THE SPARKING VOLTAGES BETWEEN SPHERICAL ELECTRODES.*

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In the paper on "The Grading of Cables" which the author read before the Institution in 1908, short tables were given by means of which the maximum electric stress between two spherical electrodes could be computed at the instant of discharge, whatever the potentials of the spheres might be. These tables have been used by several experimenters. In practice the sparking potentials are either found by fixing the spark-gap and gradually increasing the voltage until the spark ensues, or by keeping the potential difference constant and slowly bringing the electrodes nearer to one another. Using either of these methods practically constant values have been obtained for the maximum electric stress at the instant of discharge, provided that the potentials of the spheres are equal and opposite. When the requisite corrections for the temperature and pressure of the air are applied, the results obtained by different experimenters practically agree, the maximum inaccuracy being about 1 per cent. When the electrodes are further apart than 1 mm. the maximum electric stress at a given pressure and temperature of the air depends only on the size of the electrodes, being very great for small spheres and approaching a limiting value of about 27.5 kilovolts per centimetre for large spheres when the temperature is 25°C. and the height of the barometer is 76 cm.

When one of the electrodes is earthed, then apparently in some cases the maximum electric stress is not constant but increases as the length of the spark-gap increases.

It is desirable therefore to investigate and see whether some simple formula can be found which will give the value of the maximum electric stress when the potentials are equal and opposite, as this will enable us to calculate the sparking voltages between any sized spherical electrodes whatever their distance apart may be. It is desirable also to consider possible causes which would explain the results obtained when one of the electrodes is earthed.

New experimental results are not given in this paper but the results obtained by A. Heydweiller† 26 years ago and the results published in the "Standardization Rules of the American Institute of Electrical Engineers" (1918) are examined and various deductions are made from them. Many other experimental results might have been chosen as the groundwork of our analysis, but these were chosen as being perhaps the best known

to electricians. Most of the other experimental results, however, lead to the same conclusions.

FORMULÆ FOR THE MAXIMUM ELECTRIC STRESS BETWEEN EQUAL SPHERICAL ELECTRODES.

If the radii of the spherical electrodes be each equal to a , and E_1 and E_2 be their potentials, than if x be the distance between them, the value of the maximum electric stress on the dielectric is given by*

$$R_{\max.} = \frac{E_1 - E_2}{x} f_1 + \frac{2 E_2}{x} (f_1 - f) \quad (1)$$

$$\text{where } f = \sinh a \cosh \left(\frac{a}{2} \right) \left\{ \frac{\sinh (a/2)}{\cosh^2 (a/2)} + \frac{\sinh (3a/2)}{\cosh^2 (3a/2)} + \frac{\sinh (5a/2)}{\cosh^2 (5a/2)} + \dots \right\}$$

$$f_1 = 2 \sinh a \cosh \left(\frac{a}{2} \right) \left\{ \frac{\sinh (a/2)}{\cosh^2 (a/2)} + \frac{\sinh (5a/2)}{\cosh^2 (5a/2)} + \frac{\sinh (9a/2)}{\cosh^2 (9a/2)} + \dots \right\}$$

and $\cosh a = 1 + x/(2a)$.

When the potentials of the spheres are equal and opposite

$$E_1 = -E_2 = \frac{E_1 - E_2}{2}$$

$$\text{and hence by (1), } R_{\max.} = \frac{E_1 - E_2}{x} f \quad (2)$$

When $E_2 = 0$, we have similarly,

$$R_{\max.} = \frac{E_1}{x} f_1 \quad (3)$$

The values of f and f_1 are determined accurately by the formulæ given above. The computation, however, is very laborious, especially when x is small. To simplify the calculation of these quantities the following formulæ have been obtained.

Calculation of f .—When x/a is not greater than 0.8 we can use the formula

$$f = 1 + \frac{1}{3} \cdot \frac{x}{a} + \frac{1}{45} \cdot \frac{x^2}{a^2} + \frac{2}{945} \cdot \frac{x^3}{a^3} + \frac{17}{14175} \cdot \frac{x^4}{a^4}$$

The maximum inaccuracy introduced by using this formula occurs when $x/a = 0.8$ and it is then about 1 in 2,000. For values of x/a less than 0.5 it is accurate enough for all practical purposes.

* A. RUSSELL, "Alternating Currents," vol. 1, p. 258.

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† "Entladungspotentiale," *Annalen der Physik und Chemie*, 1893, vol. 48, p. 235.

When x/a is greater than 0.8 we can use the formula

$$f = \frac{1}{2} \left(\frac{x}{a} + 1 \right) + \frac{1}{x/a + 2} + \frac{x/a}{2(x/a + 2)^3} + \frac{x/a}{2(x/a + 2)^4} + \frac{x/a}{2(x/a + 2)^5} - \frac{x/a}{(x/a + 2)^7} - \frac{2x/a}{(x/a + 2)^8}$$

When x/a equals 0.8, the error introduced by using this formula is about 1 in 2,000, but it diminishes very rapidly as x/a increases.

Calculation of f_1 .—When x/a is not greater than $\frac{1}{4}$, $f_1 = f$.

When x/a lies between $\frac{1}{4}$ and 1,

$$f_1 = 0.0465 + \frac{x}{a} + \frac{1}{1 + x/a} - 0.03 \frac{x}{a}$$

and when x/a is not less than unity,

$$f_1 = \frac{x}{a} + \frac{1}{x/a + 1} + \frac{1}{(x/a + 1)(x/a + 2)^3}$$

The second of these formulæ is purely empirical. Nearly all the calculations made in this paper were computed by these approximate formulæ.

SPARKING VOLTAGES IN AIR.

The experimental results given in the next three tables were obtained by A. Heydweiller in 1893 (*l.c. ante*). He experimented with equal spherical electrodes, the diameters being 0.5, 1 and 2 cm. respectively. It has to be remembered that what happens when the spark-gap is less than 1 mm. is not considered in this paper.

TABLE 1.

Two equal spheres. Radius = $a = 0.25$ cm. Both insulated. Temperature = $\theta = 18^\circ$ C. Barometric height = $h = 74.5$ cm. E = maximum sparking pressure between the electrodes.

E Kilovolts	Spark-gap, x , in cm.	$\frac{x}{a}$	f	$R_{\max.} = (E/x)f$
4.830	0.1	0.4	1.137	54.9
8.370	0.2	0.8	1.283	53.7
11.34	0.3	1.2	1.437	54.3
13.77	0.4	1.6	1.599	55.1
15.72	0.5	2.0	1.770	55.7
17.19	0.6	2.4	1.945	55.7
18.30	0.7	2.8	2.124	55.5
19.02	0.8	3.2	2.306	54.8
20.19	1.0	4.0	2.678	54.1
22.32	1.5	6.0	3.632	54.1

The mean value of $R_{\max.}$ for spheres $\frac{1}{2}$ cm. in diameter is therefore 54.8 kilovolts per cm. The mean potential gradients vary from 48.3 to 14.88, but when multiplied by the correcting factor f they give practically the same number. As f is calculated from purely geometrical data, these results verify the theory.

TABLE 2.

Two equal spheres. Radius = $a = 0.5$ cm. Spheres insulated. Temperature = $\theta = 18^\circ$ C. Barometric height = $h = 74.5$ cm.

E Kilovolts	Spark-gap, x , in cm.	$\frac{x}{a}$	f	$R_{\max.} = (E/x)f$
4.80	0.1	0.2	1.068	51.3
8.37	0.2	0.4	1.137	47.6
11.37	0.3	0.6	1.209	45.8
14.55	0.4	0.8	1.283	46.7
17.31	0.5	1.0	1.359	47.1
19.92	0.6	1.2	1.437	47.7
22.05	0.7	1.4	1.515	47.7
24.09	0.8	1.6	1.599	48.2
25.59	0.9	1.8	1.685	47.9
27.00	1.0	2.0	1.770	47.8

As the first result is obviously too high we neglect it. The mean value of $R_{\max.}$ for the nine remaining results is 47.4 kilovolts per centimetre. Using this value we find that E for 1 mm. spark-gap is 4.44 kilovolts. For spheres of half the size we see from Table 1 that for a millimetre spark-gap the pressure was 4.83 kilovolts. The smaller electrodes therefore require an appreciably higher sparking voltage for 1 mm. gap.

TABLE 3.

Two equal spheres. Radius = $a = 1$ cm. Spheres insulated. Temperature = $\theta = 18^\circ$ C. Barometric height = $h = 74.5$ cm.

E Kilovolts	Spark-gap, x , in cm.	$\frac{x}{a}$	f	$R_{\max.} = (E/x)f$
4.710	0.1	0.1	1.034	48.7
8.100	0.2	0.2	1.068	43.3
11.37	0.3	0.3	1.102	41.8
14.49	0.4	0.4	1.037	41.2
17.49	0.5	0.5	1.173	41.0
20.37	0.6	0.6	1.209	41.0
23.25	0.7	0.7	1.245	41.3
26.04	0.8	0.8	1.283	41.8
31.29	1.0	1.0	1.359	42.5
35.49	1.2	1.2	1.437	42.5
38.64	1.4	1.4	1.515	41.8
41.28	1.6	1.6	1.599	41.3

Neglecting the first result we get 41.7 for the mean value of $R_{\max.}$ Substituting this value in the formula $E = R_{\max.} (x/f)$, we find that for 1 mm. gap the sparking pressure is 4.04 kilovolts, which is less than in either of the preceding cases.

We shall now analyse the results given in the "Standardization Rules" of the American Institute (1918), page 55.

TABLE 4.

Two equal spheres. Radius = $a = 3.125$ cm. Both insulated. Temperature = $\theta = 25^\circ$ C. Barometric height = $h = 76$ cm. $E/\sqrt{2}$ = effective kilovolts.

$E/\sqrt{2}$ Kilovolts	Spark-gap, x , in cm.	$\frac{x}{a}$	f	$R_{\max.} = (E/x)f$
10	0.42	0.1344	1.045	35.2
20	0.86	0.2752	1.093	35.9
30	1.35*	0.4320	1.148	36.1
40	1.92	0.6144	1.213	35.7
50	2.50	0.8000	1.283	36.3
60	3.20	1.024	1.372	36.4
70	3.95	1.264	1.462	36.6
80	4.90	1.568	1.588	36.7
90	6.05	1.936	1.743	36.7

The mean value of $R_{\max.}$ found from this table is 36.2.

TABLE 5.

Two equal spheres. Radius = $a = 6.25$ cm. Both insulated. Temperature = $\theta = 25^\circ$ C. Barometric height = $h = 76$ cm. $E/\sqrt{2}$ = effective kilovolts.

$E/\sqrt{2}$ Kilovolts	Spark-gap, x , in cm.	$\frac{x}{a}$	f	$R_{\max.} = (E/x)f$
30	1.41	0.2256	1.076	32.4
40	1.91	0.3056	1.104	32.7
50	2.44	0.3904	1.134	32.9
60	3.00	0.4800	1.165	33.0
70	3.60	0.5760	1.199	33.0
80	4.20	0.6720	1.234	33.2
90	4.90	0.7840	1.275	33.1
100	5.50	0.8800	1.311	33.7
120	7.10	1.136	1.412	33.8
140	8.80	1.408	1.521	34.2
160	11.0	1.760	1.668	34.3
180	13.8	2.208	1.861	34.3

The mean value of $R_{\max.}$ for spherical electrodes of radius 6.25 cm. is therefore 33.4. It will be seen that the values of $R_{\max.}$ increase slightly as the gap is increased. This is probably due to a slight want of symmetry between the two sides of the sparking apparatus. It has to be remembered that in calculating $R_{\max.}$ we must take the maximum value E for the pressure and not the effective value $E/\sqrt{2}$.

It is necessary to consider whether we are justified in assuming that the potentials of the spheres are equal and opposite. With direct voltage the leakage currents determine the potentials. Hence if the insulation resistance of each side of the supply is the same, the symmetry shows us that the potentials of the electrodes must be equal and opposite.

With alternating voltage we have also to take the capacities of the electrodes into account. We can exactly imitate the effects of the capacities of two equal spheres in space by assuming that they have no

capacity but are joined directly by a condenser of capacity $-k_{12}$ and are each connected to earth by a condenser $k_{11} + k_{12}$, where k_{11} and k_{12} are the self and mutual capacity coefficients. These coefficients can be readily calculated.* For instance when the spheres of radius 6.25 cm. are 3.125 cm. apart, so that x/a is 0.5, we find that

$$k_{11} + k_{12} = 4.55 \text{ cm.} = 4.55/(9 \times 10^5) \text{ mfd.}$$

$$-k_{12} = 3.28 \text{ cm.} = 3.28/(9 \times 10^5) \text{ mfd.}$$

It is also easy to see that the value of $k_{11} + k_{12}$ varies slowly as x increases.

If the potential difference of an electrode to earth was 100 kilovolts, and the frequency were 50, then the capacity current to earth would be

$$2\pi \times 50 \{4.55/(9 \times 10^5)\} 10^5 \times 10^{-6}$$

which equals 0.00016 of an ampere approximately. If the leakage currents be much smaller than this then the potentials of the spheres will be equal and opposite, their values being determined by the capacity currents.

It has to be remembered, however, that if one of the electrodes is nearer to a wall than the other, then the capacity coefficients of the two spheres are different and their potentials will no longer be equal and opposite. Even the presence of an experimenter in the neighbourhood of a sphere will alter its coefficients, the values of the artificial condensers connecting them to earth becoming $k'_{11} + k'_{12}$ and $k'_{22} + k'_{12}$ respectively. The ratio of the potentials of the electrodes will numerically be inversely as these capacities when the leakage currents are negligibly small. We shall return to this point when we discuss the experimental results obtained with one electrode earthed.

TABLE 6.

Two equal spheres. Radius = $a = 12.5$ cm. Both insulated. Temperature = $\theta = 25^\circ$ C. Barometric height = $h = 76$ cm. $E/\sqrt{2}$ = effective kilovolts.

$E/\sqrt{2}$ Kilovolts	Spark-gap, x , in cm.	$\frac{x}{a}$	f	$R_{\max.} = (E/x)f$
60	2.9	0.232	1.079	31.6
70	3.5	0.280	1.095	31.0
80	4.1	0.328	1.113	30.7
90	4.5	0.360	1.123	31.8
100	5.1	0.408	1.140	31.6
120	6.3	0.504	1.174	31.6
140	7.7	0.616	1.214	31.2
160	9.0	0.720	1.252	31.5
180	10.6	0.848	1.299	31.2
200	12.3	0.984	1.352	31.1
220	14.1	1.128	1.409	31.1
240	16.0	1.280	1.470	31.3
260	18.0	1.440	1.535	31.4
280	20.3	1.624	1.611	31.4
300	23.1	1.848	1.705	31.3
320	26.5	2.120	1.823	31.1

The mean value of $R_{\max.}$ is 31.3 kilovolts per centimetre and the constancy of the values for $R_{\max.}$ shows how carefully the experiments were made.

* As given in earlier editions. The value 1.41 given in the 1918 edition makes $R_{\max.}$ 34.5.

* 'Alternating Currents,' vol. 1, p. 243.

TABLE 7.

Two equal spheres. Radius = $a = 25$ cm. Both insulated.
Temperature = $\theta = 25^\circ$ C. Barometric height = 76 cm.

E/√2 Kilovolts	Spark-gap, x , in cm.	$\frac{x}{a}$	f	$R_{\max.} = (E/x)f$
80	4.1	0.164	1.055	29.1
90	4.5	0.180	1.061	30.0
100	5.1	0.204	1.069	29.6
120	6.2	0.248	1.084	29.7
140	7.3	0.292	1.099	29.8
160	8.3	0.332	1.113	30.3
180	9.5	0.380	1.130	30.3
200	10.6	0.424	1.145	30.6
220	11.7	0.468	1.161	30.9
240	13.0	0.520	1.179	30.8
260	14.4	0.576	1.199	30.6
280	15.8	0.632	1.220	30.6
300	17.1	0.684	1.238	30.7
320	18.7	0.748	1.261	30.5
340	20.4	0.806	1.283	30.2
360	22.1	0.884	1.312	30.2
380	23.9	0.956	1.341	30.2
400	25.7	1.028	1.372	30.2

The mean value of $R_{\max.}$ is 30.2 kilovolts per centimetre, and the analysis again shows the accuracy of the experimenters.

CORRECTING FOR TEMPERATURE AND PRESSURE.

The value of $R_{\max.}$ for air depends on the temperature and pressure, being greater the lower the temperature and the higher the pressure. If $R_{\max.}$ be the sparking potential gradient at 25° C. and 76 cm. pressure, and $R'_{\max.}$ be the sparking potential gradient at θ° C. and h cm. pressure, we have

$$R'_{\max.} = \frac{3.92}{273 + \theta} R_{\max.} \quad \text{and} \quad R_{\max.} = \frac{273 + \theta}{3.92 h} R'_{\max.}$$

For example, to convert the mean values of $R_{\max.}$ deduced from Heydweiller's experiments into their values at 25° C. and 76 cm. pressure we multiply them by $291/(3.92 \times 74.5)$, that is, by 0.9964. Hence we find that the mean values of $R_{\max.}$ at 25° C. and 76 cm. pressure for spheres whose radii are $\frac{1}{4}$, $\frac{1}{2}$ and 1 cm. respectively are 54.6, 47.2 and 41.5.

The author has found that the formula

$$R_{\max.} = 27.4 + 14.1/\sqrt{a} \quad (4)$$

is very approximately correct for spherical electrodes varying in radius from $\frac{1}{4}$ to 25 cm. For instance, the following table shows how the experimental results given above compare with the values computed by this formula.

Radius a in cm.	0.25	0.5	1	3.125	6.25	12.5	25
$R_{\max.}$ (observed)	54.6	47.2	41.5	36.2	33.4	31.3	30.2
$R_{\max.}$ (calculated)	55.6	47.3	41.5	35.4	33.2	31.4	30.2

It will be seen that the agreement is quite satisfactory. In experiments on spark voltages, the electrodes are generally put at a fixed distance apart and the voltage is gradually increased until the spark takes place. Before the spark ensues we have convection currents of electrified air flowing past the electrodes.

When we have a hot sphere in cold air the heat carried away by the convection currents per square centimetre of surface is greater for small spheres than for large ones. Similarly the electricity carried away per square centimetre of the electrodes by the air convection currents is greater for small than for large spheres. We know in the case of the hot sphere that the heat per square centimetre carried away by the convection currents varies as $1/a^{\frac{1}{2}}$, where a is the radius of the sphere. Pursuing the analogy, the electricity per square centimetre carried away by the convection currents flowing round the spheres will also vary as $1/a^{\frac{1}{2}}$ and this tending to prevent an electric discharge will introduce a term $B/a^{\frac{1}{2}}$ into the value of $R_{\max.}$ the force required to produce the discharge. The force required to break down the air itself will be constant. Hence we may assume that

$$R = A + B/\sqrt{a}$$

and determine as we have done above the values of A and B .

It will be noticed that formula (4) is very nearly identical with the formula

$$R_{\max.} = 27.2 + \frac{14.7}{\sqrt{a}}$$

given by F. W. Peek in his book on "Dielectric Phenomena."

The formula being undoubtedly very approximately true, it will be useful to construct by its help a table of the sparking distances between equal spheres 1 inch in diameter and also between equal spheres 10 cm. in diameter as these values are often used in practice. For 1-inch electrodes,

$$R_{\max.} = 27.4 + \frac{14.1}{\sqrt{1.27}} = 39.9 \text{ kilovolts per cm.} \\ = 101.4 \text{ kilovolts per inch.}$$

For 10-cm. electrodes,

$$R_{\max.} = 27.4 + \frac{14.1}{\sqrt{5}} = 33.71 \text{ kilovolts per cm.}$$

The values of E are equal to $(x/f) R_{\max.}$

E_1 denotes the sparking potential between the spheres when one of them is at zero potential. In this case

$$E_1 = (x/f_1) R_{\max.}$$

TABLE 8.

Two equal spheres. Radius = $a = 0.5$ inch. Temperature = $\theta = 25^\circ$ C. Barometric pressure = $h = 76$ cm. E = sparking pressure in kilovolts when both spheres insulated. E_1 = sparking pressure in kilovolts when one sphere is at zero potential. $R_{\max.} = 101.4$ kilovolts per inch.

Spark-gap, x , in inches	$\frac{x}{a}$	f	f_1	E	E_1
0.1	0.2	1.068	1.068	9.50	9.50
0.2	0.4	1.137	1.150	17.8	17.6
0.3	0.6	1.209	1.253	25.2	24.3
0.4	0.8	1.283	1.378	31.6	29.4
0.5	1.0	1.359	1.517	37.3	33.4
0.75	1.5	1.559	1.909	48.8	39.8
1.0	2	1.770	2.339	57.3	43.3
1.5	3	2.215	3.252	68.7	46.8
2.0	4	2.678	4.201	75.7	48.3
2.5	5	3.151	5.167	80.5	49.1
3.0	6	3.632	6.143	83.8	49.5

If the disruptive voltage for a 3-inch gap were 30 times that for a 0.1-inch gap it would equal 285 kilovolts. Its actual value is 83.8 kilovolts when the potentials are equal and opposite, that is, when they are 41.9 and -41.9 kilovolts respectively. This gap would also break down when one of the electrodes was at 49.5 kilovolts and the other was maintained at zero potential.

TABLE 9.

Two equal spheres. Radius = $a = 5$ cm. Temperature = $\theta = 25^\circ$ C. Barometric height = $h = 76$ cm. E = sparking pressure in kilovolts, both spheres insulated. E_1 = sparking pressure in kilovolts, one sphere at zero potential. $R_{\max.} = 33.71$ kilovolts per centimetre.

Spark-gap, x , in cm.	$\frac{x}{a}$	f	f_1	E	E_1
0.5	0.1	1.034	1.034	16.3	16.3
1.0	0.2	1.068	1.068	31.6	31.6
1.5	0.3	1.102	1.106	45.9	45.7
2.0	0.4	1.137	1.150	59.3	58.6
3.0	0.6	1.209	1.253	83.7	80.7
4.0	0.8	1.283	1.378	105.2	97.8
5.0	1.0	1.359	1.517	124.1	111.1
7.5	1.5	1.559	1.909	162.2	132.4
10.0	2	1.770	2.339	190.4	144.1
20.0	4	2.678	4.201	251.8	160.5
30.0	6	3.632	6.143	278.5	164.6

For instance, we see from Table 9 that with 10-cm. electrodes when the distance between them is 7.5 cm.

a spark discharge ensues when the potentials of the electrodes are 81.1 and -81.1 kilovolts respectively, and also when they are at potentials 132.4 kilovolts and zero respectively. For sine-shaped waves the effective values of the sparking voltages are $E/\sqrt{2}$ and $E_1/\sqrt{2}$ respectively.

SIZE OF ELECTRODES WHICH REQUIRE THE HIGHEST VOLTAGE FOR A GIVEN SPARK-GAP.

Using the formula (4) for $R_{\max.}$ we have constructed Table 10.

Looking along the horizontal column for a spark-gap of 0.2 cm. it will be seen that the sparking voltage continually diminishes as the size of the electrodes is increased, beginning with electrodes of radii each equal to $\frac{1}{4}$ cm. For equal electrodes of radii 0.1 cm. the sparking pressure for this air-gap would be 8.1 kilovolts. It will be seen therefore that for electrodes of a certain size of radius lying in value between 0.1 and 0.5, the sparking voltage for an air-gap of 0.2 cm. has a maximum value. For the other gaps given in the table it will be seen directly that a similar theorem holds.

When the air-gap is greater than 0.7 cm. it is not difficult to calculate the size of the electrodes which make the sparking voltage a maximum. In this case we may write

$$E = \frac{x(27.4 + 14.1/\sqrt{a})}{1 + x/3a + x^2/(45a^2)}$$

Equating $\frac{\partial E}{\partial a}$ to zero, we get

$$7.05(45a^2 + 15ax + x^2) = x(15a + 2x)(27.4a^{\frac{1}{2}} + 14.1)$$

The positive value of a which satisfies this equation makes E a maximum, provided that it does not make x/a greater than a half. This is ensured if x be greater than 0.7.

As a numerical example, let us put $x = 1$. The equation becomes

$$7.05(45a^2 + 15a + 1) = (15a + 2)(27.4a^{\frac{1}{2}} + 14.1)$$

By substituting, for a , 2 and 3 respectively we see that the root must lie between these values; for 2 makes the right-hand side the greater whilst 3 makes it the smaller. Assuming that $a = 2.5$ is an approximate root and applying Newton's theorem, we find that $a = 2.53$ satisfies the equation very approximately. The value of E in this case is 31.96. Since $1/2.53$ is less than 0.5 we are justified in assuming that f equals

$$1 + x/(3a) + x^2/(45a^2)$$

Hence when the air-gap is 1 cm. the sparking voltage is a maximum for spherical electrodes whose diameters are each equal to 5.06 cm., which is practically 2 inches.

By using the approximate formulæ given for f we could find in a similar way the spherical electrodes which make the sparking voltages a maximum for air-gaps less than 0.7 cm. They can, however, easily be found graphically by plotting the values of $(x/f)R_{\max.}$ against the radii of the electrodes.

TABLE IO.

Sparking Pressure in Kilovolts for Various Sized Electrodes for Gaps of 0.2, 0.5, 1, 1.5 and 2 cm. respectively.

Spark-gap in cm.	0.25	0.5	1.0	3.125	5	6.25	12.5	25	Inf.	Radii
0.2	8.7	8.3	7.8	7.1	6.7	6.6	6.2	6.1	5.5	Pressure in kilovolts
0.5	15.7	17.4	17.7	16.8	16.3	16.2	15.5	15	13.7	"
1.0	20.8	26.7	30.5	31.9	31.6	31.5	30.6	29.8	27.2	"
1.5	2.3	32.0	39.9	45.6	45.9	46.1	45.3	44.1	41.9	"
2.0	24.2	35.3	46.8	57.3	59.3	59.9	59.6	58.8	54.8	"

ONE ELECTRODE EARTHED.

We shall now consider the experimental results given in the American "Standardization Rules" when one electrode is earthed.

TABLE II.

Two equal spheres. Radius = $a = 25$ cm. One earthed ($E_2 = 0$). Temperature = $\theta = 25^\circ$ C. Barometric height = $h = 76$ cm. $E_1/\sqrt{2}$ = effective kilovolts.

$E_1/\sqrt{2}$ Kilovolts	Spark-gap, x , in cm.	$\frac{x}{a}$	f_1	$R_{\max.} =$ $(E_1/x)f_1$
80	4.1	0.164	1.055	29.1
90	4.6	0.184	1.062	29.4
100	5.2	0.208	1.070	29.1
120	6.3	0.252	1.088	29.3
140	7.4	0.296	1.105	29.6
160	8.5	0.340	1.123	29.9
180	9.7	0.388	1.143	30.0
200	10.8	0.432	1.168	30.5
220	12.0	0.480	1.188	30.8
240	13.3	0.532	1.215	31.0
260	14.8	0.592	1.249	31.0
280	16.3	0.652	1.284	31.2
300	17.7	0.708	1.319	31.6
320	19.4	0.776	1.362	31.8
340	21.4	0.856	1.416	31.8
360	23.4	0.936	1.471	32.0
380	25.5	1.020	1.531	32.2
400	27.6	1.104	1.595	32.7

It will be seen that the values of $R_{\max.}$ are not constant but increase gradually in value from about 29.1 to 32.7. As the experiments were carefully made there must be some reason for this increase. A water-tube resistance of about 1 ohm per volt was probably inserted in series with the insulated electrode so as to damp the high-frequency oscillations at the time of breakdown and also limit the current which will flow. This probably introduced a slight lack of symmetry but

not enough, in our opinion, to have any appreciable effect.

Before discussing other possible causes let us make the assumption that the potentials of the electrodes at the instant of breakdown, instead of being E_1 and

TABLE 12.

Two equal spheres. Radius = $a = 25$ cm. $E_2 = -0.43$ ($E_1 - E_2$), $m = 0.43$.

$(E_1 - E_2)/\sqrt{2}$ Kilovolts	Spark-gap, x , in cm.	f	f_1	$R_{\max.} = \frac{E_1 - E_2}{x} f_1$ $-\frac{2m(E_1 - E_2)}{x} (f_1 - f)$
80	4.1	1.055	1.055	29.1
90	4.6	1.062	1.062	29.4
100	5.2	1.070	1.070	29.1
120	6.3	1.086	1.088	29.3
140	7.4	1.101	1.105	29.5
160	8.5	1.116	1.123	29.8
180	9.7	1.133	1.143	29.8
200	10.8	1.148	1.164	30.2
220	12.0	1.165	1.188	30.3
240	13.3	1.184	1.215	30.6
260	14.8	1.206	1.249	30.4
280	16.3	1.227	1.284	30.4
300	17.7	1.248	1.319	30.6
320	19.4	1.263	1.362	30.5
340	21.4	1.304	1.416	30.3
360	23.4	1.335	1.471	30.2
380	25.5	1.367	1.531	30.1
400	27.6	1.400	1.595	30.2

This table makes the mean value of $R_{\max.} = 30.0$. This compares with 30.2 when the potentials are equal and opposite.

zero, are E_1 and E_2 , so that $E_1 - E_2$ is the sparking pressure. We shall also make the assumption that the ratio E_1/E_2 is constant. We may therefore write

$E_2 = -m(E_1 - E_2)$ where m is a constant. In this case we get, by (1)

$$R_{\max.} = \frac{E_1 - E_2}{x} f_1 - \frac{2m(E_1 - E_2)}{x} (f_1 - f)$$

There are two unknowns in this equation, $R_{\max.}$ and m , we therefore need two experimental results to determine them. Taking the experimental results for sparking pressures of 200 and 400 kilovolts respectively from the table given above, we get

$$R_{\max.} = 30.5 - m(400/10.8)(0.016)$$

and $R_{\max.} = 32.7 - m(800/27.6)(0.195)$

Solving these equations we find that

$$R_{\max.} = 30.2 \text{ and } m = 0.4348$$

This value of $R_{\max.}$ is the same as the mean result obtained when the potentials of the spheres were equal and opposite. It will therefore be of interest to see what value we get for $R_{\max.}$ from the other experiments on the assumption that $m = 0.43$ whatever the distance apart of the electrodes.

If all the tests were made with the same apparatus then it is probable that m is approximately equal to 0.43 in all cases. We shall make this assumption.

TABLE 13.

Two equal spheres.

Radius = $a = 12.5$ cm.

$$E_2 = -0.43(E_1 - E_2); E_1 = 0.57(E_1 - E_2);$$

$$R_{\max.} = \frac{E_1 - E_2}{x} f_1 - \frac{0.86(E_1 - E_2)}{x} (f_1 - f)$$

$(E_1 - E_2)/\sqrt{2}$ Kilovolts	Spark- gap, x , in cm.	$\frac{x}{a}$	f	f_1	$\frac{E_1 - E_2}{x} f_1$	$R_{\max.}$
60	2.9	0.232	1.079	1.080	31.6	31.6
70	3.5	0.280	1.095	1.099	31.1	31.1
80	4.1	0.328	1.113	1.118	30.8	30.7
90	4.6	0.368	1.126	1.134	31.4	31.3
100	5.2	0.416	1.143	1.156	31.5	31.3
120	6.4	0.512	1.177	1.205	32.0	31.5
140	7.8	0.624	1.217	1.267	32.2	31.4
160	9.2	0.736	1.258	1.336	32.9	31.7
180	10.9	0.872	1.310	1.427	33.3	31.7
200	12.8	1.024	1.373	1.534	33.9	31.7
220	15.0	1.200	1.437	1.668	34.6	31.7
240	17.7	1.416	1.525	1.840	35.3	31.6
260	21.0	1.680	1.633	2.061	36.1	31.5
280	25.0	2.000	1.770	2.339	37.1	31.6

The mean value of $R_{\max.}$, therefore, is 31.5 kilovolts per centimetre. This compares with 31.3 given by Table 6.

TABLE 14.

Two equal spheres.

Radius = $a = 6.125$ cm.

$$R_{\max.} = \frac{E_1 - E_2}{x} f_1 - \frac{0.86(E_1 - E_2)}{x} (f_1 - f)$$

$(E_1 - E_2)/\sqrt{2}$ Kilovolts	Spark- gap, x , in cm.	$\frac{x}{a}$	f	f_1	$\frac{E_1 - E_2}{x} f_1$	$R_{\max.}$
30	1.41	0.2256	1.077	1.077	32.4	32.4
40	1.91	0.3056	1.104	1.109	32.4	32.4
50	2.44	0.3904	1.134	1.145	33.2	33.0
60	3.00	0.4800	1.170	1.188	33.6	33.3
70	3.60	0.5760	1.200	1.240	34.1	33.4
80	4.20	0.6720	1.234	1.296	34.9	34.0
90	4.90	0.7840	1.276	1.368	35.5	34.1
100	5.60	0.8960	1.321	1.443	36.4	34.5
120	7.97	1.275	1.473	1.727	36.8	33.5
140	10.8	1.728	1.655	2.102	38.5	32.4
160	15.0	2.400	1.945	2.698	40.7	33.8

The mean value of $R_{\max.}$ is 33.3 kilovolts per centimetre. This compares with the value 33.4 found from Table 5.

TABLE 15.

Two equal spheres.

Radius = $a = 3.125$ cm.

$$R_{\max.} = \frac{E_1 - E_2}{x} f_1 - \frac{0.86(E_1 - E_2)}{x} (f_1 - f)$$

$(E_1 - E_2)/\sqrt{2}$ Kilovolts	Spark- gap, x , in cm.	$\frac{x}{a}$	f	f_1	$\frac{E_1 - E_2}{x} f_1$	$R_{\max.}$
10	0.42	0.1344	1.045	1.045	35.2	35.2
20	0.86	0.2752	1.092	1.094	36.0	35.9
30	1.35	0.4320	1.148	1.164	36.6	36.3
40	1.92	0.6144	1.214	1.262	37.2	36.3
50	2.55	0.8160	1.288	1.389	38.5	36.8
60	3.45	1.104	1.400	1.595	39.2	36.3
70	4.60	1.472	1.488	1.886	40.6	35.4
80	6.20	1.984	1.763	2.324	42.4	36.2

The mean value of $R_{\max.}$ is 36.1 kilovolts per centimetre. This compares with the value 36.2 given by Table 4.

From the results given above we conclude that the spheres act as if their potentials were 0.57 ($E_1 - E_2$) and $-0.43(E_1 - E_2)$ respectively.

EXPLANATION OF THE POTENTIAL VALUES.

The author does not know the nature of the apparatus that was used for the test or how it was arranged. The simplest explanation he can suggest of the anomalous

results obtained when one electrode was earthed is the following. A transformer with a high-tension winding may have been employed. The centre of this winding may have been earthed. When the electrodes were insulated the electromotive forces generated in the halves of the transformer winding would have been equal and the electrodes would have been at equal and opposite potentials. If, however, one of the electrodes was earthed through a high resistance one half of the transformer winding would be loaded with this resistance and the other half would be practically on open circuit, as the capacity current is negligibly small. Let us suppose that the voltage drop on the loaded side was 25 per cent. Then it would always have this value whatever the voltage, whatever sized spheres were used as electrodes and whatever their distance apart. Hence if 57 kilovolts was the potential of the insulated electrode

the potential of the other would be $-(57 - \frac{25}{100} \times 57)$,

that is, —43 kilovolts. On this assumption, therefore, from what we have shown above it would follow that the maximum electric stress between the two electrodes determines in all cases the disruptive discharge.

The above explanation is only offered tentatively and possibly some other explanation is the correct one.

UNEQUAL SPHERES.

When the spherical electrodes are unequal the formulæ given in "Alternating Currents," vol. I, chap. viii, enable the maximum stress at the surfaces of the two spheres to be computed. It is not easy to see which of them determines the disruptive voltage. As the potentials of the two spheres will not be equal and opposite the calculation of the maximum stresses will be laborious. It offers a promising field for experimental research as curious results will probably be found.

TABLES.

The author has found the numerical calculations in this paper rather laborious. He would suggest that the time has now come when complete tables of the values of f and f_1 should be constructed. He would also suggest that Kelvin's tables of the electrostatic capacity coefficients between equal spheres and of the

coefficients which enable their attractions to be found be expanded and completed. The labour involved would not be great and the tables would be at least as valuable and useful as many of those recently computed by the British Association.

SUMMARY.

(1) Simplified formulæ are given by means of which the maximum electric stress between equal spherical electrodes can be readily found.

(2) Experimental results on sparking voltages when the potentials of the equal spherical electrodes are equal and opposite are analysed. It is found that if a be the length of the radius of each electrode in centimetres, the maximum potential gradient $R_{\max}$ at the instant of discharge is given by

$$R_{\max} = 27.4 + \frac{14.1}{\sqrt{a}} \text{ kilovolts per cm.}$$

very approximately when the temperature of the air is 25° C. and the height of the barometer is 76 cm.

(3) The sparking voltage E between two spherical electrodes, the radius of each being a , and x being the distance between them, is given by

$$E = \frac{x}{f} \left(27.4 + \frac{14.1}{\sqrt{a}} \right) \text{ kilovolts}$$

at 25° C. and 76 cm. pressure, where f is a factor which can be easily calculated from the geometrical dimensions.

It has to be remembered that these formulæ do not apply to air-gaps less than 1 mm. or air-gaps so great that brush discharges take place.

(4) It follows from the formulæ given above that in general for a fixed spark-gap there are certain sized spherical electrodes which offer the maximum resistance to sparking. Increasing or diminishing the size of the electrodes from this value makes the spark take place more readily.

(5) The results when one electrode is earthed given in the "Standardization Rules" of the American Institute are analysed. From the results it appears that the potentials of the electrodes were not E and 0, but were $0.57E$ and $-0.43E$. These values of the potentials make the values of $R_{\max}$ deduced the same as the values found when the potentials are equal and opposite.