A METHOD OF ANALYZING THE ELECTRO-CARDIOGRAM *

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The electrocardiogram as generally taken consists of three leads, obtained by using the two arms and the left leg as the contact or leading off points. Einthoven¹ has shown that these three leads bear a definite mathematical relationship to one another and has used an equilateral triangle to express this relationship graphically. For the better understanding of the succeeding parts of this paper we shall consider this mathematical relationship and its graphic representation at some length.

The mathematical relationship is expressed by the equation: lead II equals lead III plus lead I. This means that the height of the ordinate in lead II of the electrocardiogram at any instant is the sum of the heights of the ordinates in leads III and I. The reason for this relationship can be appreciated if we consider the string galvanometer simply as an instrument for measuring differences of potential.² If now we wish to measure the difference of potential between the right arm and the left leg (lead II) at any instant, we can do it either directly by connecting the galvanometer with the right arm and left leg (lead II) or indirectly by taking some other point, as for example the left arm, finding the difference of potential between the right arm and the left arm (lead I) and between the left arm and the left leg (lead III) and adding them together. In a similar manner, if one wishes to find the difference in elevation (potential) between two towns, A and B, one may either subtract the elevation of A from the elevation of B (lead II), or one may take a third town, C, find the difference in elevation between A and C (lead I) and between B and C (lead III) and add them together. It is clear that the answer must be the same whichever way one proceeds. Thus, if the E. K. G. gives us a true record of differences of potential, it must follow that for

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^{1.} Einthoven, W.: The Different Forms of the Human Electrocardiogram and Their Signification, Lancet 1:853, 1912.

^{2.} The string galvanometer, because of the fact that the resistance of its circuit is not infinite, has a slight tendency to decrease the difference of potential between the leading off points, but ordinarily the error from this source is negligible, and the mathematical relationship between leads holds within the limits of experimental error.

any and every moment during the cardiac cycle lead II equals lead III plus lead I; or, as it is more convenient for practical purposes to put it, lead II minus lead I equals lead III.

To express this relation graphically Einthoven^{1, 3} makes use of an equilateral triangle. The peculiar fitness of the equilateral triangle is due to the fact that the projections on the sides of an equilateral triangle of any straight line drawn within the triangle have a relationship similar to the relationship between the leads of an electrocardiogram. Figure 1 will demonstrate this point.

O A is any straight line drawn within the equilateral triangle R L F. The projections of O A on the three sides correspond to the three leads of the electrocardiogram and are labeled ϵ_1 , ϵ_2 and ϵ_3 , respectively. The geometrical proof given shows that $\epsilon_2 = \epsilon_3 + \epsilon_1$. This equation will hold whatever be the position of O A, provided we give ϵ_1 , ϵ_2 and ϵ_3 negative values when they would have negative values in the electrocardiogram.

Since Einthoven first demonstrated these mathematical and geometrical relationships, numerous investigators⁴ have made use of them in analyzing electrocardiograms. The methods used have been various minor modifications of the original method of Einthoven, Fahr and de Waart,³ which consists in finding the values of the three leads at any chosen instant and in substituting these values in certain formulas, thereby obtaining a linear value called "E" or the "manifest value," and an angular value, "a." These two values determine the length and direction of a vector which corresponds to the line O A in Figure 1; O A in Einthoven's formula having the linear value E, and the angle C A O being a. The length, E, of the vector gives the manifest value of the potential difference as shown by the three leads, but the vector as drawn by Einthoven in his original paper,¹ and thereafter by numerous users of this method^{3, 4} is in a direction directly away from the point at which the center of negativity is located.

Carter, Richter and Greene⁵ have developed an interesting modification of Einthoven's method. By means of an equilateral triangle,

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^{3.} Einthoven, W., Fahr, G., and deWaart, A.: Ueber die Richtung und die manifeste Grösse der Potentialschwankungen im menschlichen Herzen und über den Einfluss der Herzlage auf die Form des Elektrokardiogramms, Arch. f. d. ges. Physiol. **150**:275, 1913.

^{4.} Pardee, H. E. B.: Form of the Electrocardiogram, J. A. M. A. 62:1311 (April 25) 1914. Fahr, G.: Simultaneous Records of Heart Sounds and the Electrocardiogram, Heart 4:147, 1912.

^{5.} Carter, E. P.; Richter, C. P., and Greene, C. H.: A Graphic Application of the Principle of the Equilateral Triangle for Determining the Direction of the Electrical Axis of the Heart in the Human Electrocardiogram, Bull. Johns Hopkins Hosp. **30**:162 (June) 1919.



Fig. 1.—This is a simple geometrical demonstration of those properties which make the equilateral triangle peculiarly suitable for use in analysis of electrocardiograms.

The values of the rectangular coordinates (X, Y) of a point (A) are found algebraically in terms of the three leads $(\epsilon_i, \epsilon_2 \text{ and } \epsilon_3)$.

PROPOSITION: The longest projection on the sides of an equilateral triangle of any straight line drawn within an equilateral triangle equals the sum of the projections on the other two sides.

LET A O* be any straight line drawn within the equilateral triangle R L F. Let ϵ_1 , ϵ_2 , and ϵ_3 be the projections of A O on the three sides of the triangle, obtained by dropping perpendiculars (AA₁, AA₂, AA₃, OO₁, OO₂, OO₃) from the ends of the line A O upon the sides of the triangle.

TO PROVE THAT $\epsilon_2 = \epsilon_3 + \epsilon_1$.

CONSTRUCTION: 1. Produce A₂A until it meets OO₁ at B.

- 2. Draw BB₃ perpendicular to L F.
- 3. Draw A H parallel to R L cutting B O at C.
- 4. Draw K B₃ parallel to A H.
- 5. Draw R O₃* the perpendicular bisector of L F.

PROOF

$A_2O_2 = B_3O_3$	for they are the projections of OB and the projections on the sides
	of an isosceles triangle of any line perpendicular to the base are equal.

A_2O_2	$= A_3O_2$	3 + A3	B_3 for $B_3O_3 = A_3O_3 + A_3B_3$.
€2	$= \epsilon_3$	+ A ₃	B ₃ for $A_2O_2 = \epsilon_2$ and $A_8O_3 = \epsilon_3$.
	$= \epsilon_3$	+ ½K	B ₃ for $A_3B_3 = \frac{1}{2}KB_3$ because triangle K A_3B_4 is similar to triangle R L O ₃ and L O ₃ = $\frac{1}{2}$ R L since F L=R L and L O ₃ = $\frac{1}{2}$ F L.
	$=\epsilon_3$	+ ½A	H for K $B_3 = A$ H because parallel lines included between parallel lines are equal.
	Ξ 63	+ A	C for $\frac{1}{2}$ A H=A C because A C=C H since triangle A B C= triangle H B C (The two triangles have the common side B C, angle A C B=angle H C B=a right angle, and angle A B C= angle H B C=60° since perpendiculars to the sides of an equilateral triangle intersect at angles of 60°.)
€2	$=\epsilon_3$	+ ει	for A $C = \epsilon_1$, because parallel lines included between parallel lines are coual.
			Q. E. D.

^{*} In the figure the point O has been made the center of the triangle for the sake of simplicity in construction but the proof does not depend on this fact and will hold whatever be the position of the line O A. As a matter of fact the point O is taken at the center of the triangle in the subsequent part of this paper.

to find the value of the rectangular coordinates (x, y) of the point a in terms of $\varepsilon_1,~\varepsilon_2,$ and ε_3

Draw NO₂ and MO₃ parallel to OO₁ Then X = A C = A₁O₁ = ϵ_1 Y = C O = (B O - B C) = (D O + D C) = $\frac{1}{2}$ (B O - B C + D O + D C) = $\frac{1}{2}$ (B O + D O) for B C=D C because triangle A B D is an equilateral triangle (having its three sides perpendicular to the sides of the equilateral triangle R L F) and A C is the perpendicular bisector (being parallel to R L and therefore perpendicular bisector (being parallel to R L and therefore perpendicular bisector (being parallel to B = $\frac{1}{2}$ (N O₂ + M O₃) for B O=N O₂ and D O=M O₃ because parallel = $\frac{1}{2} \left(\epsilon_2 \frac{2}{\sqrt{3}} + \epsilon_3 \frac{2}{\sqrt{3}} \right)$ for angle N O₂A₂=30°=angle M O₃A₃ and the secant Y = $\frac{\epsilon_2 + \epsilon_3}{\sqrt{3}}$

accurately drawn, with its center at the center of a graduated circle, they have evaluated E and the angle a geometrically instead of algebraically. In practice, this method is considerably easier than the original method.

It will be evident to the student of mathematics that Einthoven and his followers have used what is known as the polar system of coordinates, designating as E and a what the mathematician ordinarily calls ρ and θ . In July, 1916, after perusing H. B. Williams' paper,⁶ I was led to investigate the properties of Einthoven's triangle and its application to electrocardiography. In this investigation a system of rectangular coordinates was employed, and the results have been so interesting and suggestive that it seems advisable to publish this method of analysis as a preliminary to the publication of the results that we have obtained by its use.

The basic principle of the method can be demonstrated by means of Figure 1. Instead of finding values E and a which correspond to ρ and θ in the polar system of coordinates, we have found two values which correspond to what are known as X and Y in the rectangular system of coördinates. These two values locate the point A, and, since the point O is fixed at the center of the triangle,⁷ give us the same information that was given by E and the angle a.

But, although our values for X and Y give us the same information that was given by E and a, they give it in a much more useful form. In the first place, it is easier to visualize points located by rectangular than by polar coördinates. It is simpler to think of a point with the value X = 4, Y = 3 than it is to think of the same point with the value $\rho = 5$, $\theta = 37^\circ$, or E = 5, $a = 37^\circ$. Again, it is simpler to chart a point

^{6.} Williams, H. B.: On the Cause of the Phase Difference Frequently Observed Between Homonymous Peaks of the Electrocardiogram, Am. J. Physiol. **35**:292 (Oct.) 1914

^{7.} See footnote to Figure 1.

located by rectangular coördinates than the same point located by polar coördinates. Ordinary cross section paper can be used, and there is little possibility of error. Furthermore, the mathematics involved in locating a point are simpler with rectangular coördinates, viz:

rectangular coördinates⁸ polar coördinates³ $X = \epsilon_{1} \qquad a = \tan^{-1} \frac{2 \epsilon_{2} - \epsilon_{1}}{\epsilon_{1} \sqrt{3}}$ $Y = \frac{\epsilon_{2} + \epsilon_{3}}{\sqrt{3}} \qquad E = \frac{\epsilon_{1}}{\cos a}$

But the greatest advantage to be obtained by the use of rectangular coördinates is due to the fact that it is comparatively easy with this system to plot consecutively the successive values of X and Y through a complete cardiac cycle and to connect these consecutive points with a fairly smooth curve which we shall call the "monocardiogram," for reasons which will appear later.

In order to understand the significance of this "monocardiogram," we shall revert for a moment to Einthoven's original discussion of the equilateral triangle. Einthoven,⁹ speaking of the Q R S wave of the electrocardiogram, says: "The curve must represent, under all circumstances and in every moment the algebraic sum of all the potential differences which at that moment are developed in the heart." We shall try to express the same thought by saying that the point X, Y (A in Figure 1), which we locate by our system of coördinates, represents the "center of negativity" of the heart at that instant; meaning by "center of negativity" a point somewhat analogous to "center of gravity" and "center of mass." If at any moment there are present in the heart several (negative) electrical charges which have value (intensity, voltage) and position (direction) then the center of negativity is that point which represents the algebraic sum of all the potential differences. The line, E, in Einthoven's drawings, which connects this point with the center of the equilateral triangle will, by its projections on the three sides of the triangle, give the values of the galvanometer deflections (ordinates) for the three leads at that particular instant. By finding the location of the center of negativity at consecutive instants, and connecting the points thus found by a continuous line, we obtain a curve which represents the successive algebraic sums of the potential differences that develop in the heart during the cardiac cycle. This curve we have called the monocardiogram to distinguish it from the ordinary electrocardiogram, which is really a "tricardiogram" or a threefold derivative of the monocardiogram.

^{8.} See Figure 1.

^{9.} Lancet 1:856, 1912.

The monocardiogram is really a fusion of the three leads of the electrocardiogram into a single curve by an algebraic reversal of the process by which three leads are obtained from one heart. Its study brings us much nearer the real electrical events of the cardiac cycle than does the study of the ordinary E. K. G., or tricardiogram which is derived from it by our present method of leading off. Figure 2 illustrates this fact. It shows the monocardiogram of the Q R S deflection taken from an electrocardiogram published by Einthoven (Fig. 8)¹ and used by Einthoven as a demonstration in his original description of



Fig 2.—This figure shows the monocardiogram which is derived from an electrocardiogram published by Einthoven (Fig. 8).¹ It can be seen that the three leads of the electrocardiogram are really derivatives of the monocardiogram, obtained by successive projections of the monocardiogram on the three sides of the equilateral triangle.

Note that in this monocardiogram, as in all the others shown in this article, the right side is on the observer's left. This is in accordance with ordinary cardiographic usage as regards Einthoven's triangle and facilitates interpretation. the equilateral triangle. It can be seen that leads I, II and III are really derivatives obtained by successive projections of a vector connecting the center of our triangle with the successive positions of the center of negativity. Thus the monocardiogram represents, as nearly as can be represented in a plane figure, the actual electrical events of a cardiac cycle.

The direction and shape of the monocardiogram have an anatomic significance which is indicated only obscurely by the direction and shape of the ordinary electrocardiogram. The shape of the ordinary E. K. G. may be considered the result of the projection of the monocardiogram (M. C. G.) on the sides of an equilateral triangle, and thu. the anatomic significance becomes distorted and obscured, but the M. C. G. itself owes its shape to the actual electrical events of the cardiac cycle and shows the successive relative positions of the center of negativity. Thus, it affords us a method of localizing, in a plane, various parts of the cardiac musculature; of analyzing an E. K. G. with regard to its anatomic significance; of determining what part of the cardiac musculature is responsible for various types of bizarre and abnormal E. K. G.'s, and of locating the site of origin of extrasystoles. By means of three leads taken in a horizontal plane and similarly analyzed we can obtain a "transverse monocardiogram" and get a three dimensional view of events during the cardiac cycle. It is our intention in further communications to discuss the points just mentioned and more especially to use the monocardiogram for the analysis of E. K. G.'s in which it is assumed that there are defects in various localized regions of the cardiac musculature: i. e., in cases of subendocardial myocarditis as described by Oppenheimer and Rothschild.¹⁰

For the application of our method to the analysis of an electrocardiogram it is not absolutely necessary to have simultaneous E. K. G.'s of the several leads. Provided the string has been carefully standardized it is quite practicable to proceed as follows:

1. By means of a camera lucida drawing, photographic enlargement, or by examination of the film with a microscope a series of careful measurements of a complete heart cycle is made for each lead. If there is evidence of respiratory or sinus arrhythmia care must be taken to select cycles which are in the same phase. The values of the ordinates are measured in millimeters for every hundredth of a second or less.

2. By the method of trial and error the three series of measurements are so arranged that for every moment during the cardiac cycle

^{10.} Oppenheimer, B. S., and Rothschild, M. A.: Electrocardiographic Changes Associated with Myocardial Involvement, J. A. M. A. 69:429 (Aug. 11) 1917.

lead II minus lead I equals lead III. In most curves, especially those with high peaks, this is a fairly easy matter.

3. The value for X for each moment (0.01 second) is known, for it is the value of lead I (Fig. 1), but the value of Y must be calculated as follows: divide the algebraic sum of leads II and III by the square root of 3. This is done by means of Table 1, and gives us the successive value of Y.

4. The successive positions of the point X, Y are plotted and connected with a fairly smooth curve, thus giving the monocardiogram.

Figure 3 illustrates the method of procedure. The camera lucida enlargement of the E. K. G. is shown; the values of the three leads for every hundredth of a second are shown properly arranged; the values of Y are shown calculated, and the monocardiogram resulting is shown.

Table 1.—This table is used for the calculation of the value $Y\left(\frac{\epsilon_2 + \epsilon_3}{\sqrt{3}}\right)$ It is used like a logarithm table. For example: at the instant labelled 20 in the table in Figure 3, the ordinate in lead II measured 9.7 mm., and the ordinate in lead III measured —.8 mm. The algebraic sum of 9.7 and —.8 is 8.9. If we look this number up in our table we use the horizontal row labelled 8 and the vertical row labelled 9. The number which we find at the intersection of these two rows is 5.138 and thus our value for Y is approximately 5.1. The values of Y at other instants were found in the same manner by adding the values of leads II and III and using the table to simplify the process of dividing by $\sqrt{3}$.

$\frac{\epsilon_2 + \epsilon_3}{\sqrt{3}}$	0	1	2	3	4	5	6	7	8	9	Differ- ences
0 1 2 3 4 5 6 7 8 9	$\begin{array}{c} 0.57735\\ 1.1547\\ 1.7321\\ 2.3094\\ 2.8868\\ 3.4641\\ 4.0415\\ 4.6188\\ 5.1962\\ \end{array}$	$\begin{array}{r} 0.0577\\ 0.6351\\ 1.212\\ 1.790\\ 2.367\\ 2.944\\ 3.522\\ 4.099\\ 4.677\\ 5.254\end{array}$	$\begin{array}{r} 0.1155\\ \textbf{0.6928}\\ 1.270\\ 1.848\\ 2.425\\ 3.002\\ 3.580\\ 4.157\\ 4.734\\ 5.312\end{array}$	0.1732 0.7506 1.328 1.905 2.483 3.060 3.637 4.215 4.792 5.369	$\begin{array}{r} 0.2309\\ 0.8083\\ 1.386\\ 1.963\\ 2.540\\ 3.118\\ 3.695\\ 4.272\\ 4.850\\ 5.427\end{array}$	0.2887 0.8660 1.443 2.021 2.598 3.175 3.753 4.330 4.907 5.485	0.3464 0.9238 1.501 2.656 3.233 3.811 4.388 4.965 5.543	$\begin{array}{c} 0.4041\\ 0.9815\\ 1.559\\ 2.136\\ 2.714\\ 3.291\\ 3.868\\ 4.446\\ 5.023\\ 5.600\\ \end{array}$	$\begin{array}{c} 0.4619\\ 1.039\\ 1.617\\ 2.194\\ 2.771\\ \textbf{3.349}\\ \textbf{3.926}\\ 4.503\\ 5.081\\ 5.658\end{array}$	$\begin{array}{c} 0.5196\\ 1.097\\ 1.674\\ 2.252\\ 2.829\\ \textbf{3.406}\\ 3.984\\ \textbf{4.561}\\ \textbf{5.138}\\ \textbf{5.716} \end{array}$	1. 0.00577 2. 0.01155 3. 0.01732 4. 0.02309 5. 0.02887 6. 0.03464 7. 0.04041 8. 0.04619 9. 0.05196

 $\sqrt{3} = 1.7320 +$

The use of simultaneous electrocardiograms is theoretically desirable, but we are not confined to such simultaneous records in the employment of this method. Successive heart cycles in an individual are so nearly alike that, provided proper precautions are exercised in the choice of cycles, the rule, lead II minus lead I equals lead III, holds for practical purposes. In practice we have found it possible to apply this method of analysis to the ordinary E. K. G. as usually taken.

Figure 4 shows a monocardiogram derived from one of the rare electrocardiograms in which all three leads were taken simultaneously. The original E. K. G. was published by Einthoven, Bergansius and

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Fig. 3.—This figure shows the method of obtaining a monocardiogram. Above, to the left, is shown a camera lucida drawing of the three leads of an electrocardiogram. The ordinates represent 0.04 second and the abscissae represent 0.1 millivolt. To the right is shown a table of values of the ordinates of the three leads for every 0.01 second properly arranged so that lead II minus lead I equals lead III for every instant. The fourth column of figures in the table gives the values of Y calculated by means of Table 1. Below is shown the monocardiogram plotted by connecting successive values of the point, X, Y, with a smooth curve. The P, QRS, and T deflections are plotted separately. Positive values of X are plotted to the observer's left, for this is taken as the right side. Positive values of Y are plotted above the line, as is usual.



Fig. 4.—This shows the monocardiogram derived from an electrocardiogram published by Einthoven, Bergansius, and Bijtel¹¹ (Fig. 3, Cycle 2). From this E.K.G. (which is one of the very rare published electrocardiograms taken simultaneously in all three leads by means of three galvanometers) we have derived the monocardiogram, shown above. Below the monocardiogram are shown eight pairs of derived electrocardiograms. These were *derived* from the monocardiogram by a method similar to that illustrated in Figure 2 and they show the effect that rotation of the heart's axis would have on the electrocardiogram. We have followed customary trigometrical notation and called clockwise rotation negative and counterclockwise rotation positive. Thus by $+30^{\circ}$ we mean that the heart is so rotated that the apex approaches the patient's left shoulder.



Fig. 5.—The uppermost monocardiogram is that of a normal heart. The second monocardiogram was derived from an E.K.G. which showed a characteristic *left* ventricular preponderance (main deflection upright in lead I and inverted in lead III).

The third monocardiogram was derived from an E.K.G. which showed a characteristic *right* ventricular preponderance.

The lowermost monocardiogram was derived from the electrocardiogram of a ventricular extrasystole. It points definitely to the right ventricle (observer's left). Bijtel.¹¹ Our figure shows a series of derived E. K. G.'s which demonstrate the effect that rotation of the heart's axis would have on the ordinary electrocardiogram. The series of E. K. G.'s was obtained by rotating the monocardiogram and plotting the various resulting E. K. G.'s by a method similar to that illustrated in Figure 2. In Figure 2 the M. C. G. is derived from the E. K. G. By reversing our procedure, we can plot the E. K. G. from the M. C. G. By taking the M. C. G. and rotating it 15 degrees we can plot out the E. K. G. that would result if the heart itself were rotated 15 degrees. In this way we can obtain a series of E. K. G.'s that will show the effect of rotation of the heart's axis on the normal E. K. G. During normal respiration changes occur in the E. K. G. which are probably due to a combination of rotation of the heart's axis and nervous effects.³

Monocardiograms of a normal heart, of left ventricular preponderance, of right ventricular preponderance, and of a ventricular extrasystole are shown in Figure 5. It is interesting to note that in ventricular preponderance, the preponderant or thickened ventricle shows a tardiness in electrical response which causes a shifting of the center of negativity to the side opposite the preponderant ventricle and thus gives E. K. G.'s which are characteristic of preponderance. It can be seen that the different electrocardiograms which are characteristic of various types of cardiac abnormality give monocardiograms of every different form. The anatomical significance of the monocardiogram gives us reason to hope that this method of analysis will lead to closer correlation between cardiographic and pathologic findings.

SUMMARY

1. A new method is presented by which the ordinary three leads of the electrocardiogram are combined in a single curve, the monocardiogram.

2. Usefulness of this method of analyzing the electrocardiogram is explained.

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^{11.} Einthoven, W., Bergansius, F. L., and Bijtel, J.: Die gleichzeitige Registrierung elektrischer Erscheinungen mittels zwei oder mehr Galvanometer und ihre Anwendung auf die Elektrokardiographie, Arch. f. d. ges Physiol. **164**: 167, 1916.