 - v^2}{c^2} \log \frac{c - v \cos \vartheta}{c - v} - \frac{4c^2 - v^2}{c^2} \log \frac{2c - v \cos \vartheta}{2c - v} \right. \\ &\quad \left. + \frac{c^2 - v^2}{c} \left[\frac{1}{c - v \cos \vartheta} - \frac{1}{c - v} \right] \right\}. \end{aligned}$$

The function

$$f(\vartheta) = \frac{\sin^3 \vartheta}{(2c - v \cos \vartheta)(c - v \cos \vartheta)^2}$$

or

$$f(\vartheta) = \frac{\sin^3 \vartheta}{\left(2 - \frac{v}{c} \cos \vartheta \right) \left(1 - \frac{v}{c} \cos \vartheta \right)^2}$$

has been plotted in Fig. 4 for the values $v/c = 9/10$ and $v/c = 99/100$. This figure shows clearly that with increasing velocity the energy is radiated away in a direction which approaches more and more the direction of motion of the electron. The bearing of this conclusion on the fluctuations of Schweidler and perhaps on the difference in the

photoelectric effect according to the incidence of the beam of light, or Roentgen rays is obvious. If we attribute to the electromagnetic field not only energy, but also momentum and mass, then it follows, that the electromagnetic mass of the electron, when it comes to rest, is thrown

Fig. 4.

forward more and more with increasing velocity. This electromagnetic mass and momentum concentrated in a comparatively small space, is not so very different from the notion of light particles in the old emission theory.

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