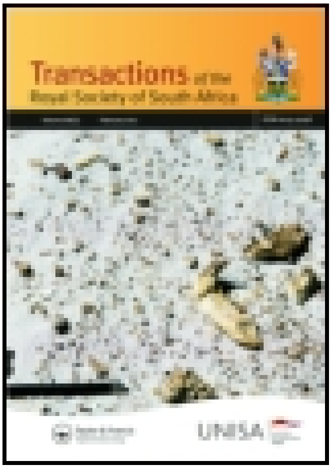

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ON A STEREOSCOPIC METHOD OF PHOTOGRAPHIC SURVEYING

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ON A STEREOSCOPIC METHOD OF PHOTOGRAPHIC SURVEYING.

By H. G. Fourcade.

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In the method proposed in this paper, photographs are taken, with a surveying camera, at a pair of points, the plates being exposed in the vertical plane passing through both stations. A *réseau*, or a graduated back frame, gives the means of measuring the co-ordinates of any point on the plates with reference to the optical axis of the camera. After development and fixing, the negatives, or positives from them, are viewed in a stereoscopic measuring machine, which, by combining the pictures, renders possible the instant identification of any point common to the pair of plates. Movable micrometer wires traverse each field, and pointings may be made simultaneously with both eyes. The readings of the micrometers, referred to the *réseau*, give the three co-ordinates of the point by direct multiplication by, or division from, constants for the plates which depend only on the focal length of the camera lens and the length of the base. When a sufficient number of points have been plotted from their co-ordinates, contour lines may be drawn.

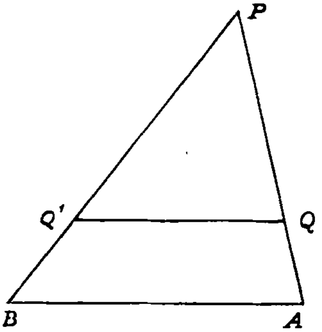

Fig. 1.

Theory of the Method.—Let A and B (Fig. 1) be the ends of the base and Q, and Q' the positions on the photographs of any point P.

Take A as origin and $A\,B$ as positive direction of x-axis.

Let (X, Y, Z) be the co-ordinates of P; $(x_a, f, z_a)(x_b, f, z_b)$ the co-ordinates of Q and Q'.

The equation of $A\,P$ is

$$\frac{x}{X}=\frac{y}{Y}=\frac{z}{Z},$$

and if we put $y=f$, we get

$$x_a=\frac{f}{Y}X$$

$$z_a=\frac{f}{Y}Z$$

Similarly the equation of $B\,P$ is

$$\frac{x-b}{X-b}=\frac{y}{Y}=\frac{z-h}{Z-h},$$

where b and h are the x and z co-ordinates of B.

Whence

$$x_b=\frac{f}{Y}(X-b)+b$$

$$z_b=\frac{f}{Y}(Z-h)+h.$$

From these equations we find

$$x_a-x_b+b=\frac{bf}{Y}=e.$$

e is the *stereoscopic difference*, constant for points in any plane perpendicular to $A\,y$ and vanishing for points at infinity.

The values of the co-ordinates of P follow

$$Y=\frac{b}{e}f$$

$$X=\frac{b}{e}x_a$$

$$Z=\frac{b}{e}z_a.$$

A check is afforded by the values of X and Z derived from $B\,P$.

$$X=\frac{b}{e}x'_b-b$$

$$Z=\frac{b}{e}z'_b-h,$$

x'_b and z'_b denoting here the co-ordinates of Q' referred to B.

The measurement of the co-ordinates of a point being made independently on each plate, although simultaneously, it will be a sufficient condition for the viewing apparatus to make corresponding portions of the two pictures combine with or without change of perspective.

Using a magnifying optical system to view the pair of plates, the condition for distinct vision is that the two images of any point appear in a *corresponding plane of vision*, so that the visual rays meet in space. This condition evidently remains satisfied when the images are magnified, or when they are brought nearer together along a line parallel to that joining the nodal points of the two eyes, and for different distances between the viewing lenses or the eyes, since in all these cases the lines joining the two images of a point remain parallel to the eyes.

Surveying Camera.—The essential features are a camera on a theodolite base, and a telescope with its line of collimation at right angles to the optical axis of the camera, so that by changing pivots the orientation of the pair of plates is not affected by errors of inclination, collimation, or graduation.

The photographic plate is pressed, during exposure, against a back frame in the focal plane of the camera lens by a spring contrivance similar to those used in other surveying cameras, which permits the shutter of the dark slide to be drawn and replaced. The *réseau* is hinged in front of the plate, its correct register being determined by

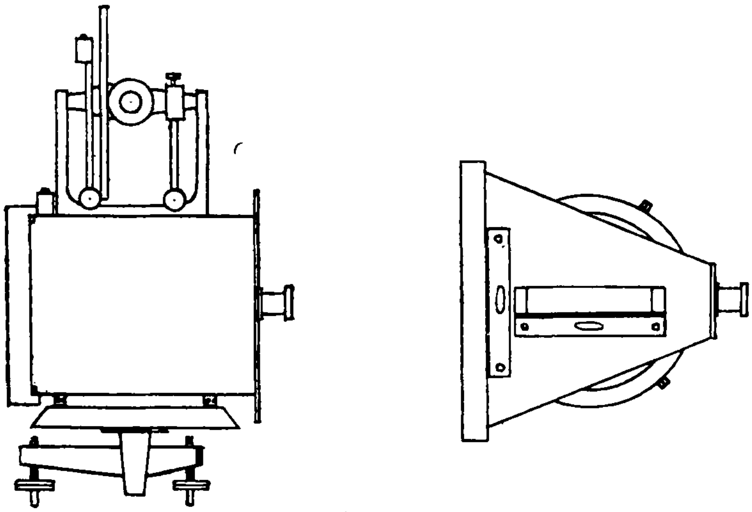

FIG. 2.

geometrical contacts. It is impressed upon the plate by exposure to sky-light reflected through the camera lens, and then moved out of the way for the exposure of the picture itself. A graduated front slide is used to displace the horizon line by moving the lens, but in normal circumstances it is set at the zero of its scale. Fig. 2 shows the general arrangement of the instrument.

Conditions to be satisfied.—One instrumental condition, sufficiently satisfied in construction, is that the front slide be parallel to the vertical *réseau* lines. Any defect in this respect is eliminated by determining the origin of the *réseau* co-ordinates and the focal length for different readings of the front scale.

The camera adjustments are: (1) *Plane of réseau to be vertical.* (2) *Horizon line of réseau to be horizontal.* These adjustments are made with the aid of a level, fitted with a Bohnenberger eyepiece.

The auxiliary level having been placed directly in front of the camera, and its line of collimation made horizontal, the vertical axis of the camera is set vertical by reference to the level of the vertical circle. Then (1) is effected by turning the camera in altitude with the foot-screws, and in azimuth, until the cross-wires of the level coincide with their image reflected from the silvered back surface of the *réseau*, when the bubble of the longitudinal level on the camera is adjusted to the centre of its run. Replace the front slide and lens and set again the vertical axis vertical. (2) is now effected by making the ends of the horizon line of the *réseau* coincide with the cross-wires of the level in two positions, using for the purpose the side capstan-headed screw in the base. The transverse level on the camera is then adjusted, and the longitudinal level made perpendicular to the vertical axis by means of the front capstan-headed screw under the camera.

The theodolite adjustments, effected by ordinary methods, are: (3) *Horizontal axis made perpendicular to vertical axis.* (4) *For collimation.* (5) *Horizontal axis made parallel with optical axis of camera.* An approximate adjustment of (5) is sufficient.

Instrumental Constants.—These are: (1) The *zero of the front scale;* (2) the *zero of the réseau* and co-ordinates of the R-points, and (3) the *focal length.* They may be determined in the usual manner, but it is convenient to first make the centre R-point coincide with the zero of the *réseau* co-ordinates, by collimating directly upon the *réseau* plate when adjusting the camera with the help of an auxiliary level, as already explained. In that case, the lens requires to be adjustable horizontally as well as vertically.

The focal length f is found from the measurement of exposed plates containing the images of well-defined points of which the angular distances are known. Call α the angle between two points of which the horizontal co-ordinates are a and b. Then—

$$f=\frac{a-b}{2\tan\alpha}+\sqrt{\frac{(a-b)^2}{4\tan^2\alpha}-ab}.$$

Measurement of the Plates.—It is unnecessary in a preliminary note such as this is, to enter into the construction of the measuring apparatus in much detail, as a description of actual instruments with examples of their use may fitly be given in a subsequent paper. A suitable machine would generally resemble those which have been used for the measurement of celestial photographs, and like such may be of various types.

In the type now considered, the plates are set side by side at an inclination corresponding to that of the base line and at heights such that corresponding R-points are horizontal. Both plate-carriers can slide about in a horizontal direction on a stage formed of a sheet of plate glass *g* (Fig. 3), which itself can be moved vertically by a double rack and pinion. Any small error in the setting of the plates and in the fitting of the slides will be automatically corrected by the position of the eyes in front of the eyepieces of the viewing microscopes, and by their power of accommodation, and does not affect the accuracy of the measurements.

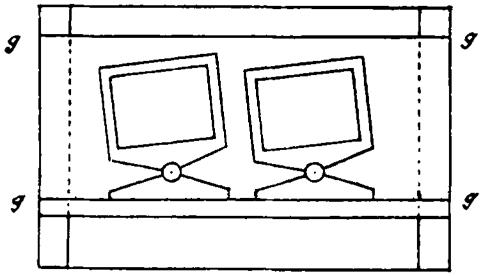

FIG. 3.

The measuring microscopes are of low power, and include in their field at least one clear R-square of 1 centimetre side. Their distance apart is adjustable to suit the eyes of the observer. One is fitted with a pair of micrometers at right angles capable of rotation in order to bring the horizontal and vertical wires parallel to the R-lines. The other is similarly fitted with the exception that one horizontal micrometer is sufficient. The runs are adjusted on a scale.

The centres of the plates are separated to a sufficient distance by

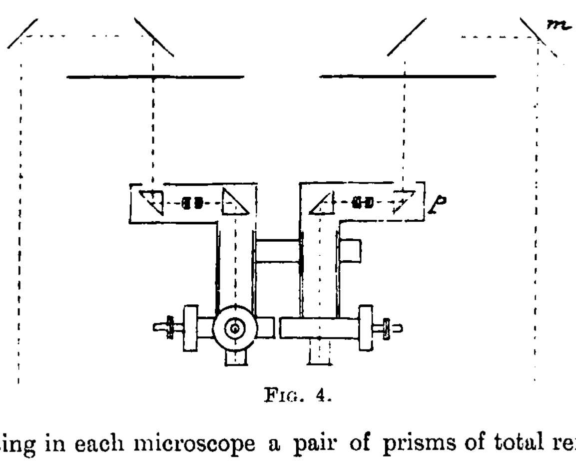

FIG. 4.

introducing in each microscope a pair of prisms of total reflection *p* (Fig. 4).

The micrometers might also be used in the position of the plates, giving more room for the screws and greater facility in the reading of their heads, and the plates themselves set further back, behind an additional lens, as in the Cambridge measuring machine recently described by Mr. Hincks (*Monthly Notices*, lxi. p. 444).

The zero wires form a frame fitting an R-square, as in Sir David Gill's machine used at the Cape Observatory (*Monthly Notices*, lix. p. 61).

For convenience, the whole arrangement is tilted at an angle of 45°, and the light illuminating the plates reflected by mirrors *m* from a window at the back of the observer.

The setting of the plates may be effected by turning a micrometer to the inclination of the base by means of a graduated circle, and making both sets of R-lines agree in inclination and height with the micrometer wires. The second micrometer is then set by making its wires parallel to the vertical R-lines on either plate.

The vertical R-lines are combined by the microscopes, but the horizontal lines only when the distance between the centres of the pictures is equal to that between the microscope object-glasses.* In making a measurement the plates are moved by the slow motion screws on the slides of their carriers and of the stage until the zero square of one microscope fits an R-square of the corresponding plate, and the zero wires of the other microscope coincide with a pair of vertical R-lines on the second plate. The points in the field of view may then be bisected without disturbing the zero settings.

The co-ordinates of any point on the plates are given by the direct readings of the micrometer heads added to the value of the R-lines considered. The stereoscopic difference results from the difference of the x's on the two plates.

Range of the Method.—In practice, the range of the method would be limited by the blurring of distant detail by light diffused in the atmosphere. This "aerial perspective" is reduced by the use of orthochromatic plates and an orange screen cutting off the rays of shorter wave-length which form the blue haze, but even then the effective range would probably not exceed some 5 miles or 8 kilometres.

On the other hand, the difference in phase of the objects would prevent their ready combination at distances less than three to four times the length of the base. The view would then correspond to that of a model seen with the eyes at a distance of 10 inches from the nearer edge.

* [Should be "between the lower optical axes of the microscopes."]

Let $2b$ be the length of the base and α the angle subtended by it at a distance y. Then—

$$y = b \cot \frac{\alpha}{2}$$

$$\frac{dy}{y} = -\frac{b}{y} \cdot \frac{d\alpha}{2 \sin^2 \frac{\alpha}{2}}$$

$$= -\frac{d\alpha}{\sin \alpha}.$$

Let $\frac{1}{100}$th of an inch or 0·25 mm. be the admissible error on the plan, 8 kilometres the limiting value of y, and $\Delta\alpha = 20''$. On the scale of the Canadian photographic surveys, $\frac{1}{40000}$, the maximum error allowable will be 10 metres at 8 kilometres, or $\frac{\Delta y}{y} = \frac{1}{800}$. Then $\alpha = 4° \ 27'$ and $2b = 620$ metres.

By increasing the base to 2 kilometres a maximum possible accuracy at 8 kilometers of $\frac{1}{2500}$ of the distance or 3 metres, would be attained, but the area mapped would be reduced to a narrow strip.

With the base of 620 metres, the area mapped with a plate of diameter equal to the focal length of the lens would be contained between the limiting circles at 8 and 2·5 kilometres shown at d and n (Fig. 5) and would amount to 22 square kilometres on either side of the base, or, more correctly, to that portion not masked by the nearer topographical features.

The error in x will be due to that in y and that of the x co-ordinate on the plate. We may write—

$$(\Delta x)^2 = \left(\frac{y}{f}\Delta l\right)^2 + \left(\frac{x}{y}\Delta y\right)^2.$$

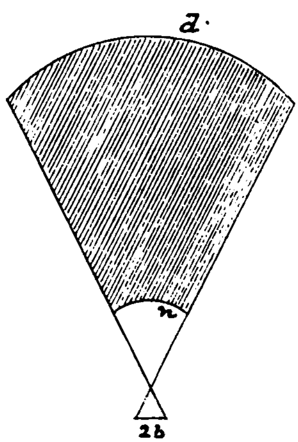

Fig. 5.

With a lens of 150 mm. focal length and an error of ·025 mm. in the plate x's, the maximum error is, for the base and the scale of plan considered, 5 metres, or on the plan 0·12 mm.

The error in height is given by the same expression. At the maximum distance the second term cannot exceed $\left(\frac{1}{4}\Delta y\right)^2$ if the difference in height between the base and the distant points does not exceed 2,000 metres. In absolute amount the total error for points at extreme distances would be $\pm$ 2·75 metres.

The contour lines should then, in the case already considered, be accurate to 0·25 mm. on slopes greater than 15°, but the actual accuracy will be reduced to some extent by the uncertainty of the correction for refraction. This correction, combined with that for curvature, can be applied at sight from a small table with y-argument.

By reducing the base, pairs of photographs may be taken within a confined space, as when mapping hidden valleys. The method can also be combined to any extent with the ordinary methods of photographic surveying. It would be of particular advantage in the mapping of large areas of mountainous country.