Rate Splitting for MIMO Multibeam Satellite Systems

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Abstract—**This paper deals with the problem of precoding in multibeam satellite system with rate splitting (RS). In contrast to the single-stream (SS) case where a unique private frame is transmitted towards each beam, in RS we consider the simultaneous transmission of a public frame to all intended users superimposed with each private frames. In this context, every user terminal (UT) firstly decodes the public frame which contains data from all UTs at all beams and; posteriorly, its intended private frame which is only decodable by a set of users. With this, each UT receives information from both the public and the private frame, leading to a system sum-rate increase in some cases. This performance increase is evaluated by computing an upper bound of the attainable rates. Moreover, a low-complexity precoding alternative is proposed considering a decoupled design of the precoding of the private frames and public frames. This technique is evaluated considering a real multibeam satellite system. A substantial gain with respect to the current benchmark technique is identified.**

I. INTRODUCTION

The offered rate of current multibeam satellite systems is limited by its employed spectrum. This is because in order to reduce the interference of adjacent beams, neighboring beams use disjoint frequency bands leading to a reduction of the user available spectrum.

Promoting an aggressive frequency reuse among beams entails the implementation of interference mitigation techniques for obtaining sufficiently large signal-to-interference-plus-noise ratio (SINR) in the presence of strong inter-beam interference. These interference mitigation techniques are differentiated depending on where they are implemented: either at the user terminal (UT) [1], [2] or at the on-ground Earth station gateway (GW) [3] (precoding).

As in terrestrial systems, the use of precoding relies on how accurate the channel state information (CSI) is fed back from the UT to the GW. Despite in fixed satellite services the UTs channel vectors present a very slow time variation, certain degradation is expected. An additional crucial challenge is the synchronization of the transmitted feed signals as the precoded signals can be generated by different GW and travel through different waveguides in the satellite payload.

While receive-based interference mitigation techniques do not present such a strict transmit problems compared to precoding, its rejection capability is strongly related to the UT cost. Bearing this in mind, attending to the UT final prize, future systems will only allow the detection of one or two inter-beam interference signals. This supposes a remarkable disadvantage as the resulting SINRs will be substantially lower with respect to the case when precoding is used.

Recent works have proposed the combination of precoding and multiuser detection techniques [4], [5]. In both works, it is considered an overloaded scenario where at each beam two simultaneous frames are transmitted. In this context, it is shown that the simultaneous non-unique decoding receiver strategy [6] offers the largest attainable rates.

In contrast to the mentioned works, this paper explores the use of rate-splitting (RS) strategy [7]. In RS, along with each beam transmitted frame, a public frame containing information for all users is transmitted through all beams. This latter frame is coined as public message whereas each beam frame is coined as private messages. While private frames are only decodable by a set of users that are located in the same beam, the public frame is decodable by all users. In this context, every UT performs a successive interference cancellation (SIC) of the public frame and, posteriorly, it detects the frame devoted to the beam it is located at.

Guided by the sum-rate achievable rates in the max-min multigroup multicast optimization described in [8], we propose the use of RS in the sum-rate optimization of the multigroup multicast case. Note that multibeam satellite systems is a multigroup multicast transmission as each frame contains information from all intended UTs located in the same beam [3].

We first obtain an upper bound of the attainable rates. This is done considering the semidefinite program relaxation (SDR) of the original non-convex sum-rate optimization problem. With this, we point out that by using RS, certain gain can be obtained compared to the single stream (SS) case. Bearing this result in mind, we conceive a low complexity precoding design that is able to balance both the public and private messages transmit power. The design differs to the precoding method presented in [9] which does not consider the multigroup multicast transmission and the per-antenna power constraints as we propose in here.

The numerical results of the proposed precoding scheme show a certain gain compared to the benchmark SS design. The results are obtained considering a close-to-real scenario with a real multibeam satellite deployment.

The rest of the paper is organized as follows. Section II presents the system model. Section III describes the optimization problems to be solved for obtaining the achievable rates of RS in multibeam satellite systems. Section IV proposes a low-complexity scheme for precoding systems with RS. Section V contains the numerical results and Section VI concludes.

Notation: Throughout this paper, the following notations

are adopted. Boldface upper-case letters denote matrices and boldface lower-case letters refer to column vectors. $(.)^H$, $(.)^T$, $(.)^*$ and $(.)^+$ denote a Hermitian transpose, transpose, conjugate and diagonal (with positive diagonal elements) matrix, respectively. $\mathbf{I}_N$ builds $N \times N$ identity matrix and $\mathbf{0}_{K \times N}$ refers to an all-zero matrix of size $K \times N$. If $\mathbf{A}$ is a $N \times N$ matrix. $[\mathbf{X}]_{ij}$ represents the (i-th, j-th) element of matrix $\mathbf{X}$. $\otimes$, $\circ$ and $||.||$ refer to the Kronecker product, the Hadamard product and the Frobenius norm, respectively. Vector $\mathbf{1}_N$ is a column vector with dimension N whose entries are equal to 1. $\text{vec}(\cdot)$ denotes the vectorization operator.

II. SYSTEM MODEL AND PROBLEM STATEMENT

Let us consider a multibeam satellite system where the satellite is equipped with an array fed reflector antenna with a total number of feeds equal to N. These feed signals are combined to generate a beam radiation pattern forming a total number of K beams, which is considered fixed. For each frame, we assume that a total number of N_u users are simultaneously served belonging to the same user beam (i.e. the total number of served users by the satellite is KN_u). These users have been scheduled according to certain criteria based on the spatial signature and rate requirements.

A multibeam satellite system can be cast as a multi-group multicast multiple-input-single-output (MISO) transmission [3]. In this context, the satellite acts as a transmitter with N antennas, sharing information towards K groups where each group is composed by N_u users.

For the sake of the analytical tractability, it is common practice to gather users in disjoint sets. In the multibeam satellite context, we propose to create K groups of N_u users. The users of each group are always located in different beams. Following this approach, the objective is to mitigate inter-beam interference to the highest possible extent in all the K groups. It is worth mentioning that there are a lot of combinations to group the users. Ideally, the user selection and the interference mitigation strategy should be jointly designed . However, the complexity of the solution may not be affordable. To reduce the complexity, we will consider a two-step design, where the user selection is computed beforehand according to a given criteria. Then, the interference mitigation technique will be built upon this selection. User selection algorithms are not within the main scope of the paper and will be left for future work.

Considering that all beams radiate in the same frequency band, the received signal at the i-th user terminal of each beam in and arbitrary time instant can be modeled as

$$\mathbf{y}^{[i]} = \mathbf{H}^{[i]}\mathbf{x} + \mathbf{n}^{[i]}, \quad i = 1, \ldots, N_u, \tag{1}$$

being $\mathbf{y}^{[i]} \in \mathbb{C}^{K \times 1}$ the vector containing the received signals at the i-th UT (i.e. the value $[\mathbf{y}^{[i]}]_k$ refers to the receive signal of the i-th UT from the k-th beam). Vector $\mathbf{n}^{[i]} \in \mathbb{C}^{K \times 1}$ contains the noise terms of each i-th UT. The entries of this vector are assumed to be Gaussian distributed with zero mean, unit variance and uncorrelated with both the desired signal and the rest of noise entries (i.e. $E\left[\mathbf{n}^{[i]}\mathbf{n}^{[i]H}\right] = \mathbf{I}_K \quad i = 1, \ldots, N_u$).

The (k, n)-th entry of matrix $\mathbf{H}^{[i]} \in \mathbb{C}^{K \times N}$ can be described as follows

$$\left[\mathbf{H}^{[i]}\right]_{k,n} = \frac{G_R a_{kn}^{[i]} e^{j\psi_{k,n}^{[i]}}}{4\pi \frac{d_k^{[i]}}{\lambda} \sqrt{K_B T_R B_W}} \tag{2}$$

for $k = 1, \ldots, K$, $n = 1, \ldots, N$, $i = 1, \ldots, N_u$. $d_k^{[i]}$ is the distance between the i-th UT at the k-th beam and the satellite. λ is the carrier wavelength, K_B is the Boltzmann constant, B_W is the carrier bandwidth, G_R^2 the UT receive antenna gain, and T_R the receiver noise temperature. The term $a_{kn}^{[i]}$ refers to the gain from the n-th feed to the i-th user at the k-th beam. The time varying phase due to beam radiation pattern and the radiowave propagation is represented by $\psi_{k,n}^{[i]}$. For the sake of notation clarity, we denote

$$\mathbf{H}^{[i]} = \left(\mathbf{h}_1^{[i],T}, \ldots, \mathbf{h}_K^{[i],T}\right)^T. \tag{3}$$

In here we consider a rate splitting (RS) approach which separates the transmit information into a public symbol s_0 and K private symbols $\{s_k\}_{k=1}^K$. The public symbol, s_0, shall be decoded by all UTs (even though it may not be intended to all UTs) while the k-th private symbol is only required to be decoded by the UTs of the k-th beam. Figure 1 shows an example scenario with $K = 2$ and $N_u = 2$.
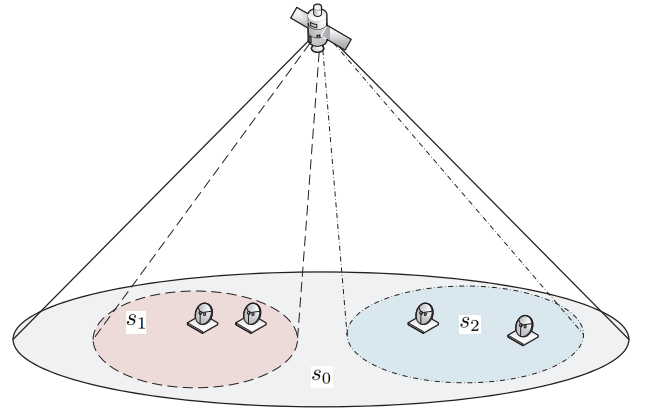

Fig. 1. A 2-beam satellite system scheme with RS.

The transmitted signal can be written as follows

$$\mathbf{x} = \mathbf{w}_0 s_0 + \sum_{k=1}^K \mathbf{w}_k s_k. \tag{4}$$

It is assumed that all symbols are unit energy. Then, the attainable rates are ruled by the common message rate

$$R_0 = \min_{k=1,\ldots,K} \min_{i=1,\ldots,N_u} R_{0,k}^{[i]} \tag{5}$$

where

$$R_{0,k}^{[i]} = \log_2\left(1 + \frac{|\mathbf{h}_k^{[i],H}\mathbf{w}_0|^2}{\sum_{k=1}^K |\mathbf{h}_k^{[i],H}\mathbf{w}_k|^2 + 1}\right) \tag{6}$$

and each private message data rate

$$R_p^k = \min_{i=1,\ldots,N_u} R_k^{[i]}, \tag{7}$$

for $k > 0$ and where

$$R_k^{[i]} = \log_2\left(1 + \frac{|\mathbf{h}_k^{[i],H}\mathbf{w}_k|^2}{\sum_{j\neq k,j>0}^K |\mathbf{h}_k^{[i],H}\mathbf{w}_j|^2 + 1}\right). \quad (8)$$

In this context, under Gaussian signaling, the sum-rate becomes

$$\mathcal{SR} = R_0 + \sum_{k=1}^K R_p^k. \quad (9)$$

It is important to remark that RS entails the transmission of s_0, which shall be decoded by all users. The optimization problem to be solved is the following

$$\begin{aligned}
&\underset{\{\mathbf{w}_k\}_{k=0}^K,}{\text{maximize}} \quad \mathcal{SR} \\
&\text{subject to} \\
&\left[\sum_{k=0}^K \mathbf{w}_k\mathbf{w}_k^H\right]_{nn} \leq P \quad n = 1,\ldots,N,
\end{aligned} \quad (10)$$

where P is the maximum available power per feed.

The optimization problem in (10) is a non-convex large scale problem. In order to observe the benefits of RS, we opt to compute the SDR of optimization problem in (10). The SDR provides an upper bound on the attainable rates of the system. Recent results in non-convex quadratically constraint quadratic programs (QCQP) techniques [10], [11] show that it is possible to obtain solutions with a performance very close to this upper bound. Yet a promising alternative would be the use of the weighted minimum mean square error technique (WMMSE) [12].

III. Upper Bound Computation

Prior to conceiving low complexity precoding techniques for the considered scenario, in this Section we show the prospective gains of employing RS. This is done by computing an upper bound of the optimization problem in (10).

We can formulate the SDP relaxation of the optimization problem in (10) so that

$$\underset{\{\mathbf{W}_k\}_{k=0}^K,\{t_k\}_{k=0}^K}{\text{maximize}} \quad \sum_{k=0}^K t_k$$

subject to

$$\left(2^{t_0} - 1\right)\left[\sum_{j=1}^K \text{Tr}\left\{\mathbf{G}_k^{[i]}\mathbf{W}_j\right\} + 1\right] - \text{Tr}\left\{\mathbf{G}_k^{[i]}\mathbf{W}_0\right\} \leq 0$$

$$\left(2^{t_k} - 1\right)\left[\sum_{j\neq k,j>0}^K \text{Tr}\left\{\mathbf{G}_k^{[i]}\mathbf{W}_j\right\} + 1\right] - \text{Tr}\left\{\mathbf{G}_k^{[i]}\mathbf{W}_k\right\} \leq 0$$

$$k = 1,\ldots,K, i = 1,\ldots,N_u$$

$$\left[\sum_{k=0}^K \mathbf{W}_k\right]_{nn} \leq P \quad n = 1,\ldots,N \quad (11)$$

where $\mathbf{G}_k^{[i]} = \mathbf{h}_k^{[i]}\mathbf{h}_k^{[i],H}$. The optimization problem in (11) is a biconvex problem. This is, fixed $\{\mathbf{W}_k\}_{k=0}^K$, the optimization problem becomes a linear optimization problem and when fixing $\{t_k\}_{k=0}^K$, the optimization problem becomes a SDP.

Biconvex problems can be solved via the alternating optimization method which is guaranteed to converge to an stationary point of the original problem [13]. Bearing in mind that the final solution depends on the initial point, we consider multiple initial random points and elect the solution with the largest sum-rate.

Note that when fixing $\{t_k\}_{k=0}^K$, the optimization problem in (11) becomes a feasibility problem as follows

$$\text{find} \quad \{\mathbf{W}_k\}_{k=0}^K$$

subject to

$$\left(2^{t_0} - 1\right)\left[\sum_{j=1}^K \text{Tr}\left\{\mathbf{G}_k^{[i]}\mathbf{W}_j\right\} + 1\right] - \text{Tr}\left\{\mathbf{G}_k^{[i]}\mathbf{W}_0\right\} \leq 0$$

$$\left(2^{t_k} - 1\right)\left[\sum_{j\neq k,j>0}^K \text{Tr}\left\{\mathbf{G}_k^{[i]}\mathbf{W}_j\right\} + 1\right] - \text{Tr}\left\{\mathbf{G}_k^{[i]}\mathbf{W}_k\right\} \leq 0$$

$$k = 1,\ldots,K, i = 1,\ldots,N_u$$

$$\left[\sum_{k=0}^K \mathbf{W}_k\right]_{nn} \leq P \quad n = 1,\ldots,N. \quad (12)$$

The overall alternating optimization is described in Algorithm 1. It is important to remark that ϵ controls the stopping criteria of the optimization.

Data: $\mathbf{G}_k^{[i]}$ for $k = 1,\ldots,K$, $i = 1,\ldots,N_u$
Result: $\{\mathbf{W}_k^*\}_{k=0}^K, \{t_k\}_{k=0}^K$
Initialize $\{t_k^{(0)}\}_{k=0}^K$;
while $\left|\sum_{k=0}^K t_k^{(n)} - \sum_{k=0}^K t_k^{(n-1)}\right| \geq \epsilon$ **do**

    Compute $\{\mathbf{W}_k^{(n)}\}_{k=0}^K$;
    Set up $t_0^{(n)} =$
    $\min_{k=1,\ldots,K;i=1,\ldots,N_u} \log_2\left(1 + \frac{\text{Tr}\left\{\mathbf{G}_k^{[i]}\mathbf{W}_0^{(n)}\right\}}{\sum_{j=1}^K \text{Tr}\left\{\mathbf{G}_k^{[i]}\mathbf{W}_j^{(n)}\right\}+1}\right)$;

    Set up $t_k^{(n)} =$
    $\min_{i=1,\ldots,N_u} \log_2\left(1 + \frac{\text{Tr}\left\{\mathbf{G}_k^{[i]}\mathbf{W}_k^{(n)}\right\}}{\sum_{j\neq k,j>0}^K \text{Tr}\left\{\mathbf{G}_k^{[i]}\mathbf{W}_j^{(n)}\right\}+1}\right)$;

    $n \leftarrow n + 1$;
end
Output the final solution;
Algorithm 1: Alternating optimization for obtaining an efficient solution of (11).

For the case where RS is not employed (i.e. $\mathbf{W}_0 = \mathbf{0}$), a similar alternating optimization method can be used as reported in [14].

Despite its efficiency, the mentioned optimization SDR approach presents a large computational complexity which limits its applicability in real systems. In this paper we propose a low-complexity alternative based on closed-form linear precoding techniques. We tackle the problem by first considering the precoding vectors associated to the private messages, $\{\mathbf{w}_k\}_{k=1}^K$, and; posteriorly, the precoding vector devoted to the public message transmission, i.e. $\mathbf{w}_0$.

IV. Precoding in RS Multibeam Satellite Systems

Based on previous studies, a variation of the minimum mean square error (MMSE) precoding under a simple scaling factor power allocation [3], [15] offers the best complexity-performance trade-off to design the precoders of the private messages. This design, $\mathbf{W} = (\mathbf{w}_1, \ldots, \mathbf{w}_K)$, can be written as

$$\mathbf{W} = \gamma \left(\widehat{\mathbf{H}}^H \widehat{\mathbf{H}} + \frac{1}{NP_{\text{private}}} \mathbf{I}_N \right)^{-1} \widehat{\mathbf{H}}^H, \qquad (13)$$

where γ controls the transmit power. The channel matrices are mapped into a single metric, namely

$$\widehat{\mathbf{H}} = \frac{1}{N_u} \sum_{i=1}^{N_u} \mathbf{H}^{[i]}, \qquad (14)$$

and P_{private} is the maximum transmit power that the private transmission can have. In other words, γ is elected so that

$$\gamma^2 = \frac{P_{\text{private}}}{\max_{n=1,\ldots,N} \left[\mathbf{WW}^H \right]_{nn}}. \qquad (15)$$

Equation in (14) generates an equivalent channel matrix $\widehat{\mathbf{H}}$ based on the average channel matrix from all N_u users simultaneously served. Note that $P_{\text{private}} \leq P$ and its value shall be elected a priori. For the sake of simplicity, we consider

$$P_{\text{private}} = \alpha P, \qquad (16)$$

where $0 \leq \alpha \leq 1$ controls the amount of power devoted to the private messages transmission.

It is noteworthy that in general only one feed out of N will transmit with P_{private}. The rest will transmit with less power. In the following we will demonstrate how RS can take advantage of the unused power to multiplex one addition message and thus, increase the spectral efficiency. To the best of authors' knowledge this strategy has not been previously adopted in the literature.

Once $\mathbf{W}$ is computed, the optimization of $\mathbf{w}_0$ becomes

$$\underset{\mathbf{w}_0}{\text{maximize}} \underset{\substack{k=1,\ldots,K \\ i=1,\ldots,N_u}}{\min} \frac{|\mathbf{h}_k^{[i],H} \mathbf{w}_0|^2}{\tau_k^{[i]}}$$

subject to

$$\left[\mathbf{WW}^H + \mathbf{w}_0 \mathbf{w}_0^H \right]_{nn} \leq P, \quad n = 1, \ldots, N, \qquad (17)$$

where

$$\tau_k^{[i]} = \sum_{l=1}^{K} |\mathbf{h}_k^{[i],H} \mathbf{w}_l|^2 + 1. \qquad (18)$$

The optimization problem in (17) is non-convex but its concave-convex relaxation shows a performance very close the optimal design [16]. It is important to remark that despite $\alpha = 1$, there might be the case where $\mathbf{w}_0$ takes a non-zero value as not all the feed will work on full power transmission.

V. Numerical Results

Prior to evaluate the potential of RS in multibeam satellite systems, we consider an example of Rayleigh distributed channel. This is depicted in Figure 2 where we have consider an scenario with $K = 3$, $N = 4$ and $N_u = 3$. The upper bound on the attainable rates has been obtained considering the alternating optimization method described previously. It can be observed that RS offers a substantial potential gain with respect to the benchmark case which for this case we consider the SDR upper bound of the sum-rate multigroup multicast optimization problem.
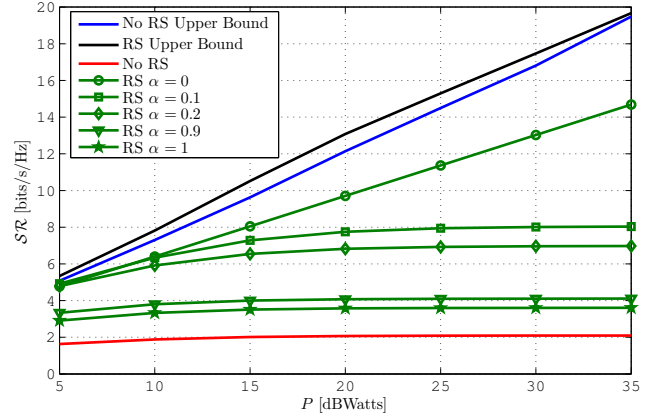

Fig. 2. Achievable rates example with Rayleigh distributed channel scenario. We consider $K = 3$, $N = 4$ and $N_u = 3$ for different P.

On the other hand, it is evident that the low complexity precoding scheme yields a poor performance. Alternatively, when combining this low complexity scheme with the optimized $\mathbf{w}_0$, certain gain can be obtained even with $\alpha = 1$. For this case, $\alpha = 0$ (i.e. pure public message transmission) attains the maximum sum-rate. The reason for this is the tentative strong channel co-linearity of users belonging to different groups which negatively impacts the data rates of the private message transmission. The results have been obtained in a Monte Carlo simulation with 500 runs.

We evaluate the considered technique in a multibeam satellite system with $K = 8$. For evaluating the aforementioned technique, a real coverage area provided by a geostationary satellite is considered. This data has been obtained in a study performed by the European space agency (ESA). We assume that at each time instant all bandwidth is shared by all beams. The simulation parameters are summarized in Table I. The considered figure of merit is the throughput defined as

$$\mathcal{TH} = B \times \mathcal{SR}. \qquad (19)$$

As a benchmark, we consider the design in (14) with $P_{\text{private}} = P$.

In order to observe the variation of $\mathcal{TH}$ over α, we consider a fixed maximum per feed available power of $P = 55$ Watts. This can be observed in Figure 3. The maximum throughput is attained via $\alpha = 0.5, 0.09, 0.05, 0.04$ for $N_u = 2, 3, 4, 5$ respectively.

In light of this simulation it is clear that in the considered multibeam satellite system the public message plays a central

Parameter	Value
Satellite height	35786 km (geostationary)
Satellite longitude, latitude	$10°East, 0°$
Earth radius	6378.137 Km
Feed radiation pattern	Provided by ESA
Number of feeds N	8
Number of users N_u	2,3,4 and 5
Number of beams	8
User location distribution	Uniformly distributed
Carrier frequency	20 GHz (Ka band)
Total bandwidth B	500 MHz
Roll-off factor	0.25
User antenna gain	41.7 dBi
G/T in clear sky	17.68 dB/K

In both Figures it is clear that the proposed method yields to a larger throughput compared to the benchmark technique of pure private transmission, where all messages are private. Remarkably, the difference between the proposed low complexity scheme and the benchmark increases as the N_u increases. The main reason of this behavior is that as long as we increase the total number of users $N_u K$, the broadcasting part of the transmission is more relevant and the pure private transmission becomes inefficient.

Fig. 5.   Throughput analysis for a multibeam satellite systems with $N_u = 4$ and 5.

Fig. 3.   Throughput analysis for a multibeam satellite systems for different N_u and varying α. The available power is set to $P = 55$ Watts.

VI.   CONCLUSIONS

The RS sum-rate optimization in multibeam satellite systems was investigated in this paper. First, an optimization technique based on SDR which required an alternating optimization was presented. With the aim of obtaining a low complexity technique, we resort to an ad-hoc precoding design based on a decoupled optimization of beamforming designs for public and private messages. The numerical results show that the potential of RS in multibeam satellite systems when comparing it with the current satellite multibeam precoding benchmark. In this way, the proposed solution allows that the flagship broadcast satellite services coexist with the unicast ones, which are enabled by the multibeam architecture.

role in the sum-rate as the optimal α is very low. Note that we obtain larger optimal α values compared to the scenario with Rayleigh distributed channel realizations. This is due to the characteristics of the satellite channel: UTs belonging to the same beam present certain channel correlations that can get benefited from the private frame transmission.

Figures 4 and 5 show the throughput for $N_u = 2, 3$ and $N_u = 4, 5$,respectively, given considering the optimal α value for each case. The upper bound rates are also depicted.
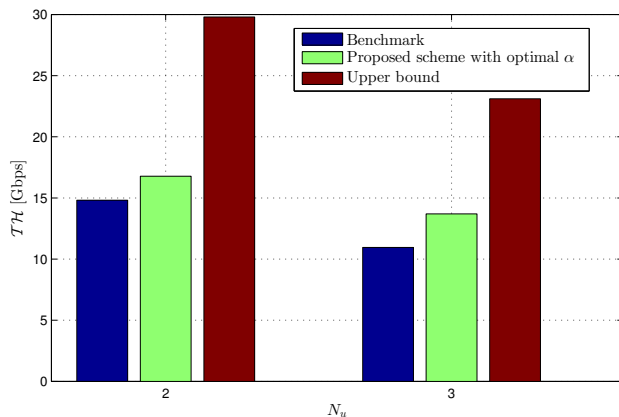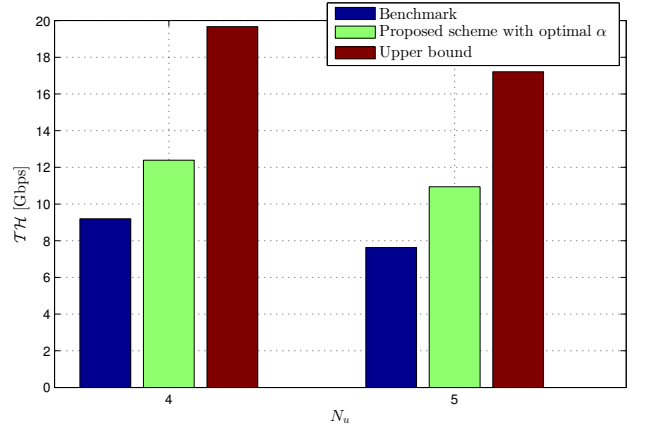
VII.   ACKNOWLEDGMENT

Fig. 4.   Throughput analysis for a multibeam satellite systems with $N_u = 2$ and 3.

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