308. On a Certain Form of Definite Integral

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physical ones on Variation on p. 281 should be more numerous, and the true utility of the approximation $(1+x)^n = 1+nx$ is better appreciated when used in such problems as finding the loss in beats of a seconds pendulum during a day when g varies, than in merely calculating such things as $(1.04)^6$ to 3 places of decimals.

Taschenbuch für Mathematiker und Physiker. 1 Jahrgang, 1909. By F. AUERBACK, with others. Pp. 450+44 (introduction). 6 marks. (Teubner.)

This is the first publication of a reference book for mathematics and mathematical physics, which is intended to appear annually. It measures $7\frac{1}{2}$ by 5 inches, so is literally a 'pocket-book.' It is astonishing to find how much information it contains. There is a frontispiece of Lord Kelvin and an appreciation of his work by the editor. Then there follow astronomical statistics and tables of 4-figure logarithms, etc.; all this forms the introduction. The main part of the book consists of the definitions, theorems, and formulae, which are of greatest importance in nearly every branch of mathematics and physics, and in the last twenty pages even a little general chemistry is added. It is interesting to notice how little space the really important formulae in some subjects take up; thus Plane Trigonometry, adequately treated, fits into almost six pages, yet the Curvature of Surfaces requires eight. Plane Elementary Geometry appears to have suffered most in the process of condensation, and may be said to have almost shrunk out of existence.

There is an appendix containing lists of scientific periodicals, new books (in various languages), an index, and an obituary.

The authors hope to extend the scope of the book in future editions, and the indications given here and there promise that the extension may be considerable, and may, among other subjects now omitted, include Invariant Theory and Crystallography.

The book, which does not appear to contain many misprints, is well printed, and should be very useful to the competent mathematician. W. M. ROBERTS.

MATHEMATICAL NOTES.

308. [E. 5.] *On a certain form of Definite Integral.*

From the formula of reduction,

$$(m+1) \int x^m (\log x)^n \cdot dx = x^{m+1} (\log x)^n - n \int x^m (\log x)^{n-1} \cdot dx,$$

we get $(m+1) \int_0^1 x^m (\log x)^n \cdot dx = -n \int_0^1 x^m (\log x)^{n-1} \cdot dx,$

and hence, by a repeated application of this result, we deduce

$$\int_0^1 x^m (\log x)^n \cdot dx = (-1)^n n! / (m+1)^{n+1}. \dots\dots\dots (A)$$

Now since

$$x^{cx} = e^{cx \log x},$$

expanding by the exponential theorem, and using (A), we shall get

$$\int_0^1 x^{cx} \cdot dx = 1 - \frac{c}{2^2} + \frac{c^2}{3^3} - \frac{c^3}{4^4} + \dots \dots\dots (1)$$

Further, multiplying by x^a the series which is the expansion of x^{cx} , we shall get, in like manner,

$$\int_0^1 x^{a+cx} \cdot dx = \frac{1}{a+1} - \frac{c}{(a+2)^2} + \frac{c^2}{(a+3)^3} - \frac{c^3}{(a+4)^4} + \dots,$$

from which we have $\int_0^1 x^{1+cx} \cdot dx = \frac{1}{2} - \frac{c}{3^2} + \frac{c^2}{4^3} - \frac{c^3}{5^4} + \dots \dots\dots (2)$

Again, writing down the expansion of x^{cx^a} , and proceeding as before, we shall obtain

$$\int_0^1 x^{cx^a} \cdot dx = 1 - \frac{c}{(a+1)^2} + \frac{c^2}{(2a+1)^3} - \frac{c^3}{(3a+1)^4} + \dots,$$

from which we deduce

$$\int_0^1 x^{cx^2} \cdot dx = 1 - \frac{c}{3^2} + \frac{c^2}{5^3} - \frac{c^3}{7^4} + \dots, \dots \dots \dots (3)$$

while, further, we get

$$\int_0^1 x^{2+cx^2} \cdot dx = \frac{1}{3} - \frac{c}{5^2} + \frac{c^2}{7^3} - \frac{c^3}{9^4} + \dots \dots \dots (4)$$

In the above four forms of series, putting $c=1$, the numerators are all equal to unity; we may also express as definite integrals series with the same denominators but where the numerators may be in succession 1, 2, 3, ..., 1, 3, 5, For example, by using (1) and (2) we shall find the series

$$\frac{1}{2^2} \pm \frac{2}{3^3} + \frac{3}{4^4} \pm \frac{4}{5^5} + \dots$$

to be equal to $1 + \int_0^1 (x-1)x^{-x} \cdot dx$ and $\int_0^1 (x+1)x^x \cdot dx - 1$

respectively; and in like manner obtain expressions for the series

$$1 \pm \frac{3}{2^2} + \frac{5}{3^3} \pm \frac{7}{4^4} + \dots;$$

while, further, by using (3) and (4) we may deduce corresponding values for the series

$$1 \pm \frac{2}{3^2} + \frac{3}{5^3} \pm \frac{4}{7^4} + \dots$$

If we take the more general form of expression x^{x^x} , and write down its expansion by the exponential theorem, the general term of the series is $(x^x \log x)^n/n!$. To find the integral of this expression, multiplying by $(\log x)^n$ the series which is the expansion of x^{x^x} , we shall get

$$\begin{aligned} & \frac{1}{(-1)^n n!} \int_0^1 x^{x^x} (\log x)^n \cdot dx \\ &= \frac{1}{1^{n+1}} - c \cdot \frac{n+1}{2^{n+2}} + \frac{c^2}{2!} \cdot \frac{(n+1)(n+2)}{3^{n+3}} - \frac{c^3}{3!} \cdot \frac{(n+1)(n+2)(n+3)}{4^{n+4}} + \dots, \end{aligned}$$

and putting $c=n$, we have the corresponding series for

$$\frac{1}{n!} \int_0^1 (x^x \log x)^n \cdot dx.$$

We thus, by putting $n=0, 1, 2, 3, \dots$, can obtain the value in series form of each term in the general integral

$$\text{Queen's Coll. Cork.} \quad \int_0^1 x^{x^x} \cdot dx. \quad \text{A. H. ANGLIN.}$$

309. [K. 21. d.] A Geometrical Construction for π .

There are many experimental methods of finding π , such as wrapping cotton round cylinders or rolling inky pennies on paper. When the circumference of a circle drawn on paper has to be estimated it is usual to step out the circumference with dividers or with transparent paper and a pricker. The multiplication of errors is however inevitable, and no more than two-figure accuracy can be expected in such an experiment. An examination of the well-known Vieta-Euler product formula in the shape

$$\pi = 2 \sec \pi/2^2 \sec \pi/2^3 \sec \pi/2^4 \dots$$

shews that it lends itself to geometrical construction, and further, that it has a simple interpretation. On taking unity as the diameter of a circle it is seen that $2 \sec \pi/2^2$ represents the perimeter of an inscribed square, $2 \sec \pi/2^2 \sec \pi/2^3$ that of an inscribed octagon and so on: the limit of the product is naturally the limiting perimeter, i.e. the circumference of the circle. Using these ideas we may proceed as follows: