Ninety-three presents were announced as having been received since the last meeting, including, amongst others:—

Publications of the Astrophysical Laboratory, Groningen, No. 20; J. C. Kapteyn and W. De Sitter, Parallaxes of 3650 stars, presented by the Laboratory; A. S. D. and E. W. Maunder, The Heavens and their Story, presented by the authors; Oxford University Observatory, Astrographic Catalogue, vol. vi., presented by the Observatory; Australian Meteorology, by Dr. W. J. S. Lockyer, presented by the Solar Physics Committee.

Astrographic Chart; 36 charts, presented by the Royal Observatory, Greenwich; Photographs of the 60-inch reflector of the Mount Wilson Observatory (6 prints), presented by Dr. G. W. Ritchey; Drawing of the lunar crater Archimedes, presented by the Rev. F. B. Allison.

Solar Parallax Papers. No. 7.

The General Solution from the Photographic Right Ascensions of Eros, at the Opposition of 1900. By Arthur R. Hinks, M.A.

§ 1. Earlier papers of this series have dealt with the construction of the standard photographic catalogue of stars (*M.N.*, 1906 June, Nov., 1907 Dec.). This catalogue is now complete, with the exception that it may be possible to incorporate the Algiers results before publication. It contains about 6000 star places; and it serves as the standard to which I have reduced all the photographic and micrometric observations of Eros.

§ 2. *The Published Photographic Observations.*—These are found in Paris Circulars 10–12; they are given in a form which is apparently uniform, but is in reality not quite so. It may be convenient to mention the exceptional cases.

The epochs are in general the mean epochs of observation. But in the case of Bordeaux they have been antedated by the aberration time.

The positions of the planet are in general geocentric. But in the cases of Bordeaux, Northfield, and Toulouse the reductions to the Earth's centre have not been applied.

The positions are in general reduced to apparent place. But in the case of Toulouse the whole effect of the aberration of light has been accidentally omitted from the published positions.

The quantities in the columns headed "parallax" are the *corrections*, which have in most cases been already applied to the quantities in the preceding columns.

The quantities (O – C) are from comparison with the ephemeris of Circular 9.

The positions of the planet are referred to a number of star systems differing slightly from one another. In most cases they have been reduced to standard by the process described in § 4.

§ 3. *The Unpublished Photographic Observations.*—The principal unpublished series is that made with the Crossley reflector of the Lick Observatory, of which a brief discussion has been published within the last few weeks (*L. O. Bulletin* 150).

The measurable field of the Crossley reflector is small—about 24′ in diameter—and the choice of comparison stars consequently restricted. It was of great importance that this fine series should be reduced with well determined stars. I ventured, therefore, to propose to Professor Campbell and Professor Perrine that I should send them charts showing the stars whose places I should eventually be able to supply from the standard catalogue, and that they should select the stars for measurement from these charts. They agreed to do so; and in due course I was able to send them good places of the greater part of the stars which they had used. In a few cases bad weather in Europe had so interfered with observation that the star places were rather weak; and occasionally they have been supplemented from Lick photographs.

Correspondence with Professor Perrine revealed a difference of opinion as to the principle upon which the stars should be selected. The motion of Eros during one night was sufficient to carry the planet half-way across the field of the Crossley reflector. If, then, the same stars were used evening and morning, they could not be symmetrically placed on the plate, which was always centred upon the planet, and the planet would be displaced evening and morning towards opposite edges of the group. If, on the other hand, the comparison stars are chosen to be symmetrical about the planet, different stars must be used evening and morning, and errors in the assumed places of the stars will enter into the parallax determination.

Professor Perrine preferred the first plan. I thought that the second gave a better chance of eliminating systematic error, though at the expense of introducing accidental. So Professor Perrine very kindly undertook that the measures and reductions should be made in both ways.

The second unpublished series consists of the greater part of the Cambridge observations, of which a small part only had been reduced and published in Circular 12. The measurement of the rest of the plates had been delayed until it was possible to select stars from the standard catalogue, on the completion of which the plates were measured and reduced to this system.

We may expect, then, that the Lick and Cambridge series will be in perfect accord with the standard system.

§ 4. *Reduction to Standard.*—The remaining series must, where necessary, be reduced to the standard system.

To effect this without the labour of complete re-reduction, I have adopted the device mentioned in *S.P.P.* No. 6 (*M.N.*, 1907 Dec., p. 96). The programme of M. Loewy provided for the measurement of all stars within the 20′ square centred on the planet, and the separate publication for each plate of the concluded star places. When it was first proposed, this plan seemed extravagant; but it has proved to be of the greatest value. For among the stars of the square one can generally find ten or a dozen whose places are given in the standard catalogue; and we can compare the positions derived from the individual plate with the catalogue positions. If anything abnormal has happened to the plate, either in making or in reduction, the effect upon the mean of the group of stars distributed over this small square is approximately the same as the effect upon the planet situated at its centre. And so if we apply, as a correction to the place of the planet, the mean of the differences "standard catalogue minus stars of the square," it seems that we should get rid, in a very simple manner, of all discordances of a systematic nature.

This reduction has been applied to the results from Bordeaux, Helsingfors, Paris, Toulouse, and San Fernando. I have not been able to apply it to the results from Minneapolis, Northfield, Pulkowa, and Upsala, because these observatories have not published the stars of the square separately for each plate. In these cases we must rely on general tests for absence of systematic error.

The two series of Greenwich plates are reduced to a system of comparison stars determined at Greenwich, but based upon the same fundamental system (Loewy's) as is the standard catalogue. These stars are included in fields of about 25′ radius; they are therefore closer than the *repère* stars, but not so close as the stars of the square. Many of them are included in my catalogue, and the two systems are in excellent agreement. It seems to me that there can be no doubt that the Greenwich results are practically homogeneous with those reduced directly to my system.

The Oxford results, reduced by me at Cambridge, are referred to my standard system. The same has already been said of the Lick and Cambridge results.

The reduction of the whole material to one standard star system is thus practically complete.

§ 5. *Correction for Refraction.*—In a determination of parallax by the diurnal method, the terms of the second order in the refraction are of great importance, since they change sign with the parallax.

The results from Bordeaux, Cambridge, Greenwich, Oxford, and Pulkowa have been corrected for the second order terms. The field of the Crossley reflector is so small that these terms are insignificant. In the Minneapolis and Northfield plates the field is also comparatively small and the correction hardly sensible; it has apparently not been applied in the reductions. In the remaining series the correction has been omitted, except in one, where it has been computed from an inapplicable formula. It is not permissible to neglect this correction altogether. But fortunately the reduction to standard, explained in § 4, will automatically, and almost completely, effect the necessary correction. I have therefore not computed it separately.

§ 6. *The Aberration of Light.*—The effects of aberration have been brought to the notice of the Society so recently that it is unnecessary to do more than refer to them very briefly here.

The mean places of the stars, to which the plate is reduced, contain the eccentricity term of the aberration, since it has never been taken out.

Hence the process of reduction of the plate removes the circular aberration terms from the planet's place, but leaves the eccentricity terms.

Hence, to bring the planet up to apparent place, for comparison with the ephemeris, we must add the precession, nutation, and circular aberration.

The resulting apparent geocentric place of the planet contains the whole aberration due to the motion of the Earth's centre.

The whole aberration is equivalent to the alteration in the place of the planet during the light-time.

Hence the usual process of antedating gives us the true comparison between observation and ephemeris, free from the effects of both circular and eccentricity terms.

This is the usual way of presenting the theory. But we may observe that nothing has been said of diurnal aberration. The process of antedating the epoch for interpolation in the ephemeris removes the whole aberration, including the diurnal. The latter is not explicitly restored in the reduction, as are the other aberration terms. Yet the comparison with the ephemeris shows no diurnal variation corresponding with the diurnal aberration. In what way has it been eliminated?

The answer to this question is simple, but not quite obvious. It is connected with the answer to another question which might be propounded. Why do we not antedate the epoch of observation in computing the parallax factors, as we do in interpolating?

In the simple geometry of planetary aberration it is usual to represent the Earth as a point. If it is drawn of finite size, and its rotation during the light-time is considered, we see at once that we ought to antedate the epochs in reducing to the Earth's centre, and at the same time we see that we ought to add the diurnal aberration to the reduction to apparent place. The two quantities are precisely equivalent. We neglect both, and the result is correct.

§ 7. *The Reduction to Apparent Place.*—The reduction of the planet to apparent place affords unexpected opportunities of introducing systematic or semi-systematic discordance.

My attention was called to this, in the first place, by finding that some of the American series contained the short period nutation terms, which were, of course, excluded from the ephemeris of the planet. The tables of Besselian and independent day numbers of the American Ephemeris include these terms—contrary to the practice of other ephemerides—and the fact is not stated with the prominence which is desirable under the circumstances.

This led to an examination of the whole question. The tabular quantities for the reduction from mean to apparent place, as given in the national ephemerides for the year 1900, are computed with diverse values of the constants of precession, nutation, and aberration (owing to the adoption by the American ephemeris of new constants a year before the other almanacs), and a change was made in all the ephemerides for the year 1901.

In 1908 October I addressed a circular to my colleagues, asking for information as to the method they had followed. This circular specified nineteen different ways in which the reduction might have been made; but the ingenuity of my colleagues had discovered several others, and I think that the number of methods actually pursued is equal to the number of observatories engaged in the reductions. But it happens, fortunately, that the nett difference between the Struve-Peters and the Paris Conference reductions is scarcely sensible in the region of the sky where the planet was; the small differences cancel one another. Hence it was necessary only to remove the short period nutation terms from those series into which they had been inadvertently or deliberately introduced—that is to say, from the Pulkowa, Toulouse, and part of the Minneapolis photographic, and from the Lick, Padua, Pulkowa, and Teramo micrometric series.

The sum of these terms in the R.A. varies between $+0^{s}{\cdot}021$ and $-0^{s}{\cdot}016$, as computed from the formulæ employed in the British *Nautical Almanac*. The almanacs are not consistent in the terms they include, and their tables are not competent to give accuracy in the third place of decimals. Hence discordances of some thousandths are common, but they have been treated as accidental.

§ 8. *The Ephemerides.*—All the values of (O – C) published in the Paris Circulars are derived from one or other of the two ephemerides published in Circular 9, based on the elements of Professor Millosevich.

The first gives, in the usual form, the R.A. and Decl. of the planet referred to the true equinox of the date.

The second, which was given by M. Loewy at the request of Professor Turner and myself, gives separately the rectangular ecliptic co-ordinates of the Sun and the planet, referred to the mean equinox for 1900·0. This ephemeris was used in my experimental reduction (*M.N.*, 1904 June). In the course of this I found that the two ephemerides were inconsistent; the first could not be reconstructed from the second. The fact was submitted to M. Loewy, who found that different values of the obliquity of the ecliptic had been employed in different parts of the work; the equatorial rectangular co-ordinates of the planet had been calculated at the Paris Observatory with Leverrier's obliquity; the corresponding co-ordinates of the Sun had been calculated at the Bureau des Longitudes with Newcomb's obliquity.

In Paris Circular 12 M. Loewy published—

(1) The table necessary to correct the ephemeris of Circular 9 for the confusion in the value of the obliquity.

(2) A statement that this ephemeris does not include the whole of the planetary perturbations.

(3) Four new systems of elements calculated by Dr. Gustav Witt of Berlin, the discoverer of the planet.

(4) A table of corrections calculated by Dr. Witt to reduce the old ephemeris to the new elements.

(5) The resulting ephemeris, obtained from the old by simple addition of the tables (1) and (4).

I am indebted to Dr. Witt for the following information on the subject of his elements and corrections.

They are based on new elements calculated for the epoch 1898 Aug. 20 M.T. Berlin (*Untersuchung über die Bewegung des Planeten* (433) *Eros*, von Dr. Gustav Witt, Berlin. 1905). The three systems of osculating elements for 1900 Oct. 31·0, 1900 Dec. 10·0, and 1901 Feb. 8·0 are deduced from the first, with a complete calculation of the planetary perturbations. When the complete perturbations had been applied, it was found that the three portions A, B, C of the ephemeris were still discordant at the points where they overlapped. After a long search Dr. Witt discovered the simple cause. In the elements of Millosevich the values of μ were given to nine significant figures, and the value of log a only in seven-figure logarithms. When it was decided to compute the ephemeris with eight-figure logarithms, the computer had simply added a zero to each value of log a, and had overlooked the fact that the resulting values of log a were irreconcilable with the values of μ. Further, there was a small mistake at the beginning of section B.

For my first general solutions I used the published values of O – C, reduced to conform with the ephemeris of Circular 12. This ephemeris was, however, still open to criticism in several respects.

No details as to its construction had been published. In particular it was desirable to know how the lunar equation in the Earth's motion had been computed. Soon after the appointment of M. Baillaud to the Directorship of the Paris Observatory, I ventured to apply to him for information on this point, and he immediately responded to my request with the ready kindness for which I have had to thank him on this and many other occasions. It appeared that the lunar equation had been computed at the Bureau des Longitudes by the rigorous formulæ given by Newcomb (Tables of the Sun: *Papers of the American Ephemeris*, t. vi. p. 18). But the process of smoothing and of interpolating to smaller intervals had been conducted after the lunar equation had been introduced, which is unsatisfactory. Moreover, there was reason to fear that defects existed in the method of interpolation employed.

§ 9. *The New Paris Ephemeris.* — Eventually M. Baillaud decided, in 1908 November, that it would be best to recompute the entire ephemeris. He entrusted this task to the care of M. Lagarde, chief of the Computing Bureau of the Paris Observatory, who worked with so much energy that the complete new ephemeris was placed in my hands by the middle of February last.

M. Lagarde has kindly given me the following details as to the construction of the new ephemeris.

The ephemeris, in rectangular co-ordinates, for the centre of gravity of the Earth-Moon system, has been computed and polished by smoothing the run of the differences. A similar ephemeris has been made for the planet.

The two combined give an ephemeris in R.A. and Decl. of the planet, as seen from the centre of gravity.

This has been reduced to the true equinox of date with the Struve-Peters constants throughout; has been smoothed again, and reduced to six-hour intervals by interpolation. The lunar equation was computed separately as above, with the tabular values of the place and parallax of the Moon; its effect on the right ascension and declination of the planet was interpolated to six-hour intervals, and applied to the planet ephemeris to reduce to geocentric place; it is also exhibited separately, to facilitate a new determination of the mass of the Moon.

The resulting new Paris Ephemeris is a great improvement on the old. Its theoretical basis is more secure; and it is free from some peculiar irregularities, of obscure origin, which existed in the old. It is worth while to examine them for a moment, because they show very well how the smoothness of third differences in an ephemeris is no guarantee of its accuracy. The following is an extreme case:—

R.A. Ephemeris of Eros. Circular 12.

		3rd Differences.	Error of Ephemeris.
	h	s	s
1900. Dec. 2.	6	·000	·000
	12	+ ·001	− 3
	18	+ 2	− 5
3.	0	5	− 8
	6	+ 2	− 8
	12	+ 3	− 11
	18	− 2	− 13
4.	0	− 2	− 14
	6	0	− 9
	12	− 3	− 5
	18	0	0
5.	0	− 2	+ 3
	6	+ 1	+ 6
	12	− 2	+ 5
	18	+ 3	+ 3

The third differences are sufficiently good; they do not indicate any correction greater than a unit in the last place, while the actual corrections required are very much larger. It is, of course, clear that differencing is good for detecting an isolated error, and no good for detecting anything at all systematic. But it is at first sight rather surprising that differencing will not detect a considerable error probably due to faulty interpolation to halves.

The new Paris Ephemeris is perfectly adapted for the determination of the solar parallax. For the mass of the Moon it is preferable that the motion of the Earth-Moon should be computed by special perturbations, instead of being taken from the Solar Tables. M. Lagarde has therefore computed still another ephemeris upon this latter principle, which will be available for the final discussion of the mass of the Moon.

May I express here my sincere thanks to M. Baillaud and to M. Lagarde for the readiness with which they undertook a severe labour, so soon as an examination of the old ephemeris revealed cause for apprehension.

§ 10. *The Systems of Weights.*—The experience gained in the formation of the Star Catalogue had shown that the contributions of different observatories were by no means of the same weight, and it became a delicate matter to decide what system of weights to adopt. I eventually found that substantial justice would be done if I gave weight unity to a single exposure with an instrument of the astrographic type; gave a slight increase of weight for instruments of greater focal length, and a small decrease for the smaller instruments.

It is quite true that there is not much *a priori* justification for giving weight directly proportional to the number of exposures. But there is evidence *a posteriori* that in this particular case it was nearly the right scale to adopt.

The principal exception is furnished by the Lick photographs. It was assumed that 4 exposures were measured on each plate, though this turns out to be not quite true. But if each plate received weight 4, the total weight of the series would be overwhelming. And since the series consists of a great number of plates made on relatively few nights, so that any error peculiar to the night is repeated a great number of times, the real weight is in this case by no means proportional to the number of exposures. In the solutions made with ephemeris 12 I had given the Lick plates weight 4. In the later solutions I reduced this weight to 2, and eventually to less.

The following table gives the adopted weights:—

Bordeaux .	2
Paris . .	3 (or rarely 2 or 1, corresponding to the number of exposures)
San Fernando	1
Upsala . .	3

Minneapolis .	1	
Poulkovo .	3	
Toulouse .	2 or 1	
Helsingfors .	2	
Northfield .	3	
Greenwich (Astrographic) .	4, 3, 2, or 1	for 4, 3, 2, 1 exposures respectively
,, (Thompson) . .	5, 4, 3, or 1	
Cambridge	5, 4, 3, or 1	
Lick	2 uniformly	

§ 11. *Determination of Tabular Error.*—In discussing the tabular error of the ephemeris in parallax determinations, it has been usual to represent the error in a series of powers of the time. There are special reasons why this would have been useless in the present case.

The planet traversed a great loop in declination, reaching as far north as $+54°$. Hence a constant error in its longitude would produce an error in its R.A. varying more or less as the secant of the declination.

The fundamental star system probably contains errors in the R.A. varying as some function of the declination, which will reappear in the tabular error of the ephemeris.

The opposition in right ascension took place nearly two months before the nearest approach to the Earth. Hence errors in the heliocentric place of the planet produce effects on its geocentric place which are quite unsymmetrical with respect to opposition.

An error in the mass of the Moon will produce a monthly inequality, and the amplitude will be unsymmetrical with respect to opposition, for the same reason as above.

It seemed best to follow the excellent example set by Greenwich, and deduce the tabular error by graphical processes.

A preliminary examination showed that the terms involving the squares and higher powers of the time were very small; that the correction required by the mass of the Moon was very small; and that the principal correction was a function of the declination, but not simply of the form $a \times b \sec \delta$.

To obtain the tabular error I have used all the observed places of the planet—not only those taken close to the meridian.

The weighted mean of all the observations of a single day gives

$$\Delta\alpha = m + \mathrm{P}f,$$

when the true parallax $= (1 + \mathrm{P}) \times$ the adopted parallax ($8''.80$), and f is the mean correction for parallax applied to the observations of the group. A preliminary solution showed that $\mathrm{P} = +0.0007$ about, and f is small except at the end of the series. Consequently the term $\mathrm{P}f$ rarely reaches the value $0^{s}.001$, and its uncertainty is insignificant compared with the other irregularities in the determination.

I have made two principal determinations of tabular error—for the ephemeris of Circular 12, and for the new ephemeris.

The representation of the first was very disappointing. It was impossible to draw a curve through the plotted points without abrupt sinuosities. After various fruitless attempts at a smooth curve, I decided to follow the larger sinuosities and to neglect the smaller.

When the new ephemeris arrived, it appeared that a good deal of this trouble was due to the considerable errors of the old, which at the worst varied by as much as $0^s{\cdot}017$ in 24 hours. With these removed, the representation of the tabular error was much more smooth, but it was exceedingly difficult to decide how to draw a smooth curve through the plotted points. If one took a stretch of, say, 14 days, it was generally impossible to decide on which side of a straight line the curvature lay. It was evident that the smooth curve must have several points of inflexion, but impossible to say how many. There was little hope, then, of finding an algebraical expression which could be fitted to the points.

Finally I used the following device. I divided the curve into sections at the points where the lunar equation vanished, and by a series of least square solutions I fitted straight lines on to each of these sections. If the correction to the mass of the Moon is significant, the convexities of the sections should lie alternately above and below these lines, and the lines themselves should lie alternately high and low. There was no certain trace of such an effect. The lines did not join up precisely, but the errors were slight and irregular. I could think of no better device than to treat these lines as a species of polygonal representation of the curve desired, and to modifiy them graphically on each side of the junctions until a smooth progression was secured from one to the other.

This should secure that the average slope of the curve is correct, which is all that is required for the parallax determination. It will be necessary to re-discuss the tabular error before the final solution for correction of the assumed mass of the Moon.

§ 12. *Formation of the Normal Equations and Solution.*—The normal equations in $\Delta\alpha$ and P were formed with a Brunsviga calculating machine, and were completely checked. The equations were solved with the determinant notation, and the solutions checked by substitution in the normal equations.

In tabulating the results I have given the coefficient of P in the normal equation for P, and also the weight of the determination of P. The difference between the two shows how much has been lost by want of balance of the parallax factors.

A first general solution was made from the comparisons with the ephemeris of Circular 12: in this solution I rejected only such equations as depended upon observations marked "through clouds," with a few that were hopelessly discordant, probably through misprints. The Lick plates received weight 4 in this solution.

The second general solution was made with the new ephemeris. In this solution equations were rejected when they gave a residual more than three times the mean error of one equation of the corresponding weight. The Lick plates received weight 2.

In all, 99 equations were rejected out of a total of 1645 equations from observations between Oct. 1 and Jan. 19. Observations outside these limits have not been used for the parallax determination.

The equations were grouped in a number of different ways, and solutions were made as follows:—

With the ephemeris of Circular 12:—

Solution I. All the material divided into four-day groups.

II. The material from each observatory separated, and grouped as far as possible in one- or two-day groups.

This left outstanding a good deal of material which could add no weight to any of these parallax groups. This was all put into a residuum and solved in four-day groups.

III. Ten contributions from each observatory in II. were combined according to their weights.

With the new ephemeris:—

IV. Solution in four-day groups, comparable with I., afterwards combined according to their weights.

V. Solutions in single-day groups; nothing corresponding to this in the first general solution.

VI. Division of solution V. into four periods.

VII. Solution for observatories separately, comparable with II.

VIII. The results of VII. combined according to their weights, comparable with III.

IX. Revision of VII. by rejecting all groups covering more than one day, and all which give a value of P differing from the mean by more than three times the mean error.

X. The contributions made by Greenwich, Lick, Paris, and Cambridge, to VII., combined with revised weights.

§ 13. In the following solutions we employ the uniform notation:—

$\Delta\alpha$ = the residual tabular error, assumed constant for the group.

P = the proportional correction required by the assumed parallax; so that the concluded correction to the parallax

$\Delta\pi = P \times 8''\cdot80$.

ϵ_1 = the mean error of one determination of P of weight unity.

The probable error of $\Delta\pi = \dfrac{0\cdot6745 \times 8''\cdot80 \times \epsilon_1}{(\text{weight})^{\frac{1}{2}}}$.

§ 14.

Table I.

Solutions from 4-day groups.

Date.	Solution I. Ephemeris of Circ. 12.		Coefft. of P.	Weight.	Solution IV. New Ephemeris.	
	Δα.	P.			Δα.	P.
	s					
Oct. 4– 7	− ·0046	− ·00345	...	...	...	...
8–11	+ 75	− 315	60	29	+ ·0077	− ·00375
12–15	− 43	− 67	238	235	− 18	− 54
16–19	− 35	− 117	128	121	− 48	− 98
20–23	+ 10	+ 252	190	189	+ 54	+ 68
24–27	− 29	+ 434	436	431	+ 28	+ 342
28–31	− 80	+ 206	192	191	− 38	+ 217
Nov. 1– 4	+ 41	+ 496	188	183	+ 147	+ 335
5– 8	+ 6	− 509	102	96	+ 5	− 128
9–12	− 4	− 179	416	414	− 1	− 40
13–16	− 11	+ 145	358	351	− 6	− 34
17–20	...	...	32	25	− 16	− 476
21–24	+ 16	− 43	199	167	+ 90	− 303
25–28	+ 30	+ 267	203	193	− 17	+ 164
Nov. 29–Dec. 2	− 75	+ 25	214	211	− 75	+ 107
Dec. 3– 6	+ 105	+ 155	345	313	+ 68	+ 274
7–10	− 49	+ 109	259	242	− 5	+ 213
11–14	+ 25	− 284	75	75	+ 20	− 357
15–18	− 46	+ 54	215	212	− 6	0
19–22	− 17	− 114	250	219	+ 14	− 136
23–26	− 44	− 588	160	115	− 14	− 328
27–30	− 18	− 201	73	44	+ 19	− 643
Dec. 31–Jan. 3	+ 29	− 338	41	30	+ 76	− 1186
Jan. 4– 7	+ 27	− 1097	151	80	+ 1	− 365
8–11	...	...	53	41	+ 66	− 901
12–15	...	...	157	109	+ 14	− 203
16–19	...	...	54	41	− 38	+ 598

The coefficient of P and the weight of the determination refer to Solution IV. Those of Solution I., on the left, are different, owing to the change in the weight assigned to Lick. The numbers serve nearly as well, however, to show the diminution of weight produced by asymmetry in the parallax factors, which is nearly the same in the two solutions.

We have, by combining these separate results according to their weights,—

	Solution I.	Solution IV.
Weight	5864	4357
ϵ_1	±0·0456	±0·0346
$\Delta\pi$	+0″·0055±0″·0036	+0″·0019±0″·0031

§ 15. It will be remembered that the Lick results are duplicated.

Series 1 is reduced with the same stars evening and morning; consequently unsymmetrical about the planet.

Series 2 is reduced with stars symmetrical about the planet; consequently not entirely the same evening and morning.

Table II.

Solutions from Lick Photographs.

	Solution from Eph. Circ. 12.						Solution from New Eph.			
	Series 1.		Series 2.		Coefft	Wt	Series 1.		Series 2.	
	Δα.	P.	Δα.	P.			Δα.	P.	Δα.	P.
	s.		s.				s.	s.		
Oct. 6	− ·0055	− ·00456	− ·0041	− ·00629	12	12	− ·0010	− ·01036	...	...
									s.	
12	− 22	− 106	− 15	+ 441	34	33	+ ·24	+ ·195	+ ·0044	+ ·00696
13	− 66	− 865	− 120	− 206	36	35	− 1	− 967	− 69	− 293
14	− 138	+ 207	− 86	+ 113	59	56	− 124	+ 96	− 67	+ 46
15	− 45	− 235	+ 10	− 262	60	59	− 13	− 288	+ 25	− 277
16	− 98	+ 634	− 25	+ 62	58	56	− 52	+ 687	+ 1	+ 115
21	+ 78	+ 194	+ 83	+ 174	74	74	+ 199	+ 209	+ 187	+ 234
24	+ 58	+ 125	+ 87	+ 579	65	60	+ 120	+ 209	+ 155	+ 609
26	− 32	+ 15	− 76	+ 318	108	108	+ 12	+ 47	+ 16	+ 352
29	+ 23	+ 338	− 54	+ 18	66	58	+ 52	+ 397	+ 14	+ 42
Nov. 3	+ 138	+ 804	+ 101	+ 603	114	114	+ 352	+ 785	+ 265	+ 512
10	− 102	− 761	+ 52	− 683	87	83	− 58	− 744	+ 68	− 701
28	+ 32	+ 460	− 30	+ 202	120	119	− 10	+ 370	− 64	+ 218
29	− 46	+ 187	− 1	+ 88	158	155	− 110	+ 145	− 133	+ 47
Dec. 5	+ 178	+ 184	+ 127	+ 263	140	136	+ 125	+ 270	+ 71	+ 356
6	+ 137	+ 220	+ 94	+ 190	138	134	+ 118	+ 346	+ 100	+ 274
7	− 36	+ 59	− 33	+ 186	133	128	+ 54	+ 150	+ 47	+ 311
24	− 122	− 226	− 129	− 523	62	55	− 86	− 87	− 100	− 209

When these are combined according to their weights they lead to the following:—

Lick Contributions to Solution II.

	Series 1.	Series 2.
Weight	2972	3016
ϵ_1	$\pm 0{\cdot}0506$	$\pm 0{\cdot}0433$
$\Delta\pi$	$+0''{\cdot}0105 \pm 0''{\cdot}0055$	$+0''{\cdot}0107 \pm 0''{\cdot}0047$

The coefficients and weights in the table belong to the later solution. It will be noticed that the Lick material consists almost entirely of complete and well-balanced series: the odd plates were not measured.

Lick Contributions to Solution VII.

	Series 1.	Series 2.
Weight	1463	1461
ϵ_1	$\pm 0{\cdot}0346$	$\pm 0{\cdot}0297$
$\Delta\pi$	$+0''{\cdot}0149 \pm 0''{\cdot}0054$	$+0''{\cdot}0148 \pm 0''{\cdot}0046$

The reduction from 4 to 2 in the weight assigned to each Lick equation should reduce the corresponding value of ϵ_1 in the ratio $1 : \sqrt{2}$. The actual reduction, as calculated, has almost precisely this value; and the probable errors of $\Delta\pi$ remain the same.

In both solutions Series 2 comes out decidedly better than Series 1. We may apparently conclude that the influence of irregularity in the star places is less serious than the influence of asymmetry of the stars with respect to the planet in the centre of the field.

There are three points in these solutions worthy of investigation:

1. The great change in the value of $\Delta\pi$ with the change of ephemeris.

2. The absence of improvement in the probable error.

3. The large size of the probable error compared with that assigned in *L.O.B.* 150 to the Lick discussion of this material. We shall return to these points.

§ 16. The Greenwich observations of the planet have been subjected to a very fine discussion in the recently published "Observations of the Planet Eros 1900–1901: Appendix to the Volume of Greenwich Observations for the year 1905." To my table of solutions of the Greenwich material I have added the Greenwich solutions for comparison. The quantities P in this last column have been formed from the fifth column of Greenwich Table 25, by dividing by 8·80. I have carried the division to one place more than is significant, in order to facilitate comparison with my own results.

Table III.

Solutions from Greenwich Photographs.

Date.	Sol. from Eph. Circ. 12.		Coefft.	Wt.	Sol. from New Ephem.		Greenwich Solution.	
	Δα.	P.			Δα.	P.	Δα.	P.
	s				s			
Oct. 4–7	− ·0206	− ·01578	39	3	...	...	...	...
8–11	+ 368	+ 1972	28	4	...	...	...	...
12–15	− 63	− 860	7	4	− ·0044	− ·00493	Oct. 14, 15	− ·00513
20	− 145	+ 1296	5	5	− 97	+ 1259		+ 1334
21	− 118	+ 116	37	28	− 56	+ 224		+ 80
26	− 52	+ 251	81	76	+ 13	+ 28		+ 68
27	− 62	+ 469	67	65	− 20	+ 409		+ 433
28, 29	− 99	− 29	62	52	− 68	− 34		− 103
Nov. 6, 7, 8	− 156	− 330	36	36	− 89	+ 206*	Nov. 8, 9	− 137
9	− 19	+ 533	33	33	− 16	+ 434	9, 10	+ 137
10	+ 6	− 38	42	42	0	− 19	10, 11	+ 137
13	− 57	− 90	115	114	− 38	− 152		− 125
14, 15	− 11	− 822	58	54	− 22	− 600*		− 764
22	− 25	+ 130	68	48	+ 112	− 367	22, 23	+ 103
23	− 24	− 276	14	8	+ 22	− 318		...
27	+ 59	− 60	46	30	+ 1	− 64		− 57
Dec. 7	− 113	+ 599	35	21	− 71	+ 714	Dec. 6, 7	+ 616
9	− 106	+ 425	12	12	...	...	9, 10	+ 125
13	− 62	− 642	33	32	− 28	− 600		− 502
15	− 39	− 257	78	70	+ 14	− 312	15, 16	− 205
17	− 74	− 623	16	1	+ 31	− 240		...
19	+ 44	+ 173	64	60	+ 24	+ 153		+ 34
21	− 176	+ 380	67	39	− 72	+ 418		+ 376
26	− 97	− 1255	4	3	...	...		...
28	− 214	+ 1613	24	3	− 169	+ 1503	28, 29	+ 376
Jan. 5	− 168	− 558	50	8	− 186	− 298		0
8	...	...	20	9	− 102	− 366	Jan. 8, 9	− 935
9	...	...	8	7	+ 102	− 508		...
14	...	...	39	21	− 48	+ 265	13, 14	+ 342
15	...	...	32	25	+ 66	− 64		+ 388
18	...	...	13	11	− 61	+ 1049	17, 18	+ [3158]

When these solutions are combined according to their weights they lead to the following:—

---

* 1 plate rejected from the second solution.

Greenwich contributions to

	Solution II.	Solution VII.
Weight	864	902
ϵ_1	±0·0249	±0·0215
$\Delta\pi$	+0″·0002±0″·0050	−0″·0003±0″·0043

The Greenwich solution of the same material gave

$$\Delta\pi = 0''{\cdot}000 \pm 0''{\cdot}0044.$$

The excellent accordance of these three solutions, referred to three different ephemerides, and three different determinations of tabular error, is most satisfactory. It shows that the Greenwich observations have an admirable stability: alterations in the method of discussion, the ephemerides, or the tabular error affect the results hardly at all.

§ 17. The data of the two solutions of the Paris photographs are given in the following table.

Table IV.

Solutions from Paris Photographs.

		Soltn. from Eph. Circ. 12.				Soltn. from New Eph.	
		Δα.	P.	Coefft.	Wt.	Δα.	P.
		s				s	
Oct.	17	−·0115	+·01868	8	7	−·0146	+·01774
	19	− 77	− 508	10	7	− 138	− 433
	22	+ 66	− 365	19	19	+ 144	− 399
	23	− 84	+ 294	18	18	− 38	+ 355
	25, 26	− 112	+ 109	18	14	− 43	+ 25
	27	− 117	+ 1691	17	16	− 15	+ 1098
Nov.	7	− 123	− 1427	35	8	− 93	− 1687
	8	− 179	− 2270				
	10	+ 7	− 133	9	7	− 2	+ 12
Dec.	7	+ 62	− 1745	37	3	...	...
	17, 18	...	...	14	14	+ 14	+ 1338
	18	+ 46	+ 1185	12	10	...	...
	19	+ 25	+ 21	13	13	+ 10	+ 41
	21	− 26	− 178	7	4	− 153	− 1817
	28	− 4	− 1687	17	8	+ 47	− 1810
Jan.	4	+ 34	+ 570	6	5	+ 11	+ 848
	5	− 269	+ 1730	24	3	− 259	+ 1731
	6	...	...	33	17	+ 37	+ 799
	9	...	...	16	12	+ 112	− 398
	13	...	...	11	9	− 71	+ 233
	14	...	...	23	15	+ 70	− 1124
	15	...	...	19	13	+ 95	− 407
	17	...	...	14	7	+ 12	− 1135

Combining these separate results according to their weight we have—

Paris contribution to

	Solution II.	Solution VII.
Weight	201	216
ϵ_1	$\pm 0\cdot0339$	$\pm 0\cdot0326$
$\Delta\pi$	$+0''\cdot0137 \pm 0''\cdot0142$	$+0''\cdot0002 \pm 0''\cdot0132$

§ 18. The Cambridge results are given in similar form :—

Table V.

Solutions from Cambridge Photographs.

	Solution Eph. Circ. 12.				Solution New Ephem.	
	Δα.	P.	Coefft.	Wt.	Δα.	P.
	s				s	
Oct. 12–15	+ ·0325	− ·01682	6	5	+ ·0241	− ·02014
27	− 66	+ 1103	24	23	− 52	+ 1060
Nov. 9	+ 16	+ 50	57	55	+ 18	+ 1
10	− 38	+ 428	57	56	− 49	+ 484
13	− 64	+ 324	28	26	− 42	+ 257
14, 15	− 120	− 7	38	35	− 109	− 14
22	+ 25	− 558	45	4	+ 95	− 610
23	+ 148	− 681	37	4	+ 159	− 697
Dec. 13	− 74	− 310	17	17	− 65	− 210
15	− 9	+ 118	35	32	+ 8	+ 89
16, 17	− 61	+ 584	22	21	− 41	+ 567
19	+ 48	− 35	7	6	− 1	− 109
21	+ 40	− 441	34	20	+ 114	− 424

These give the following :—

Cambridge contribution to

	Solution II.	Solution VII.
Weight	306	305
ϵ_1	$\pm 0\cdot0230$	$\pm 0\cdot0241$
$\Delta\pi$	$+0''\cdot0141 \pm 0''\cdot0078$	$+0''\cdot0119 \pm 0''\cdot0082$

§ 19. Inasmuch as I hope that the full details will be published at an early date, I have not thought it necessary to give the contributions of other observatories in quite such complete form. We may summarise them as follows: the series are not sufficiently complete for determination of their probable errors from internal agreement.

Contributions to	Solution II.		Solution VII.	
	Wt.	$\Delta\pi$.	Wt.	$\Delta\pi$.
Bordeaux	33	−0″·0906	23	−0″·0039
Helsingfors	22	+ 700	39	+ 262
Minneapolis	109	− 206	105	− 245
Northfield	72	− 350	35	− 165
Oxford	95	− 285	95	− 56
Poulkovo	15	+ 1100	4	− 658
San Fernando	1	+ 1408	...	...
Toulouse	33	− 134	41	− 363
Upsala	30	+ 669	39	+ 709

The residuum in Solution II., divided into 4-day groups and solved, gives—

Weight	447
ϵ_1	±0·0474
$\Delta\pi$	+0″·0239±0″·0133

The great increase in the value of ϵ_1 in this solution is no doubt partly due to the errors in star places for these scattered observations, but it must not be put down entirely to the discredit of the star places. The observatories whose results are less accurate, and have had perhaps rather too much weight given to them, contribute largely to the residuum.

§ 20. Solution V. divides the whole material into single-day groups; or occasionally, when the observations of a single day gave no solution, two or more days have been combined. In no case has a day which gave a fair solution by itself been joined with another. The whole available material is utilised in this solution. Normal equations were formed for each group and solved by least squares.

The differences between the coefficient of P in the normal for P, and the weight of the determination, give an interesting criterion of the waste of material due to want of balance of parallax factors.

Table VI.

Solutions from Single-Day Groups with New Ephemeris.

		Coefft.	Wt.	Δα.	P.
				s	s
Oct.	8	15	5	– ·0076	– ·02621
	9	11	4	+ 138	+ 1242
	10	20	11	+ 99	– 783
	11	13	8	+ 129	+ 590
	12	46	38	+ 16	+ 596
	13	46	46	+ 7	– 338
	14	60	57	– 59	+ 5
	15	86	86	+ 5	– 301
	16	66	60	– 11	+ 102
	17	11	10	– 108	+ 947
	18	17	11	– 10	– 596
	19	35	34	– 88	– 330
	20	8	8	– 79	+ 665
	21	121	121	+ 82	+ 401
	22	24	24	+ 120	– 37
	23	37	36	+ 20	+ 123
	24	77	77	+ 171	+ 499
	25	22	18	+ 73	+ 469
	26	223	217	+ 7	+ 99
	27	114	114	– 18	+ 607
	28, 30, 31	65	11	– 240	+ 1399
	29	127	88	– 8	+ 340
Nov.	1, 2, 4	49	43	+ 76	+ 4
	3	139	138	+ 206	+ 369
	5, 6	22	7	+ 163	+ 1680
	7	51	50	+ 35	– 340
	8	29	25	– 22	– 457
	9	121	111	+ 13	+ 44
	10	249	244	– 4	– 207
	11, 12	46	29	+ 3	+ 562
	13	232	231	+ 6	+ 75
	14	41	22	– 33	– 100
	15	85	46	– 48	– 460
	21, 24	14	12	+ 105	– 1475
	22	134	97	+ 56	– 124
	23	51	45	+ 102	– 149
	25, 26	32	32	+ 48	+ 189
	27	52	36	– 28	– 59
	28	120	119	– 64	+ 218
	29	166	163	– 80	+ 14
	30	29	29	– 110	+ 620
Dec.	1, 2	19	19	– 19	+ 46
	3, 4	3	2	+ 207	+ 5677
Dec.	5	140	136	+ ·0071	+ ·00356
	6	202	163	+ 73	+ 142
	7	217	169	– 6	+ 158
	8, 9	29	25	– 44	+ 96
	10	14	11	+ 68	+ 1535
	11, 12	16	14	+ 151	+ 414
	13	52	51	– 38	– 487
	14	7	2	+ 334	– 1277
	15	113	102	+ 15	– 193
	16	26	26	– 43	+ 429
	17	37	23	– 27	– 155
	18	39	31	+ 24	+ 27
	19	113	106	+ 26	– 140
	20	17	11	+ 47	– 1316
	21	110	88	– 7	+ 90
	22	9	8	+ 27	– 382
	23	16	13	+ 42	– 1349
	24	83	67	– 86	– 106
	25, 26	61	32	+ 46	– 497
	27	4	4	– 42	– 2390
	28	59	29	+ 16	– 596
	29	10	6	+ 4	+ 85
	30				
	31	22	14	– 21	– 544
Jan.	1, 2	19	15	+ 157	– 1670
	3				
	4	21	21	+ 55	– 231
	5	87	18	– 142	+ 33
	6	43	18	+ 16	+ 841
	7				
	8	20	13	– 189	+ 249
	9	28	24	+ 113	– 837
	10	5	2	+ 177	– 2935
	11				
	12	4	1	– 131	+ 408
	13	26	18	– 43	+ 509
	14	73	47	– 8	– 277
	15	53	42	+ 91	– 348
	16	7	6	– 88	– 60
	17	29	17	+ 14	+ 67
	18	14	13	– 49	+ 934
	19	4	3	– 142	+ 2583

When these solutions are combined according to their weight we have—

	Solution V.
Weight	3732
ϵ_1	±0·0274
$\Delta\pi$	+0″·0041±0″·0026

And when they are divided into four parts, and solutions in each part combined, we have—

	Solution VI.		
	Weight	ϵ_1	$\Delta\pi$
Oct. 1–Oct. 23	559	±0·0278	+0″·0039
Oct. 24–Nov. 16	1381	±0·0310	+0·0088
Nov. 17–Dec. 22	1416	±0·0245	+0·0048
Dec. 23–Jan. 19	376	±0·0274	−0·0158

§ 21. Solutions II. and VII. are composed of contributions from individual observatories, kept separate.

If we now combine these according to their weights we have—

	Solution III. Ephemeris of Circ. 12.	Solution VIII. New Ephemeris.
Weight	5244	3265
ϵ_1	±0·0354	±0·0259
$\Delta\pi$	+0″·0086±0″·0029	+0″·0073±0″·0027

Solution III. includes the residue. If we omit the residue we have—

Weight 4797 $\Delta\pi$ +0″·0072.

§ 22. Some of the individual solutions which contribute to the various parts of Solution VII. are discordant; and some of them depend upon observations made on more than one day, and involving different stars. If we omit all the latter, and also all those which give a $\Delta\pi$ differing from the mean by more than three times the mean error for a contribution of that weight, the principal series give the following revised results:—

	Solution IX.	
	Weight.	$\Delta\pi$.
Cambridge	300	+0″·0162
Greenwich	756	+ 30
Lick Series 2 . .	1378	+ 194
Paris	149	− 122

§ 23. Inspection of the internal probable errors of the principal contributions to Solution VII. suggests that the adopted weights need revision.

The mean errors for a determination of P of weight unity come out respectively—

Greenwich	±0·0215
Cambridge	241
Lick Series 2	297
Paris	326

If we bring the other three to the standard of Greenwich, we must multiply their respective contributions by factors proportional to the reciprocal of the squares of these quantities; *i.e.* by 0·794 and 0·523 and 0·435.

We thus obtain—

	Solution X.
Weight	2002
ϵ_1	±0·0215
$\Delta\pi$	+0″·0071 ± 0″·0028

§ 24. As a preliminary to bringing the final results together, we may note the following points.

Few observatories have been able to furnish a substantial independent contribution to the determination of the parallax itself, but the others have furnished the bulk of the observations for the star catalogue.

If a is the correction for parallax already applied, and the true parallax is $(1 + P) \times$ the assumed, our equations of condition become

$$\Delta a + aP + m = 0.$$

No term depending upon the time is retained; that is to say, it is assumed that the variation of the tabular error with the time has been completely determined and eliminated.

Each observation of the planet gives an equation of condition of the above form. We have next to decide how to group them.

It is probably safe to suppose that the variation of the tabular error is sufficiently well determined over intervals of four days. But different stars are used during that interval, and their errors will enter into the determination. If we restrict the grouping to intervals of one day we get rid of part of this effect, but at the expense of much loss of material. If we go further, and group separately the observations of each observatory for each day, we practically eliminate the star places, but with renewed heavy loss of material.

The adoption of the new ephemeris has somewhat diminished the probable error. But the improvement is not very great, and it is evident that the principal irregularities are inherent in the published values of O – C.

The adoption of the new ephemeris has somewhat increased the value of $\Delta\pi$. But this is due principally to the Lick series, which have shown themselves remarkably sensitive to the change.

The Greenwich series, on the other hand, are extremely stable. Whether they are reduced with the ephemerides of Circular 9, or of Circular 12, or with the new; or which determination of the tabular error is used, makes little difference.

§ 25. We will now bring together the principal results for comparison :—

	With Eph. Circ. 12.			With New Eph.		
		Δπ.			Δπ.	
4-day groups .	Sol. I.	+0″·0055	±0″·0036	Sol. IV.	+0″·0019	±0″·0029
Cambridge . .	Sol. II.	+0·0141	±0·0078	Sol. VII.	+0·0119	±0·0082
Greenwich . .		+ 2	50		− 3	43
Lick Series 2 .		+ 107	47		+ 148	46
Minneapolis .		− 206	...		− 245	...
Northfield . .		− 350	...		− 165	...
Oxford . .		− 285	...		− 56	...
Paris . . .		+ 137	142		+ 2	132
Combination of observatory solutions	Sol. III.	+0·0086	±0·0029	Sol. VIII.	+0·0073	±0·0031
1-day groups .	...	...		Sol. V.	+0·0041	±0·0026
Camb., Green., Lick., Paris, with new weights	...	...		Sol. X.	+0·0071	±0·0028

The accordance between the different combined solutions is in striking contrast with the divergence between the results from different observatories. I think that we may conclude that any outstanding improvements in ephemeris, tabular correction, or method of combination would have a very small effect upon the parallax. The serious question for discussion is—Why do Minneapolis and Northfield give a value of Δπ so low, and Lick and Cambridge a value so high?

I believe that we may find an explanation in the fact discovered at Greenwich, that the parallax deduced depends on the magnitude of the comparison stars. (See Greenwich Tables 28 and 29, *loc. cit.*) The fainter stars, at least near the centre of the plate, are at Greenwich displaced towards the zenith by comparison with the *repère* stars. Hence when bright *repère* stars are used in the reduction, the parallax comes out small. Can we extend this proposition, and suppose that when the comparison stars are fainter than the planet the parallax comes out large?

Minneapolis and Northfield used bright stars, and no reduction to standard with fainter stars has been possible. They give a small value of Δπ closely resembling that found at Greenwich from the *repère* stars.

Paris also used *repère* stars, but I have been able to make the reduction to standard. The Paris result is only a little small.

The Greenwich comparison stars were fainter than the *repère*, but probably a little brighter than the planet. The Greenwich result is a little small.

The Lick and Cambridge comparison stars were decidedly fainter than the planet. Their correction $\Delta\pi$ is decidedly large.

Our results are therefore consistent with the hypothesis that the deduced parallax depends to some extent upon the magnitude of the comparison stars.

We have decisive evidence only in the case of the Greenwich astrographic telescope; and the effect may, of course, be peculiar to the telescope. For the others it must be investigated, and the investigation is comparatively easy. We have only to find from the residuals in the solutions of the plates if the stars of outstanding magnitude, either bright or faint, have an apparent diurnal parallax. I hope to take up this question in the near future.

§ 26. But meanwhile we must decide what shall be taken as the best result from the above solutions.

After taking account of all the considerations above noted, and remembering that when we work with the mean of a number of exposures the result tends to be a little small (see Greenwich Observations of Eros, p. lx), it seems to me that we may adopt

$$\Delta\pi = +0''{\cdot}007 \pm 0''{\cdot}0027$$

as the most probable result of our solutions.

This is derived from the right ascensions alone. In the Lick and Cambridge series the declinations were not measured. At Greenwich they were discussed, and gave the same value of the parallax as the right ascensions, but with very little weight. I do not think it worth while to discuss the remaining observations of declination.

§ 27. *The Residual Tabular Errors.*—In some cases these are rather large, and might seem to require a further correction to the adopted tabular error. I think it has been sufficiently shown that this would not affect the parallax. We will therefore reserve the discussion of these residuals for the determination of the mass of the Moon, which will be undertaken as soon as the latest Paris ephemeris is received.

§ 28. *The supposed Inequality in the Place of the Planet depending on the Light-variation.*—In my early work I thought I had found evidence of such an inequality, with semi-amplitude about $0''{\cdot}03$ and period $2^h\ 38^m$. The much more extensive material now available does not confirm this at present. Perhaps the period is not precisely $2^h\ 38^m$, and there might very probably be secular terms in the epochs of minimum depending on the phase angle. Further investigation will be made; and I shall be grateful for any photometric observations of the planet made from October to December 1900. The published information is exceedingly slight.

§ 29. At the moment of completing the above solutions I received, by the kindness of Professor Perrine, an advance proof of *Lick Observatory Bulletin* 150. He gives as the result of his discussion of the Lick series,

$$\Delta\pi = +0''\cdot0067 \pm 0''\cdot0025.$$

The result is in excellent agreement with my general result, but decidedly different from the result which I have given above from my own solutions of the material which the Lick Observatory kindly placed at my disposal in advance of publication. The probable error is also very much smaller than that which I found. For the moment I am not able to suggest the reason for this divergence, which must be examined. But it may be interesting to call attention to the various probable errors for $\Delta\pi$ derived at Lick Observatory. Thus—

P.E. of $\Delta\pi$ derived from 126 equations (all)	$\pm 0\cdot0027$
,, 96 ,, (30 with large residuals rejected)	$\pm 0\cdot0018$
P.E. of $\Delta\pi$ derived from 18 daily means	$\pm 0\cdot0052$

Thus the P.E. derived from daily means, which corresponds practically to the method I have used, is actually larger than mine; I cannot suggest a reason why alternative methods should give a result so much smaller that they correspond to a weight four times as great.

§ 30. The present paper deals with the photographic results for the parallax. The discussion of the micrometric observations is practically complete, and will be presented to the Society without delay.

I hope also that the discussion of the mass of the Moon may be completed within a few months.

§ 31. In presenting the conclusions of the discussion upon which I have been engaged for so long, I must not omit to thank all my colleagues, who have shown me throughout the greatest kindness and given me the most cordial help. My best thanks are due also to the assistants who have been engaged in the calculations at Cambridge; to the Royal Society, which has made grants towards the cost of this assistance; and finally, to the Director of the Cambridge Observatory, who has permitted me to occupy a large part of my time in this work, and has given me every encouragement.

Cambridge Observatory:
1909 *May* 13.

The Constants of the Physical Libration of the Moon.

By F. J. M. Stratton, M.A.

(*Abstract.*)

The following are the results obtained from a fresh reduction of the observations of Mösting A made by Schlüter at Königsberg in the years 1841–3. The observed positions were reduced so as to give the apparent selenographical latitude (β) and longitude (λ) of the crater at each observation. Two reductions were undertaken: one was based on the previous work of Dr. Franz on these observations; in the second reduction an assumption of Dr. Franz with regard to the constancy of the focal setting of the heliometer was not adopted. One element, which had been overlooked by Dr. Franz, was taken into account in both reductions, namely, the excess (dh) of the radius of the moon to Mösting A over the observed mean radius to the limb.

Two solutions were made for each reduction. In the one case, the *unrestricted* case, the residuals between the observed β, λ and an assumed constant β, λ were analysed by the method of least squares for corrections to the assumed constants, and for certain periodic terms; no theoretical connection between the coefficients of these periodic terms was assumed in the analysis. The value of one of the unknown constants $\left(f \equiv \dfrac{(\mathrm{C}-\mathrm{B})\,\mathrm{B}}{(\mathrm{C}-\mathrm{A})\,\mathrm{A}}\right)$ was then derived from the coefficients of these periodic terms. In the second or *restricted* solution, a connection between the coefficients was assumed beforehand, according to the scheme worked out by Dr. Hayn,[*] and the values of f and of the other constants were obtained on this theoretical basis.

The following four solutions were thus obtained:—

1. Unrestricted solutions.

(*a*) With Dr. Franz's assumption:

From Latitude Equations.	From Longitude Equations.
$\beta = -3^\circ\,9'\,36'' \pm 47''$	$\lambda = -5^\circ\,9'\,18'' \pm 42''$
$\mathrm{I} = 1^\circ\,28'\,39'' \pm 70''$	$\mathrm{I} = 1^\circ\,35'\,2'' \pm 206''$
$f = 0{\cdot}51 \pm 0{\cdot}09$	$f = 0{\cdot}38 \pm 0{\cdot}04$
$dh = +8''{\cdot}8 \pm 2''{\cdot}2$	$dh = +1''{\cdot}9 \pm 2''{\cdot}2$
$\delta\beta = -106''\sin\omega - 17''\cos\omega - 77''\sin(g+\omega-\lambda)$	$\delta\lambda = -66''\sin g + 168''\sin g' - 26''\sin 2\omega$
p.e. of observation $= 61''$	p.e. of observation $= 89''$

---

[*] *Selenographische Koordinaten*, ii. p. 52.