



THE ARCHITECTONIC ENCODING OF THE MINOR LUNAR STANDSTILLS IN THE HORIZON OF THE GIZA PYRAMIDS

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ABSTRACT

The paper is an attempt to show the architectonic method of the ancient Egyptian designers for encoding the horizontal-projections of the moon's declinations during two events of the minor lunar standstills, in the design of the site-plan of the horizon of the Giza pyramids, using the methods of descriptive geometry. It shows that the distance of the eastern side of the second Giza pyramid from the north-south axis of the great pyramid encodes a projection of a lunar declination, when earth's obliquity-angle was $\sim 24.10^\circ$. Besides, it shows that the angle of inclination of the causeway of the second Giza pyramid, of $\sim 13.54^\circ$ south of the cardinal east, encodes the projection of another lunar declination when earth's obliquity-angle reaches $\sim 22.986^\circ$. In addition, it shows the encoded coordinate system in the site-plan of the horizon of the Giza pyramids.

KEYWORDS: Giza Pyramids, Causeway, Archaeoastronomy, Moon, Minor Lunar Standstills, Obliquity.

1. INTRODUCTION

The term standstill was first introduced by Alexander Thom in 1972 (Sim, 2003) and means solstice or slow motion (Judith S. Young, 2006). Similar to associating the term solstice to the sun, the term standstill refers to the slow motion of the moon that occurs every ~ 9.3 years at one of the two extremes of its observed range of declinations or azimuths. Tome *et al* (1975, 19-30) showed that some ancient European cultures recorded it in their megalithic horizons, and discussed the possibility that Stonehenge was a lunar observatory. That is in terms of encoding the azimuths of moonrise and moonset during the year of the lunar standstill. In Egypt, none has found yet any ancient lunar observatory. However, Al-Maqrizi, the medieval historian (1364-1442AD), mentioned narratives about the astronomical daily records in ancient Egypt, citing early Coptic historians. He wrote "the priest who spent seven years in studding any of the seven moving orbs: Sun, Moon, Mercury, Venus, Mars, Jupiter, and Saturn, was called *Baher*; and the one who studied all of them was called *Kater*, implying the master priest¹ (Al-Maqrizi, 1846). Recent researches support the narratives of Al-Maqrizi. Aboulfotouh (2007) showed that Dendera Zodiac² records the declination of the full moon that appears at $\sim 13.6^\circ$ west of the meridian and $\sim 30^\circ$ south of the zenith; it records the full moon as was observed from the latitude of 30° north, during the vernal equinox. In Napta in Upper Egypt, Malville *et al* (1998) had found a primitive observatory site, and concluded that it dates back to ca 4800 BC and encodes the sightline of summer solstice. Aboulfotouh (2002) showed that the design-locations of the three Giza pyramids were set based on recording the observed motion of the sun when earth's obliquity angle O_t was $\sim 24.10^\circ$, which denotes that the encoded date in the design of Giza Pyramids' horizon³ is ~ 3055 - 3065 BC. Magli (2013) showed that if one stands at the eastern end of the causeway of the second Giza pyramid at the time of the setting sun

in summer solstice, one will observe its alignment as it coincide with the mid point of the sunset angle from due west⁴. Besides, the tilts of the entrance passages of the Giza Pyramids were set based on using relativistic mathematical equations. The equations link the latitude of the pyramid λ with the obliquity of time $O_t = \sim 24.10^\circ$ and the two extreme values of earth's obliquity rang as a frame of reference: 24.30° and 21.672° , and/or its median value 22.986° , as was thought by the pyramids' designer in his days (Aboulfotouh, 2007).

This paper is an endeavor to show that the ancient Egyptian designer also used a method of descriptive geometry for encoding the horizontal projection of the lunar declinations in the site plan of Giza Pyramids plateau. It shows the encoded horizontal projection of moon's declination during the minor lunar standstill when O_t was $\sim 24.10^\circ$, which identifies the distance of the eastern side of the second Giza pyramid from the cardinal north-south axis of the great pyramid. Besides, it shows the encoding of another lunar declination in the alignment of the causeway of the second Giza pyramid, during other event of minor lunar standstill when earth's O_t reaches its mean of $\sim 22.986^\circ$, according to the opinion of the ancient designer/astronomer.

2. THE ASTRONOMICAL DESIGN OF THE HORIZON OF GIZA PYRAMIDS

Fig.1a shows a north-south cross-section in a spherical coordinate system, for the latitude λ of 30° north, and $O_t = 24.30^\circ$, and where the observer looks towards the west. Its center is imaginary the center of the earth, i.e., it is a geocentric system, while both observations and design outputs are topocentric. The lines P_1 - P_3 and P_2 - P_4 are the projections of the two planes of the daily motion of the sun during summer and winter solstices, respectively. The designer of the Great Pyramid in Giza used similar cross-section for aligning its four internal shafts, using O_t equals $\sim 24.10^\circ$ and $\sim 21.672^\circ$ (Aboulfotouh 2005), see fig.1b. If

one rotated the plane of that north-south cross-section (Fig.1a) clockwise for 30°, and afterwards rotated it around the axis Y_n-Y_s for 90°, towards the west, it will become a horizontal plane, i.e., a circular horizon

where its contents of lines and curves are imaginary projected from that plane on its ground site⁵, as shown in Fig. 2.

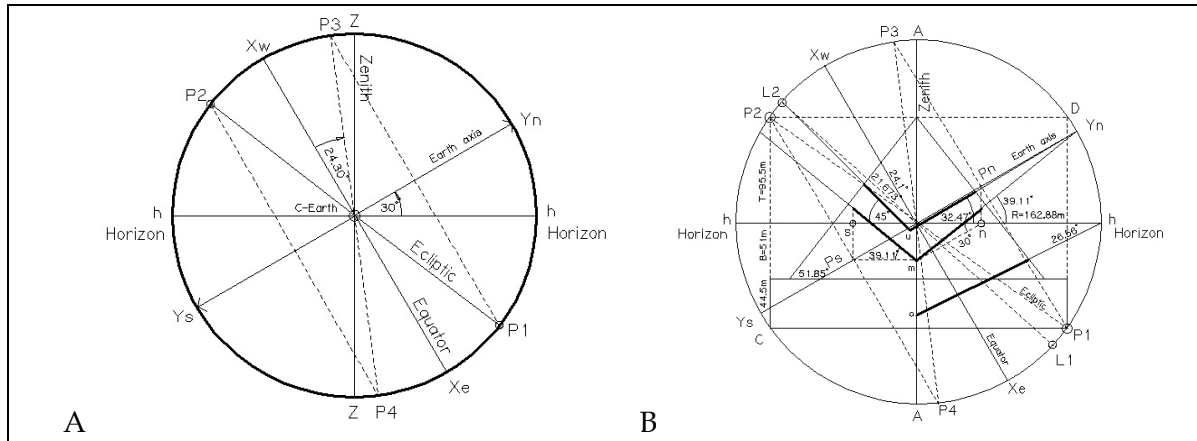


Figure 1. Using the cross section in a spherical coordinate system in pyramids design: (A) shows the north-south cross-section in a spherical coordinate system at the latitude of 30° north, for the obliquity angle 24.30°; and (B) shows the astronomical design concept of the Great Pyramid in Giza, based on a design radius equivalent to 168.88m, which it is the radius of a circle enclosing its square base; while conserving two obliquity angles: 24.10° denoted by the limits of the projection-lines P_1-P_3 and P_2-P_4 ; and 21.672° denoted by the line L_1-L_2 , and the position of the north pole at Y_n and its southern mirror, that upon which the directions and ends of its internal shafts were designed (Aboufotouh, 2005).

Accordingly, the line Y_n-Y_s that represents the earth's spin axis, in the cross section, will become the geographic north-south axis of that circular-horizon. No matter where the horizon model will be implemented any where on earth, its center c imaginary represents the center of the earth. Besides, its cardinal east-west axis X_e-X_w and the diameter P_1-P_2 , will imaginary represent the horizontal projections of the earth's equatorial plane and the plane of the ecliptic in the horizon's ground-plane, respectively. The paper shows hereafter that the ancient Egyptian designer used this cross-sectional horizon-model in the site plan of Giza Pyramids plateau.

Aboufotouh (2002) showed that the center of the horizon-model of Giza Pyramids plateau is the point of intersection between the cardinal east-west axis of the Sphinx and the cardinal north-south axis of the Great Pyramid, as shown in Fig. 3, where the horizon's design-radius R was found = ~746m. That value of R complies with the results of Petrie's survey data (Petrie, 1882, p125), and gives correct and meaningful

astronomical positions for the three pyramids, with regard to the observed daily motion of the sun, taking into consideration the value of the implementation tolerances in megalithic construction-survey, i.e., ~1/1000.

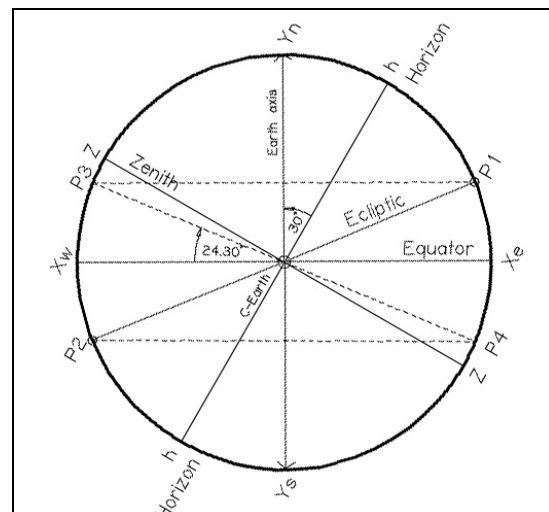


Figure 2. The generated horizon-plan after rotating the spherical cross-section, in fig.1a, 30° clockwise, and afterwards rotating it 90° anticlockwise around Y_n-Y_s . At the end, Y_n, Y_s, X_e and X_w will denote the cardinal directions: north, south, east, and west, respectively.

Besides, the same work also showed that the ancient Egyptian set the positions of the Giza Pyramids based on the shadows of an imaginary vertical-post⁶ that its height equals R and stands at the center of the horizon, when O_t was almost 24.10° . The three related corresponding findings followed, as shown in Fig.3. First, the end of the shadow of that vertical post at noon during the vernal equinox marks the center of the Great Pyramid's base. Second, the

end of its shadow at noon, during summer solstice, marks the north-south position of the east-west axis of the second pyramid. However, the justification for setting the east-west position of the north-south axis of the second pyramid was not discussed. It will be shown in the present paper, as it is closely related to the astronomical alignment of the causeway of the second Giza pyramid.

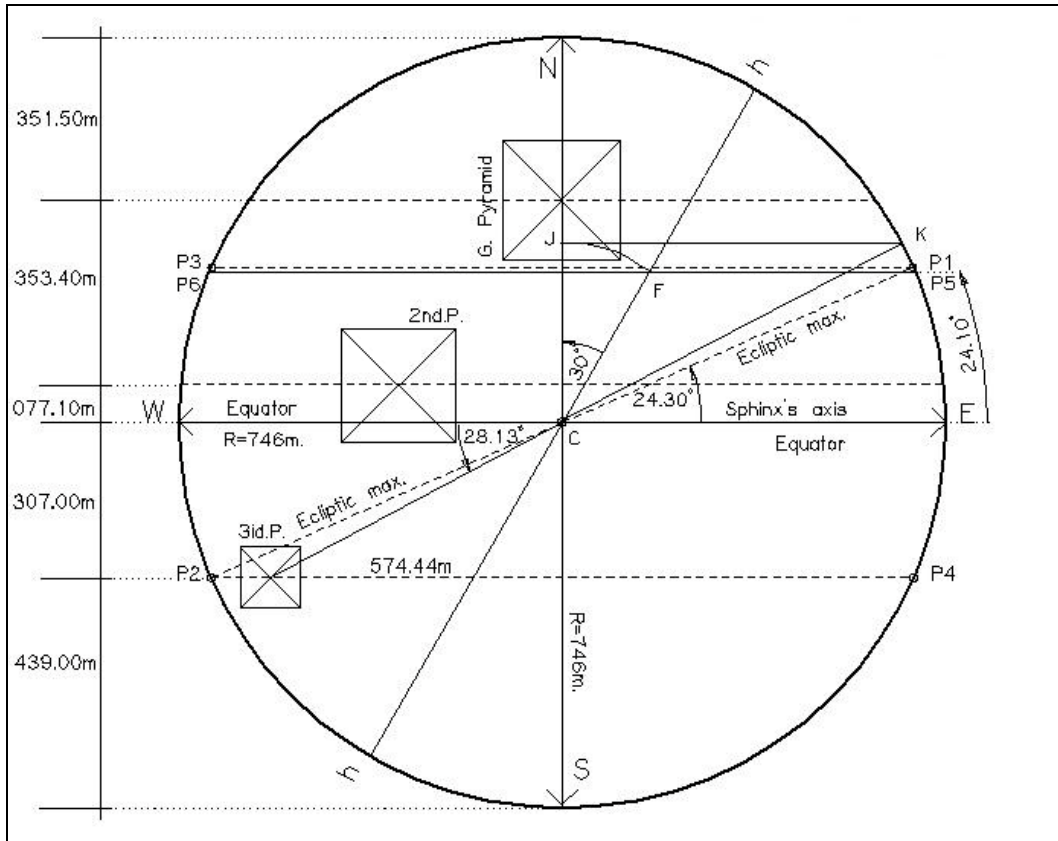


Figure 3. The site plan of the horizon of the three pyramids in Giza plateau, as was presented in Aboufotouh (2002).

Third, the line P_2-P_4 is the east-west axis of the third pyramid, and it is the projection line of the plane of the daily-motion of the sun during winter solstice, for $O_t=24.30^\circ$. Besides, the center of the third pyramid is the intersection point between the projection-line P_2-P_4 , and the shadow of the imaginary vertical-post during the sunrise of summer solstice, for $O_t = \sim 24.10^\circ$. The encoded sunrise angle a in summer solstice, from due west, approaches its geocentric value⁷. It can be derived geometrically, using the line $h-h$ in the plane of the

horizon. Starting from the intersection point F between the projection-line P_5-P_6 and line $h-h$, we rotate anticlockwise until the curve meets the north-south axis $N-S$ at point J . Then, starting from J , we draw a line towards the east and parallel to the east-west axis $E-W$; which will meet the circumference at point K . If we draw the line $K-C$, the generated angle $K-C-E$ is the geocentric value of the sunrise angle at summer solstice based on the values of the obliquity of time O_t , e.g., $O_t = \sim 24.10^\circ$, and the latitude of the place λ .

3. THE LUNAR DATA AND THE METHOD FOR RECKONING THE DECLINATIONS OF THE MOON

The current earth's obliquity to its orbits O_t is about 23.45° (Williams, 2006); and the mean geocentric inclination of the moon's orbit to ecliptic $i_g=5.15^\circ$, which is the mean between two values⁸: 4.95° and 5.33° (Malville, *et al* 1993).

Hence, the geocentric declination of the moon Δ fluctuates between two values: the minimum $\Delta_m = (O_t - i_g)$, and the maximum $\Delta_x = (O_t + i_g)$. They are the geocentric declinations of the minor and major lunar standstills, where their current approximate values are: 18.28° and 28.58° , respectively (Williams, 2006).

These values are the inclinations of the full moon, from the earth's equatorial plane, when the moon meets the meridian of the place of observation. Similarly, for $O_t=24.10^\circ$, $\Delta_m=18.95^\circ$ and $\Delta_x=29.25$, approximately.

Besides, the moon spends about 9.3 years to swing between the two declinations: Δ_{xg} and Δ_{mg} , i.e., it spends about 18.61 years between two succeeding minor standstills⁹. The last minor lunar standstill was in February 1997, and the next one will be in October 2015 (Vincent, 2005).

One cannot observe the true value of the geocentric declination of the moon while standing on earth. It is due to the effect of both the parallax and the atmospheric refraction¹⁰ that should be taken into consideration. Their values should be added to i_g , where the result is the moon's topocentric declination, when observed from the earth's surface.

For reckoning the parallax, Fig.4 shows an observer at c_1 standing at $\lambda = 29.974^\circ$ north ($\sim 30^\circ$), during the minor lunar standstill.

The angle φ is the maximum geocentric declination of the full moon from the zenith of the place and the angle θ is its associated topocentric observed value. Hence, the value of θ could be reckoned using the equation¹¹: $\text{Atan } \theta = (R_1 * \sin \varphi) / ((R_1 * \cos \varphi) - R_2)$.

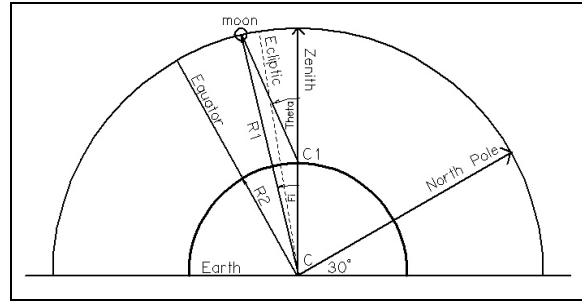


Figure 4. Half cross-section in the semi-elliptical sphere of the moon's orbit around the earth. It is perpendicular to the plane of the moon's orbit and passes through the line of apsides that part of it is being represented in the figure by the line R_1 , which it is the shortest radius of the moon's orbit. The point C is the center of the earth, and the point C_1 is the place of an observer at the latitude $\sim 30^\circ$ north of the equator, and R_2 is the mean radius of the earth. The figure is not to scale.

Where R_1 is the mean radius of the moon's orbit, R_2 is the volumetric mean radius of the earth; they are 384,469 km and 6,371 km respectively (Williams, 2006); and φ equals $(\lambda - O_t + i_g)$, where λ is the latitude of the place of observation. For $O_t = 24.10^\circ$ and $\lambda = 29.974^\circ$ north, the difference between φ and θ is $\sim 0.184^\circ$, which should be added to i_g of 5.15° , and the result would be the observed topocentric inclination i_{mp} (or i_{xp}) of the moon's orbit to the plane of the ecliptic. The effect of atmospheric refraction can be neglected at the elevations close to the zenith¹².

Accordingly, if $\lambda=29.974^\circ$ north, and $O_t = 24.10^\circ$, $i_{mp} = \sim 5.334^\circ$ for the minor lunar standstill¹³. Similarly, for $O_t=22.968^\circ$, the corresponding i_{mp} is $\sim 5.352^\circ$. In order for an ancient designer to encode the declination of the full moon during the minor lunar standstill for the obliquity angles: 24.10° and 22.986 , he might encode the corresponding Δ_{mp} as: $\sim 18.766^\circ$ and $\sim 17.634^\circ$ respectively in the design of his megalithic horizon, where $\Delta_{mp} = O_t - i_{mp}$.

However, if the first inclination was an observed value in his days, i.e., it is not derived from a reckoning process, he might assume that $i_{mp}=5.334^\circ$ for any obliquity angle, and his supposition for the encoded declination Δ_{mp} of the moon might be hence equal $\sim 17.652^\circ$ for $O_t=22.986^\circ$.

4. THE ARCHITECTONIC MAPPING OF THE MINOR LUNAR STANDSTILL.

Fig. 5 shows photos of the Giza pyramids plateau; and the causeway of the second Giza pyramid. Based on Google Earth data (of 7/12/2013) the ground length of causeway, from the western entrance of the valley temple to the eastern entrance of mortuary temple is ~496m; of which only ~90m from the valley temple remain with parapets¹⁴; and the orientation of its axis is almost 13.5° south of the cardinal east.

In 2006, Nell *et al* (2012, p19) surveyed the boundaries of the causeway, and concluded that the orientation of the north and south boundaries of the causeway is $13^\circ 26'$ and $13^\circ 33'$, south of the cardinal east, respectively.

It confirms the Google Earth data. The causeway intersects with the east-west axis of the horizon and the Sphinxes at point X where its distance¹⁵ from the north south axis of the Great pyramid is ~193m, as shown in fig-5a, and fig-6.

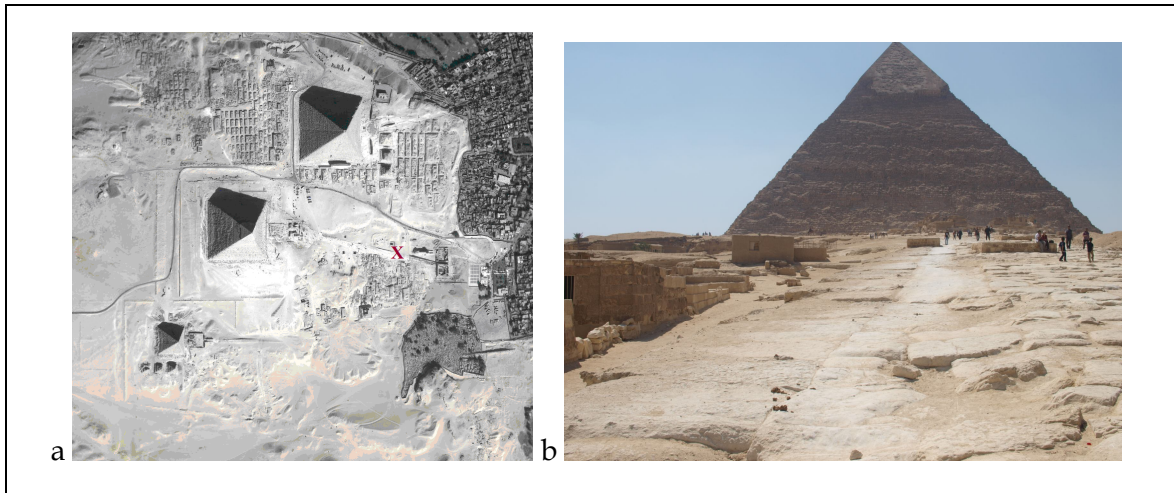


Figure 5. The aerial and ground photos of the Giza pyramids: (A) a Google earth photo of the Giza Pyramids plateau, where X in red marks the point of intersection between the axis of the causeway of the second pyramid and the east-west axis of the Sphinx; and (B) is a photo taken at X and looking towards the west, where the causeway is inclined to the right side from due west by almost 13.54° .

Fig.6 shows the circular horizon of the Giza Pyramids. It is similar to the rotated cross section in Fig.2. The points: M_1 and M_2 represent the observed positions of the full moon, when it crosses the meridian of the place, during two events of minor lunar standstill: M_1 for $O_t = \sim 24.10^\circ$; and M_2 when O_t reaches $\sim 22.986^\circ$. The line $C-M_1$ that corresponds to $O_t = \sim 24.10^\circ$, intersects with the east-west axis of the second pyramid at point Y, which it is also the center of the eastern side of the base of the second pyramid. That intersection identifies the east-west position of the second Giza pyramid, implying the encoding of the declination of the moon during the minor lunar stand still for $O_t = \sim 24.10^\circ$, in the east-west position of the second Giza pyramid. The side-dimensions¹⁶ of the right-angled triangle

YCT confirm the projected lunar declination angle $\Delta_{mp} = 18.766^\circ$; where $\tan 18.766^\circ = YT/TC = \sim 77.05/226.78$. Since Fig.6 is a cross-section and a horizon-plan in the same time, the projection of the points M_2 on the line $h-h$ is the point V_2 . From V_2 , if we draw a line towards the east, and parallel to the cardinal axis $E-W$ in the horizontal-plan (or the equator in the cross-section) it will meet the circular circumference of the horizon at point L_2 . Now, the triangle $M_2-V_2-L_2$ corresponds to the event of the minor lunar stand still at the time when $O_t = \sim 22.986^\circ$. By reckoning, if $\Delta_{mp} = O_t - i_{mp} =$ angle M_2-C-W , and β is the angle M_2-X-W , which it is the inclination of the axis of the causeway M_2-L_2 on the east-west cardinal direction; then, $\tan \beta = R \sin \Delta_{mp} / (R \cos \Delta_{mp} + CX)$. That is, if $R=746m$, $CX=193m$,

and $\Delta_{mp} = 17.65143^\circ$, then $\beta = \sim 13.544^\circ$. The value of β conforms to the survey result of Nell *et al* (2012, p19) of $13^\circ 33'$. It proves the justification of the astronomical purpose of the alignment of the causeway of the second Giza pyramid as to encode the declination of the moon during the minor lunar standstill when earth's obliquity angle reaches its mean value of 22.986° as was

thought by the ancient Egyptian designer. Concerning the encoded years that the two lunar declinations mark, counting from 2016AD, and using the interval of 18.61 years, in the past the year 3065BCE was a year of minor lunar standstill and meets $O_i = \sim 24.10^\circ$, and in the future, 5478AD will be a year of minor lunar standstill and meets $O_i = \sim 22.986^\circ$.

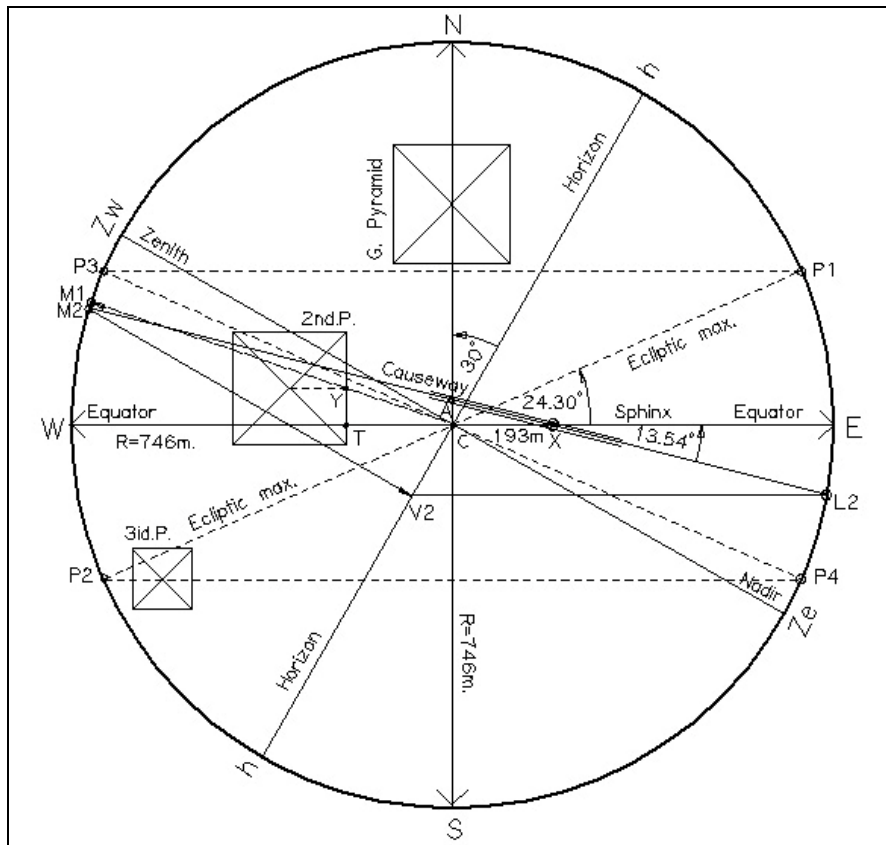


Figure 6. The circular horizon of the Giza pyramids. It is a horizon plane and a vertical cross section in the same time, and its center C is imaginary the center of the earth. Hence, the point Z_w represents the zenith of the place, the north south coordinate-line N-S represents the spin axis of the earth, and the east-west coordinate-line E-W represents the projection of the plane of the earth's equator. The line P_1-P_2 represents the projection of the plane of the ecliptic with its assumed maximum inclination of 24.30° to the earth's equatorial plane¹⁷.

4. CONCLUSION

The paper showed the architectonic method of the ancient Egyptian designers for encoding the horizontal-projections of the moon's declinations during two events of the minor lunar standstills, in the design of the site-plan of the horizon of the Giza pyramids, using the methods of descriptive geometry.

Its findings are consistent with the results of previous papers on the architecton-

ic and astronomical design of the Giza Pyramids and their horizon. It implies the ancient Egyptian designers were encoding astronomical information using the geometrical language. Using descriptive geometry and algorithms, the information they encoded were not only on the daily motion of the sun and the earth obliquity range but also on the motion of the moon, particularly encoding the moon's declinations during specific events of the minor lunar standstill.

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