

describe the tests of toil and pain, the tests of fear, the tests of seductive pleasures, and he tells us that "He who at every age as boy and youth, and in mature life, has come out of the trial victorious and pure, shall be appointed a ruler and guardian of the state. He shall be honored in life and death, and shall receive sepulcher and other memorials of honor, the greatest that we have to give."

The essentials of life and character have not changed since the days when Socrates talked of truth and justice in the groves of Academus. You graduates to-day go forth to be tested. You have in varying measure the vision of the university, the sense of obligation which your training lays upon you. You must hear, be it ever so faintly, the call to be servants of the commonwealth. Put to yourselves the question which comes down through the centuries, can you hold to this conviction that the interests of the community should be the rule of all your actions. You will face intellectual sophistry and beguiling fallacies. Have you the keenness of mind and the force of character to analyze these specious assertions and to hold steadfastly to things that are true and enduring? You will be tested by fear, fear of financial loss, fear of ridicule, fear it may be of social ostracism. Have you the courage and character to preserve your convictions of loyalty to the general good? You will be lured by pleasure, dazzled it may be by luxury and ostentation, tempted to self-indulgence and evanescent pleasures. Have you the fiber to resist these appeals and to remember that the social servant must be ever strong, clear eyed and faithful to his work?

May you hold to the vision you have caught: may it with the passing years grow ever clearer, brighter, more commanding in your lives. The university sends you

forth to-day with God speed, entrusts to you the good name of our widening community, summons you to loyalty, urges you to organize all your resources of mind and spirit into the unity of a high aim, the firm resolve to realize in your own lives the masterful purpose of the university which is to be in ever fuller measure at once the standard bearer and the servant of the state.

Go to your work and be strong, halting not in a world of men,
Basking the end half won for an instant dole of praise.
Stand to your work and be wise—certain of sword and pen,
Who are neither children nor Gods, but men in a world of men.

GEORGE E. VINCENT

COURSES IN HIGHER PURE MATHEMATICS

THE number of the objects of mathematical thought is infinite and the rapidly widening range of developed mathematics is continually directing mathematical attention to objects which were previously either practically or entirely ignored. Efforts to classify mathematics have been only partially successful and it is extremely difficult, in many cases, to draw reliable conclusions as to the nature of a course from its title. Hence the efforts to ascertain from the announcements of the leading universities of the world the relative emphasis which different countries place on the various subjects of higher mathematics can not be expected to lead to entirely trustworthy results.

The rapid development of our universities has led to such a rapid increase in the number of different mathematical courses beyond the first courses in differential and integral calculus, that many well-educated people have failed to keep informed as regards the general meaning of the titles of some of these courses. This is perhaps not surprising in view of the fact that several of our strongest universities offer their advanced courses under more than thirty different titles.

This great number of titles of courses implies that many of them are devoted to comparatively narrow fields and raises the question whether we are not in danger of specializing too much and of depriving our students of a thorough training in the more general and fundamental methods of wide contact. It would probably be generally conceded that we have gone further in the line of special courses than the universities of Germany, which were pioneers in this direction. On the other hand, it may be observed that the large German and French mathematical encyclopedias which are in the course of publication treat pure mathematics under about one hundred general headings, each of which would be sufficiently extensive for a course suitable for graduate students.

This great extent of available material makes it impossible for one student to cover even superficially the entire field during his college days and hence it tends to increase his interest in a wise choice of his courses, especially as regards such courses as are most generally given and are most likely to be very useful in his later mathematical development. One of the most reliable sources of information along this line is furnished by the number and the extent of the courses devoted to different subjects in the various leading universities.

Various journals, including the *Bulletin of the American Mathematical Society*, are publishing, from time to time, lists of announcements of the courses offered by a large number of different universities of Europe and America. A comparative study of these announcements for a period of years involves a large amount of labor in view of the changes from year to year and the uncertainty as to the real significance of some of the titles. As American mathematical activity has been so largely influenced by Germany, especially during recent years, and as the University of Berlin occupies such a prominent place among the greatest centers of learning, it may be fitting to begin with some results drawn from the announcements of this university during the last six years.

The following table gives only those courses in pure mathematics to which the equivalent of at least twelve lecture hours for one semester have been devoted during the last six years. A large number of special courses have thus been excluded. For instance, one of the courses offered only once during the period covered by the table bears the title, "Transcendental Nature of π and e ," another is announced under the title "Influence of Euler's Work on Modern Mathematics," while still another bears the title "Problems in Maxima and Minima treated by Methods of Elementary Geometry." No course bearing the title "Differential Geometry" was offered at Berlin during the period under consideration, and only one course entitled "Linear Substitutions." A large part of the subject matter of such courses was, of course, given in the courses under more general titles. Similar remarks apply to a large number of other subjects. The number of hours devoted to seminars and colloquia was not included in the following table as the subject matter of these exercises was not always clearly announced.

BERLIN UNIVERSITY

Subjects	Lecture Hours for Six Years	Totals
Curves and surfaces	4, 11, 8, 8, 8, 13 ¹	52
Theory of functions	4, 6, 0, 8, 10, 12	40
Determinants, theory and applications	4, 8, 4, 4, 8, 6	34
Theory of numbers	4, 4, 8, 8, 4, 2	30
Elliptic functions	4, 4, 4, 8, 8, 2	30
Algebraic equations	0, 4, 8, 4, 6, 4	26
Differential equations	0, 4, 6, 4, 6, 4	24
Algebra	4, 4, 0, 4, 0, 4	16
Calculus of variation	0, 4, 4, 0, 0, 4	12

It may be observed that the total number of hours devoted to the four subjects belonging to arithmetic and algebra is 106, while the total numbers of hours devoted to analysis and geometry are 98 and 52, respectively. As we shall see later, these results are not in accord with those obtained in a similar man-

¹ The last of these numbers applies to the current year, and the numbers which precede relate to the preceding years in order.

ner from many other universities and they would be slightly changed at Berlin if all the courses in pure mathematics had been tabulated. They would also have been affected by the consideration of the courses on applications of mathematics. As is well known, a large number of courses on applications of elliptic functions are given at Berlin. These are in addition to the courses on the theory of these functions as listed above, and during the six years under consideration the number of lecture hours devoted to these applications in Berlin University were, respectively, 6, 0, 10, 0, 8, 4—making a total of 28. Hence a total of 58 lecture hours for a semester were devoted to elliptic functions and their applications during these six years—an average of nearly five hours for each semester.

Among the other German universities which maintain very strong mathematical departments Göttingen should perhaps be especially mentioned in view of the facts that so many Americans have studied there and the influence of Klein and Hilbert has been so great in shaping our courses in higher mathematics. It may also be desirable to bring the mathematical courses of the universities of Berlin and Göttingen together in view of the fact that they exhibit a great difference as to emphasis on the various subjects of pure mathematics. For instance, only two courses on determinants and their applications were given at Göttingen during the last six years while at Berlin this course has been given very frequently, as may be seen from the table given above.

At Göttingen courses on differential equations have been given very much more frequently than at Berlin, while courses on elliptic functions are much more common at the latter institution than at Göttingen. Judging from the number and the extent of the courses, there is a very wide difference between Berlin and Göttingen as regards the emphasis on the subjects which are usually classed under the general headings, algebra and analysis; or arithmetic and algebra, and analysis. At Berlin the former receive very much more attention than at Göttingen and

the predominating influence of the latter institution is evident in the advanced mathematical courses of many American universities. A marked difference between Göttingen and Berlin may also be observed with respect to the tendency to give courses under a large variety of names. At Göttingen we find a larger number of courses under such titles as, encyclopedia of geometry, encyclopedia of elementary mathematics and elementary mathematics from a higher standpoint, than at Berlin.

In the following table we give again only those courses to which at least twelve lecture hours for one semester have been devoted during the last six years, excluding courses which appeared to have been devoted mainly to applications, and arranging the others in the order of the number of lecture hours.

GÖTTINGEN UNIVERSITY

Subjects	Lecture Hours for Six Years	Totals
Differential equations.....	12, 12, 12, 8, 11, 4	59
Theory of functions.....	7, 0, 8, 6, 10, 8	39
Descriptive geometry.....	4, 4, 0, 8, 0, 8	24
Algebra.....	4, 0, 4, 4, 4, 4	20
Theory of numbers.....	4, 0, 2, 4, 3, 4	17
Calculus of variations.....	4, 4, 1, 4, 0, 2	15
Curves and surfaces.....	7, 4, 2, 0, 0, 0	13
Principles of mathematics	0, 0, 4, 2, 2, 4	12

A comparative study of the courses offered in other German universities reveals wide differences as regards emphasis on the various subjects and such a study tends to explain the migration of students from one institution to another. Unfortunately, there is very little migration in American universities, and hence our students are frequently acquainted only with the courses offered by one institution. This makes it the more desirable that our large universities should aim to offer a wide range of subjects, covering the most important parts of the various developed fields of mathematics. It is evident, however, that it would be much better if our students could be induced to divide their time of graduate study among different universities and to seek instruction under the foremost men along the lines of their chief interests.

It is of great interest to consider the difference between the emphasis placed on various subjects by different countries. A comprehensive study of this difference becomes, however, quite difficult in view of the fact that the different countries vary widely as regards the line of division between their university courses and those treated earlier. It is, however, not difficult to establish certain decided differences. A slight study reveals the fact that American universities are unusually weak, on an average, with respect to courses on elliptic functions, general mathematics and theory of numbers; while the relative number of our courses in the theory of groups, theory of functions, and differential geometry is above the average. These results are deduced from a fairly extensive tabulation of the courses in mathematics which Professor J. B. Shaw recently presented before the Mathematical Club of the University of Illinois. In particular, Professor Shaw listed the courses of all German universities for a period of six years and found that during this period the number of lecture hours devoted to courses in the three great fields of pure mathematics—algebra, analysis and geometry—were in the proportion 193, 259 and 200, respectively.

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WILLIAMINA PATON FLEMING

MRS. WILLIAMINA P. FLEMING, curator of astronomical photographs at the Harvard College Observatory, was born in Dundee, Scotland, on May 15, 1857, and came to this country soon after her marriage in early womanhood. She soon drifted into the work which was destined to occupy her life, by undertaking some simple astronomical calculations at the Harvard Observatory, where, upon her death on May 21, 1911, she had just completed thirty years of service. These thirty years have covered a period of remarkable changes in the methods of attacking astronomical problems. The prism has revealed to us something of the nature of the heavenly bodies, and the photographic plate has made

a permanent record of the condition of the sky, which may be studied at any time. Celestial photography was systematically undertaken at Harvard in 1882, by Professor E. C. Pickering, the present director. The work was placed on a firm basis by the liberality of Mrs. Draper in establishing the Henry Draper Memorial, and in a short time, photographs were being taken in large numbers. The Harvard photographic library now contains over 200,000 plates.

Mrs. Fleming's duties were to qualify these plates, superintend their care, examine them for peculiar objects, and make investigations by means of them. Each plate must be so indexed that it can be found at any time, and must be carefully handled and stored, being of fragile glass, and without a duplicate. With a naturally clear and brilliant mind, Mrs. Fleming at once evinced special aptitude for this photographic investigation, which was so novel that precedents could not be found for its execution, and, in return, the photographs proved to be veritable mines of wealth for the extraction of information concerning the sidereal universe. Most of Mrs. Fleming's discoveries were made from the spectrum plates which are taken by means of a prism placed before the object glass of the telescope, and which often show the spectra of several hundred stars. She examined with a magnifying glass, all these plates taken at Cambridge and at the Harvard southern station in Arequipa, Peru, and marked all objects showing any peculiarities in their spectra. In this way, she found ten new stars and more than three hundred variable stars, because of the presence of bright lines in their spectra. She classified the spectra of 10,351 stars, which were published in 1890 in a volume of the *Harvard Annals*, called the "Draper Catalogue of Stellar Spectra." When stricken with the fatal illness, she was at work on a Memoir on Peculiar Spectra, which will give useful tables and much additional information concerning many interesting celestial objects. Much of her time was always occupied by tedious work upon the proof of the numerous volumes of the *Annals*