

single-ray velocity be taken as the axis of x'' we have

$$\left(\frac{a^2c^2}{b^2}x''^2 + b^2y^2\right)(x''^2 + y^2) - 2a^2c^2x''^2 - b^2(c^2 + a^2)y^2 + a^2b^2c^2 = 0$$

as the equation to the curves.

*Section containing the axis of single-ray velocity and
the axis of y.*

<i>Inner.</i>		<i>Outer.</i>	
x''	y	x''	y
0·00	2·00	0·00	4·00
1·00	1·85	1·00	3·85
1·16	1·80	1·16	3·80
1·59	1·59		
2·00	1·33	2·00	3·32
2·32	1·05	2·32	3·03
.		2·65	2·65
2·37	1·00	3·18	1·00
2·75	0·50	3·14	0·50
3·00	0·00	3·00	0·00
			} and 1·97

Angles are given to the nearest 10', and the other numbers to the second decimal place.

XLII. *On the Steady Motion of an Electrified Ellipsoid.*
By G. F. C. SEARLE, M.A., *Demonstrator in Experimental
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AT the Meeting of the Royal Society on 19th March, 1896, I read a paper on "Problems in Electric Convection." The first part of the paper is printed in the 'Philosophical Transactions of the Royal Society' †, and contains the principles which are required in the solution of any problems about moving charges. The second part of the paper, which deals with the motion of a charged ellipsoid, was not published by the Royal Society. A few of the results are, however, stated in an abstract published in the 'Proceedings' ‡. By the permission of the Royal Society I now publish my results for a moving ellipsoid. As frequent reference to my paper in the 'Philosophical Transactions' will be necessary, I shall use the notation §§ 5 and §(9) to indicate the paragraph or equation referred to in that paper.

* Communicated by the Physical Society: read June 25, 1897.

† Phil. Trans. vol. 187 (1896) A. pp. 675-713.

‡ Proc. Roy. Soc. vol. 59, p. 343.

When any system of electric charges moves with uniform velocity through the æther, the electromagnetic field, when referred to axes moving forwards with the charges, can be completely defined by means of a quantity Ψ , as was first shown by Prof. J. J. Thomson*. The electric force \mathbf{E} and the magnetic force \mathbf{H} are simple functions of Ψ . But besides \mathbf{E} and \mathbf{H} there is another vector of great importance, viz. the mechanical force \mathbf{F} experienced by a unit charge moving with the rest of the system. The value of \mathbf{F} I have shown {§ 10} to be given by the vector equation

$$\mathbf{F} = \mathbf{E} + \mu \mathbf{V} \mathbf{u} \mathbf{H}. \quad \dots \dots \dots (1)$$

The equations of the field are {§ 4}

$$\text{curl } \mathbf{F} = 0, \quad \dots \dots \dots (2)$$

$$\mathbf{H} = K \mathbf{V} \mathbf{u} \mathbf{E}. \quad \dots \dots \dots (3)$$

If $v = \frac{1}{\sqrt{K\mu}}$ is the velocity of light, and if α stand for $1 - \frac{u^2}{v^2}$, then when the motion takes place parallel to the axis of x , we have {§ 4}

$$F_1 = -\frac{d\Psi}{dx} \quad F_2 = -\frac{d\Psi}{dy} \quad F_3 = -\frac{d\Psi}{dz}, \quad \dots (4)$$

$$E_1 = -\frac{d\Psi}{dx} \quad E_2 = -\frac{1}{\alpha} \frac{d\Psi}{dy} \quad E_3 = -\frac{1}{\alpha} \frac{d\Psi}{dz}, \quad \dots (5)$$

$$H_1 = 0 \quad H_2 = \frac{Ku}{\alpha} \frac{d\Psi}{dz} \quad H_3 = -\frac{Ku}{\alpha} \frac{d\Psi}{dy}. \quad \dots (6)$$

From these equations, since \mathbf{E} has no divergence,

$$\alpha \frac{d^2\Psi}{dx^2} + \frac{d^2\Psi}{dy^2} + \frac{d^2\Psi}{dz^2} = 0. \quad \dots \dots \dots (7)$$

Here, and throughout the paper, the axes are supposed to move forward with the same velocity as the electrical charges.

Prof. W. B. Morton has considered the motion of an ellipsoid in a paper read before the Physical Society on 27th March, 1896 †. He obtains the two following results, viz. : (1) that the distribution of electricity is the same as if the ellipsoid is at rest, and (2) the value of Ψ when the ellipsoid moves along one of its axes.

Prof. Morton obtains his result by the assumption first

* Phil. Mag. July 1889.

† Proc. Phys. Soc. No. 71, August 1896, p. 180; Phil. Mag. xli. p. 488.

made by Mr. Oliver Heaviside, F.R.S. *, that a distribution of electricity on the surface of a charged body such as to give zero disturbance at all points inside the surface is an equilibrium distribution. Since \mathbf{F} satisfies $\text{curl } \mathbf{F} = 0$ and \mathbf{F} vanishes inside the surface, it follows that on the outside of the surface \mathbf{F} is perpendicular to the surface. This implies that Ψ is constant over the surface. But as neither the electric force \mathbf{E} nor the mechanical force experienced by each part of the charged surface (calculated from the Maxwell stress) is normal to the surface, I felt unable to accept the validity of Mr. Heaviside's assumption until I discovered {§ 15} that \mathbf{F} is the mechanical force on an isolated moving unit charge, and that the term $-\mathbf{VGD}$, which appears in the expression for the force experienced by the surface, has no influence in causing convection of electricity from one part of the surface to another. Here \mathbf{D} is the electric displacement, and \mathbf{G} the "magnetic current" $\mu \frac{d\mathbf{H}}{dt}$.

Since Ψ is a true potential for the mechanical force \mathbf{F} , I have called Ψ the "electric convection potential."

When there has been established the boundary condition that Ψ is constant over the surface, with its consequence that there is zero disturbance within the surface, it is very easy to show that the distribution on an ellipsoid is the same for motion as for rest. Suppose the ellipsoid to have the same distribution as when it is at rest, so that $\sigma = qp/4\pi abc$, where q is the charge, a, b, c the axes of the ellipsoid, and p the perpendicular from the centre upon the tangent-plane at the point. Through any internal point M as vertex draw a slender double cone intercepting two areas N, N' on the surface. Now the electric force due to a moving point-charge is still radial and still varies inversely as the square of the distance, although it alters with change of direction of the radius vector. Thus it follows just as in electrostatics, since $\sigma \propto p$, that the effects at M of N and N' are exactly equal and opposite. The whole surface can be treated in the same manner, and thus it follows that $\mathbf{E} = 0$ at all internal points. Hence $\mathbf{H} = 0$ also. Thus the assumed distribution is in equilibrium and is therefore the actual distribution. Thus the motion has no influence upon the distribution, and this result is true whatever the direction of motion with respect to the axes of the ellipsoid.

In order to find the state of the field near a charged ellipsoid moving with velocity u parallel to the axis of x , it is necessary to find a value of Ψ which shall be constant over

* 'Electrical Papers,' vol. ii. p. 514.

the surface of the ellipsoid, shall vanish at infinity, and shall satisfy (7). We see at once that if $f(x, y, z)$ satisfies $\nabla^2 f = 0$, then $f(x/\sqrt{\alpha}, y, z)$ satisfies (7). Now from electrostatics we know that

$$\Phi = \int_{\lambda}^{\infty} \frac{d\lambda}{\sqrt{(a^2 + \lambda)(b^2 + \lambda)(c^2 + \lambda)}},$$

where λ is connected with x, y, z by the relation

$$\frac{x^2}{a^2 + \lambda} + \frac{y^2}{b^2 + \lambda} + \frac{z^2}{c^2 + \lambda} = 1,$$

satisfies $\nabla^2 \Phi = 0$.

Hence

$$\Psi = \int_{\lambda}^{\infty} \frac{\Lambda d\lambda}{\sqrt{(a^2 + \lambda)(b^2 + \lambda)(c^2 + \lambda)}}, \dots (8)$$

where λ is connected with x, y, z by the relation

$$\frac{x^2}{(a^2 + \lambda)} + \frac{y^2}{b^2 + \lambda} + \frac{z^2}{c^2 + \lambda} = 1, \dots (9)$$

satisfies (7).

Writing a^2 for $\alpha a'^2$, (8) and (9) become

$$\Psi = \int_{\lambda}^{\infty} \frac{\Lambda d\lambda}{\sqrt{(a^2 + \alpha\lambda)(b^2 + \lambda)(c^2 + \lambda)}}; \dots (10)$$

$$\frac{x^2}{a^2 + \alpha\lambda} + \frac{y^2}{b^2 + \lambda} + \frac{z^2}{c^2 + \lambda} = 1. \dots (11)$$

This value of Ψ is constant over the surface of the ellipsoid a, b, c , for $\lambda = 0$ at all points of this surface; it also vanishes at infinity, and it satisfies (7). It is therefore the value of Ψ required. To find the constant Λ we make σ have its proper value $q/4\pi bc$ at the end of axis a .

Now

$$\sigma = \frac{K}{4\pi} E_n = \frac{K}{4\pi} E_1$$

at the end of the axis.

But by (5) $E_1 = -\frac{d\Psi}{dx}$.

Again, at $x = a, y = z = 0$ we have $d\lambda/dx = 2a/\alpha$ and consequently

$$\frac{d\Psi}{dx} = \frac{d\Psi}{d\lambda} \frac{d\lambda}{dx} = -\frac{\Lambda}{abc} \cdot \frac{2a}{\alpha}$$

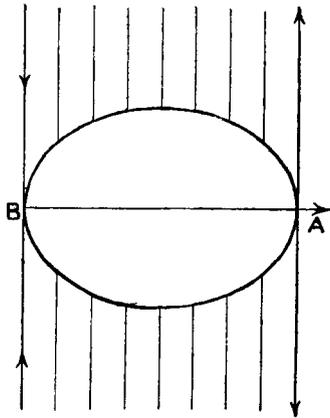
Hence $\Lambda = \frac{q\alpha}{2K}$.

Thus, as Prof. Morton has also shown by the same method,

$$\Psi = \int_{\lambda}^{\infty} \frac{q \alpha d\lambda}{2K \sqrt{(a^2 + \alpha\lambda)(b^2 + \lambda)(c^2 + \lambda)}} \dots (12)$$

Now I have shown {§ 21} that if there is a surface A carrying a charge q , and any surface B is found for which Ψ is constant, then a charge q placed upon B and allowed to acquire an equilibrium distribution will produce at all points not inside B the same effect as the charged surface A.

Fig. 1.



Hence the ellipsoid (11) when carrying a charge q produces at all points not inside itself exactly the same disturbance as the ellipsoid a, b, c with the same charge.

If we make $a=b=c=0$, the surfaces of equal "convection potential" are the ellipsoids given by

$$\frac{x^2}{\alpha} + y^2 + z^2 = \lambda.$$

They are therefore all similar to each other. Thus the ellipsoid of this form produces exactly the same effect as a point-charge at its centre, and thus an ellipsoid of this form takes the place of the sphere in electrostatics. An ellipsoid with its axes in the ratios $\sqrt{\alpha} : 1 : 1$ I have called a Heaviside Ellipsoid, since Mr. Heaviside* was the first to draw attention to its importance in the theory of moving charges. Whatever be the ratios $a : b : c$, the equipotential surfaces

* 'Electrical Papers,' vol. ii. p. 514.

approximate to Heaviside ellipsoids as λ is made very great.

The value of Ψ at the surface λ is $\frac{q\sqrt{\alpha}}{\sqrt{\lambda}}$.

Putting $c=b$ so that we have an ellipsoid of revolution, the axis of revolution being the axis of x , we see by taking $\lambda=-b^2$ that a uniformly-charged line of length $2\sqrt{a^2-b^2}$ lying along the axis of x produces exactly the same effect as the ellipsoid a, b, b . It may therefore be called its "image." When $b=a$ this length becomes $2au/v$. Thus, when a charged sphere is at rest it produces the same effect as a point-charge at its centre. When the sphere is in motion it produces the same effect as a uniformly-charged line whose length bears to the diameter of the sphere the same ratio as the velocity of the sphere bears to the velocity of light. When $u=v$, so that the sphere moves with the velocity of light, the line becomes the diameter of the sphere; and the same is true for an ellipsoid. Since when $u=v$ each element of the charged line produces a disturbance which is confined to the plane through the element perpendicular to the direction of motion {(46)}, it follows that the disturbance is entirely confined between the planes $x=\pm a$. Between them the electric force is radial to the axis of x and has exactly the same value, viz. $q/aK\rho$, as if the line had been of infinite length and had had the same line-density $q/2a$. Here ρ stands for $\{y^2+z^2\}^{\frac{1}{2}}$. The magnetic force is by (3) $qu/a\rho$. Hence the field between the planes $x=\pm a$ is independent of x . There are therefore no displacement-currents except in the two bounding-planes. There is an outward radial current in the front plane and an inward current in the back plane, the total amount of current in each case being qu , equal in amount to the convection-current carried by the ellipsoid.

It appears, however, that at the velocity of light *any* distribution on *any* surface is in equilibrium. For the value of Ψ at any point near a moving point-charge is {(43)}

$$\Psi = \frac{q\sqrt{\alpha}}{K\sqrt{x^2/\alpha+y^2+z^2}}$$

and this vanishes when $u=v$ (so that $\alpha=0$), even when $x=0$. Thus the value of Ψ for a point-charge vanishes, and the value of Ψ for *any* distribution being derivable from that for a point-charge by integration, it follows that Ψ has the constant value zero everywhere. Hence the charge is in equilibrium however it may be distributed. The same result follows from the expression {§ 19} for the force between two

moving charges. When they move parallel to each other with the speed of light the force between them vanishes.

If the ellipsoid is more oblate than Heaviside's the limiting internal surface of ellipsoidal form, whose action is the same as that of the ellipsoid, is a disk of radius $\sqrt{b^2 - a^2/\alpha}$, the axis of the disk coinciding with the axis of x .

The form of the lines of the electric force \mathbf{E} due to an ellipsoid of revolution is easily found. Putting ρ^2 for $y^2 + z^2$, the equilibrium surfaces are given by

$$\frac{x^2}{a^2 + \alpha\lambda} + \frac{\rho^2}{b^2 + \lambda} = 1. \quad \dots \quad (13)$$

Now the mechanical force \mathbf{F} is normal to this surface, and therefore

$$\frac{F_\rho}{F_1} = \frac{\rho(a^2 + \alpha\lambda)}{x(b^2 + \lambda)},$$

where

$$F_\rho^2 = F_2^2 + F_3^2.$$

But by (5),

$$E_1 = F_1 \quad \text{and} \quad E_\rho = F_\rho/\alpha;$$

so that

$$\frac{E_\rho}{E_1} = \frac{1}{\alpha} \frac{\rho(a^2 + \alpha\lambda)}{x(b^2 + \lambda)}. \quad \dots \quad (14)$$

Now consider the conic

$$\frac{x^2}{a^2 + \alpha\nu} + \frac{\rho^2}{b^2 + \nu} = 1. \quad \dots \quad (15)$$

The tangent of the angle which the geometrical tangent makes with the axis of x is

$$-\frac{x(b^2 + \nu)}{\rho(a^2 + \alpha\nu)}. \quad \dots \quad (16)$$

But if the point x, ρ lies on both (13) and (15), it follows that

$$-\frac{x(b^2 + \nu)}{\rho(a^2 + \alpha\nu)} = \frac{1}{\alpha} \frac{\rho(a^2 + \alpha\lambda)}{x(b^2 + \lambda)}.$$

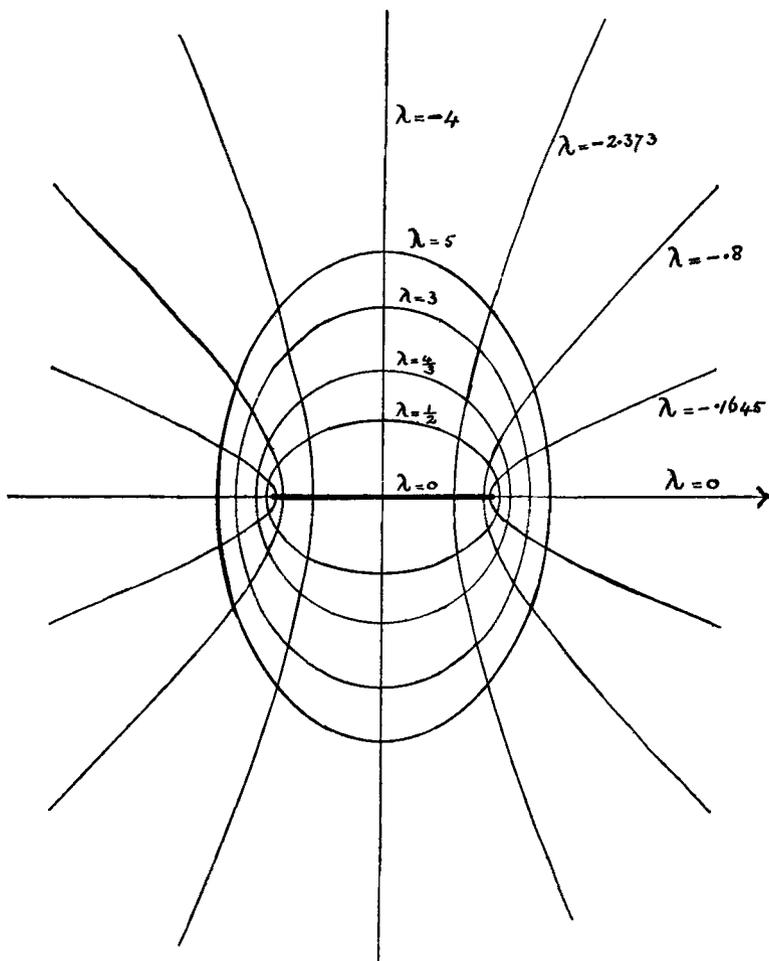
Hence by (14) and (16) the electric force is always tangential to the conic (15). But this conic has exactly the same equation as the equilibrium surfaces. Thus the single equation (13) represents both the equilibrium surfaces and the lines of electric force.

If any point x, ρ be taken, there are two values of λ which will satisfy (13) considered as a quadratic in λ . One value

corresponds to an ellipsoidal equilibrium surface ; the other to a hyperbolic surface whose lines of intersection with planes passing through the axis of x are the lines of electric force. The lines of electric force for a charged sphere in motion are not radial but form a series of hyperbolas.

Figs. 2 and 3 show the forms of the equilibrium surfaces and of the lines of electric force, for a line and a disk respec-

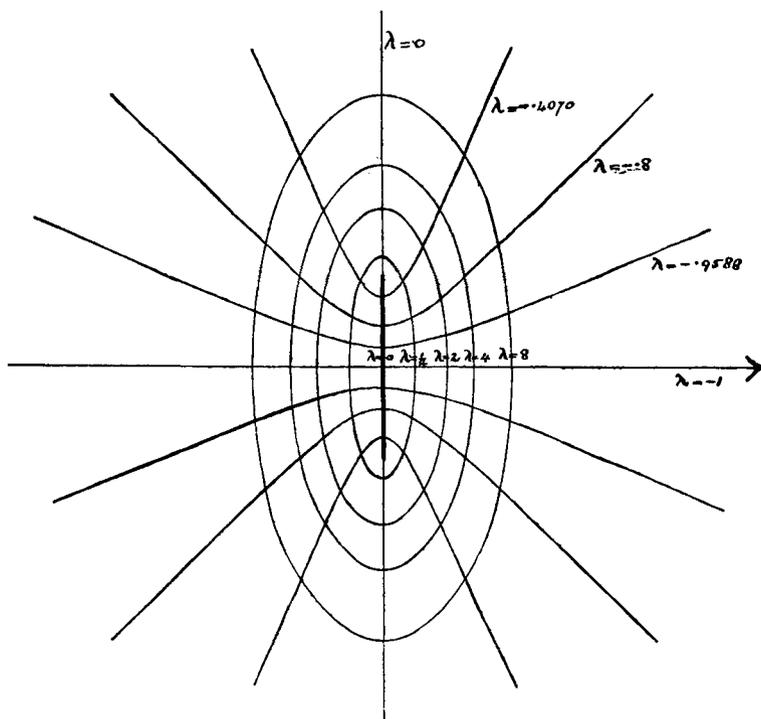
Fig. 2.



tively, when $\alpha = \frac{1}{4}$ so that $u/v = .866$. In fig. 2 the curve marked $\lambda = \frac{2}{3}$ is a circle, a section of the sphere of which the

line marked $\lambda=0$ is the "image." The semi-length of the line and the radius of the disk are each taken as unity.

Fig. 3.



I have attempted to find the lines of the mechanical force \mathbf{F} , these being everywhere perpendicular to the equilibrium surfaces. But the process involved an impracticable integration, and thus led to no result.

I will now write down the values of \mathbf{E} and \mathbf{H} at any point near the ellipsoid of revolution with axes a, b, b . Instead of λ it will be convenient to take as the parameter of any one of the equilibrium surfaces its x axis and to denote this by h . Thus

$$h^2 = a^2 + \alpha\lambda;$$

and consequently if we put l^2 for $a^2 - \alpha b^2$, so that l is the semi-length of the line which is the "image" of the ellipsoid, we have

$$b^2 + \lambda = \frac{h^2 - l^2}{\alpha}.$$

The value of Ψ in terms of h thus becomes

$$\Psi = \frac{q\alpha}{K} \int_h^\infty \frac{dh}{h^2 - l^2} \dots \dots \dots (17)$$

Equation (11) now becomes

$$\frac{x^2}{h^2} + \frac{\rho^2 \alpha}{h^2 - l^2} = 1, \dots \dots \dots (18)$$

so that instead of the cylindrical coordinates x and $\rho (= \sqrt{y^2 + z^2})$ we can take h and ϕ where

$$x = h \cos \phi, \quad \rho = \frac{\sqrt{h^2 - l^2}}{\sqrt{\alpha}} \sin \phi. \dots \dots (19)$$

From (18) we have in terms of h and ϕ

$$\frac{dh}{dx} = \frac{(h^2 - l^2) \cos \phi}{h^2 - l^2 \cos^2 \phi}, \quad \frac{dh}{d\rho} = \frac{h \sqrt{h^2 - l^2} \sin \phi \sqrt{\alpha}}{h^2 - l^2 \cos^2 \phi}.$$

Hence

$$E_1 = - \frac{d\Psi}{dh} \cdot \frac{dh}{dx} = \frac{\alpha q \cos \phi}{K(h^2 - l^2 \cos^2 \phi)}, \dots \dots \dots (20)$$

$$E_\rho = - \frac{1}{\alpha} \frac{d\Psi}{dh} \cdot \frac{dh}{d\rho} = \frac{qh \sin \phi \sqrt{\alpha}}{K \sqrt{h^2 - l^2} (h^2 - l^2 \cos^2 \phi)}, \dots (21)$$

$$H = K\alpha E_\rho = \frac{guh \sin \phi \sqrt{\alpha}}{\sqrt{h^2 - l^2} (h^2 - l^2 \cos^2 \phi)} \dots \dots \dots (22)$$

I now pass on to calculate the total energy possessed by the ellipsoid when in motion along its axis of figure. In making the calculation I shall suppose that $a^2 > ab^2$, i. e., that l^2 is positive. The case in which $a^2 < ab^2$ can be deduced by the appropriate mathematical transformation.

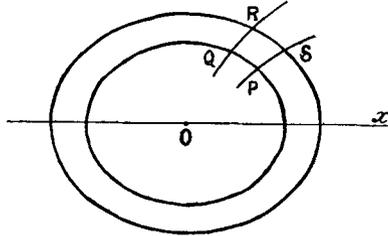
I have shown {§ 22} that the total energy, viz. the volume integral of $\frac{KE^2 + \mu H^2}{8\pi}$, due to the motion of a charge on any surface, is

$$W = \frac{1}{2}q\Psi_0 + 2T,$$

where Ψ_0 is the value of the convection-potential at the surface of the body, and T is the magnetic part of the energy, viz., the volume integral of $\mu H^2/8\pi$.

Now in fig. 4 let the ellipsoid PQ be determined by h , and the ellipsoid RS by $h + dh$. Let the angular coordinate of

Fig. 4.



P and S be ϕ , and let that of Q and R be $\phi + d\phi$. Then the area PQRS

$$\begin{aligned} &= \frac{d(x,\rho)}{d(h,\phi)} dh d\phi = \left(\frac{dx d\rho}{dh d\phi} - \frac{dx d\rho}{d\phi dh} \right) dh d\phi \\ &= \frac{h^2 - l^2 \cos^2 \phi}{\sqrt{\alpha} \sqrt{h^2 - l^2}} dh d\phi. \end{aligned}$$

Now if the area PQRS revolve about the axis Ox the volume of the ring traced out is

$$2\pi\rho \frac{d(x,\rho)}{d(h,\phi)} dh d\phi = \frac{2\pi(h^2 - l^2 \cos^2 \phi) \sin \phi}{\alpha} dh d\phi.$$

Thus for the magnetic part of the energy we have

$$\begin{aligned} T &= \iint \frac{\mu H^2}{8\pi} 2\pi\rho \frac{d(x,\rho)}{d(h,\phi)} dh d\phi \\ &= \frac{\mu q^2 u^2}{4} \iint \frac{h^2 \sin^3 \phi dh d\phi}{(h^2 - l^2)(h^2 - l^2 \cos^2 \phi)}. \end{aligned}$$

Since λ goes from 0 to ∞ h goes from a to ∞ . The limits of ϕ are 0 and π .

Now

$$\begin{aligned} \int_0^\pi \frac{\sin^3 \phi d\phi}{h^2 - l^2 \cos^2 \phi} &= -\frac{1}{l^2} \left[\cos \phi - \frac{h^2 - l^2}{2hl} \log \frac{h + l \cos \phi}{h - l \cos \phi} \right]_0^\pi \\ &= \frac{1}{l^2} \left\{ 2 - \frac{h^2 - l^2}{hl} \log \frac{h + l}{h - l} \right\}. \end{aligned}$$

Hence

$$\begin{aligned} T &= \frac{\mu q^2 u^2}{4l^2} \int_a^\infty \left(\frac{2h^2}{h^2 - l^2} - \frac{h}{l} \log \frac{h + l}{h - l} \right) dh \\ &= \frac{\mu q^2 u^2}{4l^2} \left[h - \frac{h^2 + l^2}{2l} \log \frac{h + l}{h - l} \right]^\infty. \end{aligned}$$

When h is large the quantity in []

$$= h - l \left(\frac{h^2}{l^2} + 1 \right) \left(\frac{l}{h} + \frac{l^3}{3h^3} + \dots \right)$$

vanishing when $h = \infty$.

Thus, making use of $\mu K v^2 = 1$ we have for the magnetic energy

$$T = \frac{q^2 u^2}{4lKv^2} \left\{ \frac{a^2 + l^2}{2l^2} \log \frac{a+l}{a-l} - \frac{a}{l} \right\}.$$

Now by (17) we have at the surface of the ellipsoid

$$\Psi_0 = \frac{q\alpha}{K} \int_a^\infty \frac{dh}{h^2 - l^2} = \frac{q\alpha}{2Kl} \log \frac{a+l}{a-l}.$$

Hence the total electromagnetic energy of the ellipsoid is

$$W = \frac{1}{2} q \Psi_0 + 2T = \frac{q^2}{4Kl} \left\{ \left(1 + \frac{u^2 a^2}{v^2 l^2} \right) \log \frac{a+l}{a-l} - 2 \frac{u^2 a}{v^2 l} \right\}. \quad (23)$$

Here we must remember that $l^2 = a^2 - ab^2$.

(A) *Energy of Heaviside Ellipsoid.* If we put $a/l = S$ and make S large we have

$$\begin{aligned} W &= \frac{q^2 S}{2Ka} \left\{ \left(1 + \frac{u^2 S^2}{v^2} \right) \left(\frac{1}{S} + \frac{1}{3S^3} + \dots \right) - \frac{u^2 S}{v^2} \right\} \\ &= \frac{q^2}{2Ka} \left(1 + \frac{1}{3} \frac{u^2}{v^2} \right) \text{ when } S = \infty. \quad \dots \quad (24) \end{aligned}$$

This corresponds to the Heaviside ellipsoid, for when $S = \infty$ $a^2 = ab^2$. The energy of the same ellipsoid at rest is

$$\frac{q^2 \sqrt{a}}{2Ka} \cdot \frac{v}{u} \sin^{-1} \frac{u}{v}.$$

(B) *Energy of a Sphere.* Putting $b = a$ we have $l = au/v$, and thus

$$W = \frac{q^2}{2Ka} \left(\frac{v}{u} \log \frac{v+u}{v-u} - 1 \right). \quad \dots \quad (25)$$

If u is small compared with v we have

$$W = \frac{q^2}{2Ka} \left(1 + \frac{2}{3} \frac{u^2}{v^2} + \dots \right).$$

It will be found that as far as u^2/v^2 the magnetic energy is

$$\frac{q^2 u^2}{3Kav^2} = \frac{\mu q^2 u^2}{3a}$$

as has been found by Mr. Heaviside*. It follows from this

* 'Electrical Papers,' vol. ii. p. 505.

that as far as terms in u^2/v^2 the electric part of the energy is unaltered by the motion.

(C) *Energy of a very slender Ellipsoid.* When the ellipsoid is so slender that b^2/a^2 may be neglected in comparison with unity we have

$$W = \frac{q^2}{2Ka} \left\{ \left(1 + \frac{u^2}{v^2} \right) \log \frac{2a}{b \sqrt{1 - \frac{u^2}{v^2}}} - \frac{u^2}{v^2} \right\}. \quad (26)$$

When u/v is small, this becomes

$$W = \frac{q^2}{2Ka} \left\{ \left(1 + \frac{u^2}{v^2} \right) \log \frac{2a}{b} + \frac{1}{2} \frac{u^2}{v^2} \right\}.$$

(D) *Energy of a Disk.*

When $a^2 < ab^2$ the ellipsoid is more oblate than Heaviside's, and l^2 becomes negative. In this case let us write

$$r^2 = b^2 - \frac{a^2}{\alpha},$$

so that r is the radius of the disk which is the "image" of the ellipsoid a, b . Then writing $\sqrt{-1} = i$ we have from (23)

$$W = \frac{q^2}{4Kir \sqrt{\alpha}} \left(1 - \frac{u^2 a^2}{v^2 r^2 \alpha} \right) \log \frac{1 + i \sqrt{\alpha} r/a}{1 - i \sqrt{\alpha} r/a} + \frac{q^2 u^2 a}{2Kv^2 r^2 \alpha}.$$

But

$$\frac{1}{i} \log \frac{1 + xi}{1 - xi} = 2 \left(x - \frac{x^3}{3} + \frac{x^5}{5} \dots \right) = 2 \tan^{-1} x,$$

so that (23) becomes

$$W = \frac{q^2}{2Kr \sqrt{\alpha}} \left\{ \left(1 - \frac{u^2 a^2}{v^2 r^2 \alpha} \right) \tan^{-1} \frac{r \sqrt{\alpha}}{a} + \frac{u^2 a}{v^2 r \sqrt{\alpha}} \right\}. \quad (27)$$

When $a=0$ we find for the energy of a disk of radius r moving along its axis

$$W = \frac{q^2 \pi}{4Kr \sqrt{\alpha}}. \quad (28)$$

In all these cases it will be found that when $u=v$ the energy becomes infinite, so that it would seem to be impossible to make a charged body move at a greater speed than that of light.