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XXV. *Notes on Electricity and Magnetism.*—III. *On the Behaviour of Iron and Steel under the Operation of Feeble Magnetic Forces.* *By* Lord RAYLEIGH, *Sec. R.S.*[*]

[Plate II.]

THE question whether or not iron responds proportionally to feeble magnetic forces is of interest not only from a theoretical point of view, but from its bearing upon the actual working of telephonic instruments. Considerable difference of opinion has been expressed concerning it, several of the best authorities inclining to the view that a finite force is required to start the magnetization. Prof. Ewing remarks[†]:—"As regards the hysteresis which occurs when the magnetism of soft iron is changed, my experiments confirm the idea already suggested by other observers, that when the molecular magnets of Weber are rotated they suffer, not first an elastic and then a partially non-elastic deflection as Maxwell has assumed, but a kind of frictional retardation (resembling the friction of solids), which must be overcome by the magnetizing force before deflection begins at all." In a subsequent passage[‡] Prof. Ewing treats the question as still open, remarking that though his curves suggest that the initial value of k (the susceptibility) may be finite, they afford no positive proof that it is not initially zero, or even negative.

My attention was first called to the matter about a year and a half ago in connection with the operation of iron cores

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[*] Communicated by the Author.

[†] Phil. Trans. 1886, p. 526, § 5.

[‡] *L. c.* § 61.

in the coils of an induction-balance. Experiment showed that iron responded powerfully to somewhat feeble forces; and I endeavoured to improve the apparatus in the hope of being able thus to examine the subject more thoroughly. Two similar long helices were prepared by winding fine insulated wire upon slender glass tubes. These were connected in series with a battery, a resistance-box, and a microphone-clock, so as to constitute a primary circuit. The secondary consisted of a large quantity of copper wire, mounted upon a bobbin, through the opening in which both primary coils were inserted. The circuit of the secondary was completed by a telephone. When neither primary coil contained a core, silence at the telephone could readily be obtained. The iron cores used were those described in Part II.*; and it was found that all of them (including the bundle of seventeen very fine wires) disturbed the silence until the resistance was so far increased that the magnetizing force was less than about $\frac{1}{50}$ of the earth's horizontal force (H). Moreover, there was no indication that the absence of audible effect under still smaller magnetizing forces was due to any other cause than the want of sensitiveness of the apparatus.

I did not pursue the experiments further upon these lines, because calculation showed that the feeble magnetization of a piece of iron could more easily be rendered evident directly upon a suspended needle (the magnetometric method), than indirectly by the induction of currents in an encompassing coil connected with a galvanometer. Nearly all the results to be given in this paper were obtained by a form of the magnetometric method, specially adapted to the inquiry whether or not the magnetization of iron continues proportional to the magnetizing force when the latter is reduced to the uttermost.

The magnetizing-spiral first used was one of those already referred to. It consists of a single layer of fine silk-covered copper wire wound on a glass tube and secured with shellac varnish (A, Plate II. fig. 1). The total length of the spiral is 17 centim., its diameter is about ·6 centim., and the windings are at the rate of 32 per centim. The resistance is about $5\frac{1}{2}$ ohms.

The magnetometer was simply a small mirror backed by steel magnets (B), and suspended from a silk fibre, as supplied by White for galvanometers. It was mounted between glass plates at about 2 centim. distance from the magnetizing-spiral. The earth's force was compensated by steel magnets, which also served to bring the mirror perpendicular to the helix in spite of the influence of residual magnetism in the iron core.

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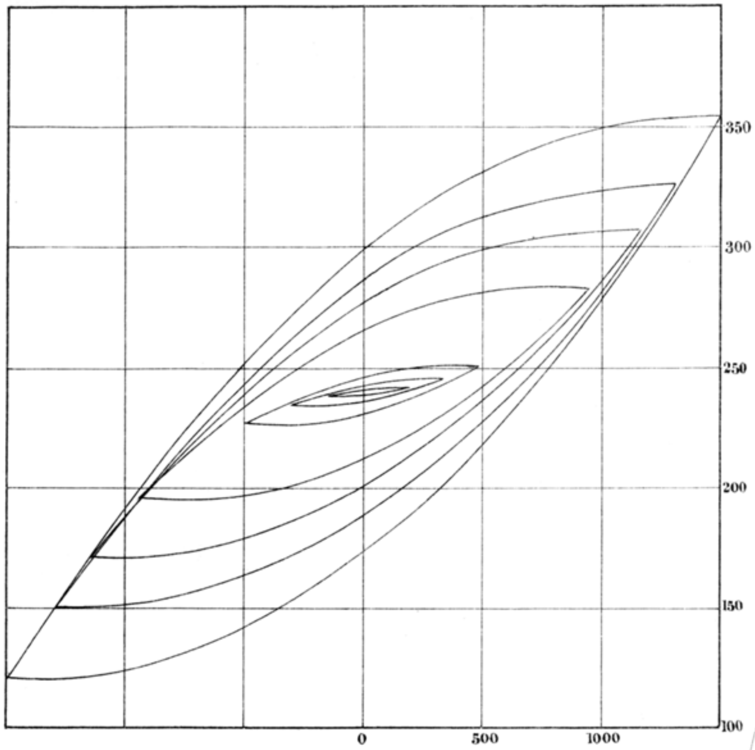
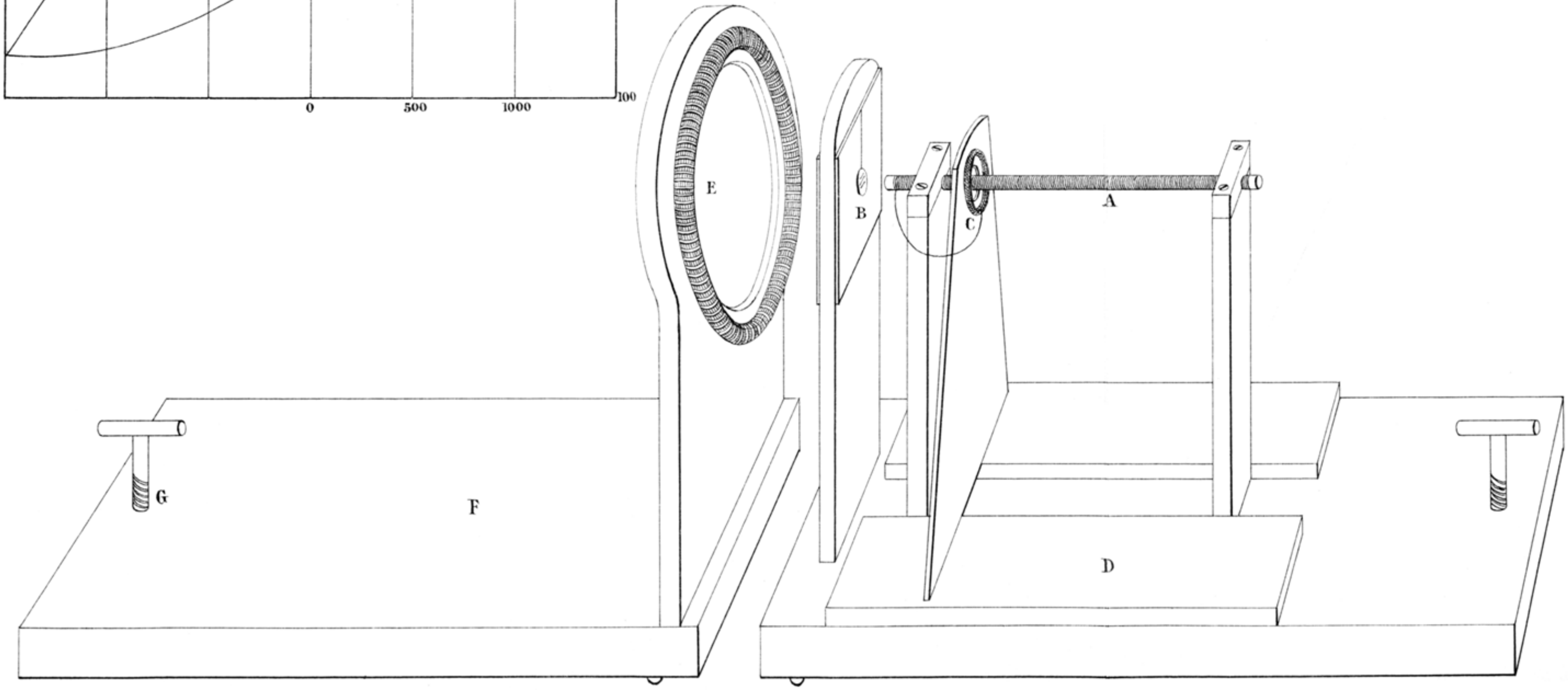
* Phil. Mag. December 1886, p. 490.



Unannealed Iron.
(Jan. 4.)

Fig. 2.

Fig. 1.



Mintern Bros. lith.

The deflections were read in the manner usual with Thomson's galvanometers, by the motion of a spot of light thrown upon a scale after reflection by the mirror. The division is in millimetres, and with the aid of a lens a displacement of $\frac{1}{10}$ of a division can usually be detected with certainty.

The direct effect of the magnetizing-spiral upon the suspended needle was compensated by a few turns of wire C, 7 centim. in diameter, supported upon an adjustable stand D. This adjunct might have been dispensed with; but what is essential is the larger coil, E, by which the effect of the *iron core* is compensated. This coil consisted of 74 convolutions, of mean diameter 18 centim., tied closely with string, and mounted upon an independent stand, F. By sliding this stand, and ultimately by use of the screw, G, the action of this coil upon the suspended needle can be adjusted with precision. All the coils are connected in series; and provided that the magnetic condition of the iron under given force is definite, matters may be so arranged that the imposition of the force produces no movement of the suspended needle, or, more generally, the compensation may be adjusted so as to suit the transition from any one magnetic force to any other. If the susceptibility (k) and permeability μ ($=4\pi k+1$) were constant, as has often been supposed in mathematical writings, the compensation suitable for any one transition would serve also for every other, and the magnetometer-needle would remain undisturbed, whatever changes were permitted in the strength of the magnetizing current*. The question now presenting itself is, How far does this correspond to fact? or, rather, How far is it true for magnetizing forces which are always very small? for we know already that, under the operation of moderate forces exceeding (say) 1 or 2 C.G.S., not only is μ not constant, but there is no definite relation at all between magnetic induction and magnetizing force, whereby the one can be inferred from the other without a knowledge of the previous history of the iron.

The magnetizing force of the spiral is of course easily calculated. The difference of potential in passing through n convolutions of current C is $4\pi n\mathrm{C}$. If the n convolutions occupy a length l, the magnetizing force is

$$4\pi\mathrm{C}\frac{n}{l};$$

or, in the present case, $128\pi\mathrm{C}$.

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* The idea of compensating the iron is not new. The method was employed by Koosen (Pogg. *Ann.* Bd. lxxxv. S. 159, 1852) to exhibit the phenomena of "saturation."

C is here expressed in C.G.S. measure, on which scale the ampere is ·1.

It may be objected that the magnetic force of the spiral is not the only external force operative upon the iron. It is true that the compensating-coils must have an influence, and in the opposite direction. But calculation shows that the influence must be small. The radius of the large coil is 9 centim., and (to take an example) the distance of its mean plane from the suspended needle in one set of experiments on hard iron was 13·6 centim. Under these circumstances the magnetic force in the spiral, even at the nearer end, is influenced less than 2 per cent. by the large compensating-coil. The effect of the smaller coil is about the same. For the present purpose it is hardly worth while to take these corrections into account.

As has been remarked, the coils of the apparatus were always connected in series; but a reversing-key (serving also to make and break) was introduced so as to allow of the reversal of the compensating-coil in relation to the others. In one position of the key (−) the action of the coil and of the magnetized iron are opposed; in the other (+) the actions conspire. When the currents to be used were not exceedingly small, the whole apparatus was in simple circuit with a Daniell cell and such resistance-coils as were necessary. Exclusive of the cell and of the added resistances, the whole resistance was 7½ ohms.

As an example, I will now give the details of some observations on December 6 made to test the behaviour of unannealed Swedish iron wire. The diameter of the wire is 1·6 millim.; it is from the same hank as a piece used in the experiments of Part II.* The compensating-coil was adjusted until it made no difference whether the key was open or closed (−), the additional resistance being 1000 ohms. In stating the result it will for the present be sufficient to give the German-silver resistances, that of the apparatus and of the battery being relatively of no importance. The corresponding current is about 10^{-4} C.G.S., and the strength of the magnetic field in the spiral is given by

$$128\pi C = \cdot 04 \text{ C.G.S.}$$

We shall have a better idea of this if we recall that, on the same system of measurement,

$$H = \cdot 18\,;$$

so that the force in action is about $\frac{1}{5}$ of that which the earth exercises horizontally.

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* *L. c.* p. 488.

When the resistance was altered to 11,000 ohms, the compensating-coil of course remaining undisturbed, contact (−) produced no visible motion, showing that the same compensation is suitable for the much smaller force. But at this point we require to be assured that the absence of disturbance is not due merely to want of sensitiveness. The necessary information is afforded at once by making reversed contact (+), which (with 11,000 ohms) gave a swing of 57 divisions.

To diminish the magnetizing force still further, a shunting arrangement was adopted. The current from the Daniell was led through 10,000 ohms and then through a box capable of providing resistances from 1 to 1000. The circuit of the apparatus included another coil of 10,000 ohms, and its terminals were connected to those of the box. The battery-current was thus about ·0001 ampere, or 10^{-5} C.G.S. If a be the (unplugged) resistance in the box, the E.M.F. at the terminals of the apparatus-circuit is $a \times 10^{-4}$ volts; and the current C through the magnetizing helix and compensating-coil is $a \times 10^{-9}$ C.G.S.

When $a = 1000$ ohms, (−) gave no visible deflection, while (+) caused a swing of 5 divisions.

At this stage recourse was had to the "method of multiplication" in order to increase the sensitiveness*. A pendulum was adjusted until its swings were synchronous with those of the suspended needle. It was then easy to make and break contact in such a way as to augment the swing due to any outstanding force. Thus, when $a = 1000$, the swing was increased by the use of the timed contacts and ruptures (+)

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* The advantage of the method of multiplication seems to be hardly sufficiently appreciated. It is not merely that the effect is presented to the eye in a magnified form. That object can be attained by optical appliances, and by diminishing the directive force upon the suspended parts, whether by using a nearly astatic system of needles, or by compensating the field. For the most part these devices augment the unavoidable disturbances (which exhibit themselves by a shifting zero) in the same proportion as the effect to be measured, or at any rate rendered apparent. The real ultimate impediment to accuracy of measurement is almost always the difficulty of distinguishing the effect under examination from accidental disturbances, and it is to overcome this that our efforts should be directed. The method of multiplication is here of great service. The desired effects are largely magnified, while the disturbances, which are not isoperiodic with the vibrations of the needle, remain unmagnified, and therefore fall into the background.

It is obvious that, in order to secure this advantage, the vibrations must not be strongly damped. No doubt a highly damped galvanometer-needle is often convenient, and sometimes indispensable. But it seems to be a mistake to use it where a null method is applicable, and when the utmost delicacy is required. In such a case the inertia of the needle, and the forces both of restitution and of damping, should all be made small.

until it measured 26 divisions instead of 5 only. But a similar series of operations with reversed currents (−) caused no swing amounting to $\frac{1}{10}$ division ; so that we may consider the compensation proved to be still perfect to about 1 per cent.

In applying the method to still smaller forces we cannot avoid a loss of sensitiveness. With $a=100$, (+) gave 3 divisions, while the effect of (−) remained insensible. The correctness of the compensation is thus verified to about 6 per cent. of the separate effects. Had the iron, even at this stage, refused to accept magnetization, the fact would have manifested itself by the equality of the swings obtainable in the two ways, (+) and (−), of making the connections.

In the last case mentioned the current was 10^{-7} C.G.S., and the magnetic force was 4×10^{-5} C.G.S. We may therefore regard the proportionality of magnetic induction to magnetic force over the range from $\frac{1}{5}$ H to $\frac{1}{5000}$ H as an experimental fact. In view of this, neither theory nor observation give us any reason for thinking that the proportionality would fail for still smaller forces.

Quite similar results have been obtained with steel. On December 13 a piece of drill steel (unannealed) was examined, the delicacy of the apparatus, as evidenced by the (+) effect, being about the same as in the above experiments on hard Swedish iron. No failure of proportionality could be detected with forces ranging from about $\frac{1}{5}$ H to $\frac{1}{10000}$ H.

Annealed iron is a much less satisfactory subject. With unannealed iron and steel the compensation for small forces may be made absolute, so that neither at the moment of closing the circuit nor afterwards is there any perceptible disturbance. This means that (so far as the magnetometer-needle can decide) the metal assumes instantaneously a definite magnetic condition which does not afterwards change. But soft iron shows much more complicated effects. The following observations were made upon a piece of Swedish iron (from the same hank as the former) annealed in the flame of a spirit-lamp. When an attempt was made to compensate for the imposition of a force equal to $\frac{1}{5}$ H, no complete balance could be obtained. When the coil was so placed as to reduce as much as possible the instantaneous effect, there ensued a drift of the magnetometer-needle represented by about 170 divisions of the scale, and in such a direction as to indicate a continued increase of magnetization. Precisely opposite effects followed the withdrawal of the magnetizing force. The settling down of the iron into a new magnetic state is thus shown to be far from instantaneous. On account of the complication entailed by the free swings of the needle, good observations on the drift could not be obtained with this apparatus; but it was evident that, whilst most of the anomalous action was over in 3 or 4 seconds, the final magnetic state was not attained until after about 15 or 20 seconds*.

The operation of feebler forces was next examined, rather with the expectation of finding the drift reduced in relative importance. But the imposition of $\frac{1}{50}$ H was followed by a drift of 13 or 14 divisions, no very small fraction of the whole action; as was seen from the observation that the (+) effect was now 300 divisions, of which 150 are due to the iron. With 20,000 ohms in circuit, giving a force equal to $\frac{1}{100}$ H, the drift was 6 or 7 divisions. By still further diminishing the force the drift could be reduced to insignificance; but it appeared to maintain its proportion to the instantaneous effect. Apart from the complication due to the drift, the magnetization was proportional to magnetizing force from $\frac{1}{10}$ H to $\frac{1}{5000}$ H or less†.

The question now presents itself, What is the actual value of the permeability which has been proved to be a definite constant for small forces? In consequence, however, of the nearness of the operative pole to the suspended needle in the preceding experiments, no moderately accurate value of μ can be deduced. But the observations described in Part II. are sufficient to show that the constant permeability for hard iron has some such value as 90 or 100, the forces then operative being within the prescribed limits. The fact that the initial value of μ is so large is obviously of great theoretical and practical importance. Further evidence will be brought forward presently in connection with observations made with an arrangement better suited to an absolute determination.

Too definite a character must not be ascribed to the above-mentioned limit of $\frac{1}{5}$H. Below this point the deviations from the law of proportionality, though mathematically existent, are barely sensible. In order to understand this, it is well to consider what happens when the limit is plainly exceeded. If a force of the order H be imposed, the compensating-coil (adjusted for small forces) appears to be overpowered, and a

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* Prof. Ewing (*loc. cit.* § 52) describes "a time lag in magnetization," especially noticeable in the softest iron and at points near the beginning of the steep part of the magnetization-curve. It should have been stated that my apparatus was very firmly supported, and, being situated underground, was well protected from vibration. The drift or creeping did not appear to be due to this cause.

† The results here set forth were announced in a discussion following Prof. Hughes's address to the Society of Telegraph Engineers on February 11, 1886 (Journ. Tel. Eng. xv. p. 39), on the strength of preliminary experiments tried towards the close of 1885.

large deflection occurs. If the force be now removed, the recovery is incomplete, indicating that the iron retains residual magnetism. Subsequent applications and removals of the force produce a nearly regular effect, and always of such a character as to prove that the magnetic changes in the iron *exceed* those demanded by the law of proportionality. As might be expected, the excess varies as the *square* of the force; and thus, when the force is small enough, it becomes insignificant, and the law of proportionality expresses the facts of the case with sufficient accuracy. But the precise limit to be fixed to the operation of the law depends necessarily upon the degree of accuracy demanded.

The readings with and without the force being tolerably definite, it would of course be possible, by pushing in the compensating-coil, to bring about an adjustment in which the application or removal of the force causes no deflection. But this state of things must be carefully distinguished from the compensation obtainable with very small forces, in that it is limited to one particular step in the magnitude of force. If we try a force of half the magnitude, we find the compensation fail. Not only so, but the reading will be different under the same force according as we come to it from the one side or from the other. The curve representing the relation between force and magnetization is a loop of finite area.

Except for the purpose of examining whether the whole magnetization is assumed instantaneously (absence of drift), there is little advantage in the compensation being adjusted for the extreme range under trial. It is usually better to retain the adjustment proper to very small forces. Even though it fails to give a complete compensation, the coil offers an important advantage, which will presently appear; and its use diminishes the displacement to be read upon the scale.

We have seen that when the forces are very small there is a definite relation between force and magnetization, of such a character that one is proportional to the other: the ratio k (the susceptibility) is a definite constant. When, however, certain limits are exceeded there is no fixed relation between the quantities; and if k is still to be retained, it requires a fresh definition. It is not merely that k, as at first defined, ceases to be constant, but rather that it ceases to exist. Upon this point the verdict of experiment is perfectly clear. There is no curved line by which the relation between force and magnetization can be unambiguously expressed, and which can be traversed in both directions. As soon as the line ceases to be straight, it ceases also to be single. I have thought it desirable to emphasize this point, because the term "magnetization-function," introduced by Dr. Stoletow, rather suggests a different conclusion.

The curves given by Stoletow and by Rowland in their celebrated researches are not exactly magnetization-curves in the more natural sense; that is to say, they do not exhibit fully the behaviour of a piece of iron when subjected to a given sequence of magnetic forces. But a number of such curves have been drawn by Ewing which afford all necessary general information. Among these we may especially distinguish the course followed by the iron in passing from strong positive to strong negative magnetization and *vice versâ*, and that by which iron starting from a neutral condition first acquires magnetization under the action of a force constantly increasing.

Attention is called by Ewing to the *loops* which are formed when the forces are carried round a (not very small) cycle of any kind. "Every loop in the diagram shows that when we reverse the *change* of magnetizing force from increment to decrement, or *vice versâ*, the magnetism begins to change very gradually relatively to the change of $\mathfrak{H}$ (the force), no matter how fast it may have been changing in the opposite direction before. So much is this the case that the curves, when drawn to a scale such as that of the figure, appear in all cases to start off tangent to the line parallel to the axis on which $\mathfrak{H}$ is measured whenever the change of $\mathfrak{H}$ is reversed in sign."

The question here raised as to the direction of the curve, after the force has passed a maximum or minimum, is one of great importance. If it were strictly true that this direction were parallel to the axis, it would follow generally that iron in any condition of magnetization would be uninfluenced by small periodic variations of magnetic force; for example, that in many telephone experiments iron would show no magnetic properties. The experiments already detailed prove that when the whole force and magnetization are small (they were not actually evanescent) very sensible proportional changes of magnetization accompany small changes of force, the ratio being such as to give a permeability not much inferior to 100. Nothing is easier than to show that this conclusion is not limited to very small mean forces and magnetizations. As regards the latter, we may apply and remove a force (say) of 5H. By this operation the iron is left in a different magnetic condition, and the zero-reading of the magnetometer is altered, probably to the extent of driving the spot of light off the scale. But if we bring the needle back with the aid of external magnets, we can examine, as before, the effect of imposing a small force (under $\frac{1}{5}$H). If this be in the opposite direction to the previous large force, it will produce, in spite of the compensating-coil, a very sensible effect; for in this case the movement from 0 to $-\frac{1}{5}$H is in continuation of the previous movement from 5H to 0. But subsequent applications and removals of $\frac{1}{5}$H produce no visible effect upon the needle, as would have happened from the first had the small force operated in the positive direction. We may conclude, then, that the compensation for small forces suitable when the iron is nearly free from magnetization is not disturbed by the presence of considerable residual magnetism.

To examine the action of a small increment or decrement, when the total force is relatively large, we must either introduce a second magnetizing helix or effect the variation of current otherwise than by breaking the circuit. I found it most convenient simply to vary the resistance taken from the box, so arranging matters that the small alteration of current required could be effected by the insertion or removal of a single plug. The corresponding change of current is obtained by inspection of a table of reciprocals; and it was readily proved that within the admissible range of the apparatus the compensation was just as effective whether a step (not exceeding $\frac{1}{5}$H) was made from zero or from a force (say) of 5H, 20 or 30 times as great as the increment or decrement itself. It need scarcely be repeated that there is an exception as regards the first step, in the case where it is in the same direction as the large movement preceding it.

At this stage the original magnetizing-coil, having been arranged for the investigation of the smallest forces, was replaced by another at a greater distance from the suspended needle. When the magnetization of the iron in its various parts fails to vary in strict proportion to the force, the effective pole is liable to shift its position; and this is an objection to the horizontal arrangement adopted in the earlier experiments. The helix was therefore placed vertically, the lower end of the iron core being a trifle below the level of the magnetometer-needle. The upper pole was at such a distance as to give but little relative effect. The length of the new helix, wound like the other upon a glass tube, is about 30 centim. The windings are in four layers, at the rate altogether of 65 per centim.; so that (under the same current) the magnetizing force is about twice as great as before. The resistance is 4·75 ohms.

A large number of observations have been made upon a core of rather hard Swedish iron, 3·30 millim in diameter. The same compensating-coil as before was found suitable, and the arrangements were unaltered, except that an additional reversing-key was introduced, by which the poles of the Daniell cell could be interchanged. The total resistance of the circuit, independently of the box, was 7 ohms. The length of the core—or, rather, of the part exposed to the magnetizing force*—being about 100 diameters, is scarcely sufficient for an accurate determination; but from the observed position necessary for the compensating-coil we can get at least a rough estimate of the susceptibility for small forces. Thus, on December 28th, there was compensation for small forces when the distances of the needle from the mean plane of the compensating-coil and from the operative pole of the iron core were respectively 17·2 centim. and 9·3 centim. The magnetic force at the needle, due to unit current in the compensating-coil, is

$$\frac{2\pi \times 74 \times 9^2}{\{9^2 + 17{\cdot}2^2\}^{\frac{3}{2}}} = 5{\cdot}15.$$

The magnetizing force in the interior of the helix for unit current is

$$4\pi \times 65 = 817.$$

If k be the susceptibility, the strength of the pole is

$$\tfrac{1}{4}\pi \times {\cdot}330^2 \times 817 \times k\,;$$

and since the distance of this from the needle is 9·3 centim., we have, to determine k,

$$k = \frac{5{\cdot}15 \times 9{\cdot}3^2}{\frac{1}{4}\pi \times {\cdot}330^2 \times 817} = 6{\cdot}36\,;$$

so that

$$\mu = 1 + 4\pi k = 81.$$

This is probably an underestimate.

In order to obtain results comparable with those of Stoletow and Rowland, the iron was submitted to a series of cycles of positive and negative force. According to Ewing, the behaviour is simplest when the iron is first treated to a process of "demagnetization by reversals." This was effected *in situ* as a preliminary to the experiments of January 4th, the resistance in the box being increased by small steps from a few ohms to a thousand ohms; while at each stage the battery was reversed several times. It must be remarked, however, that the iron was all the while under the influence of the earth's vertical force; so that the resulting condition was certainly not one of demagnetization. But even as thus

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* At the upper end the iron projected beyond the coil.

carried out, the operation was probably advantageous as obliterating the influence of the previous history of the iron core.

The compensation was in the first place adjusted so that no displacement could be detected, whether the resistance was infinity or 2007 ohms*. This, of course, was in the position of the reversing-key denoted (−). When the iron and the compensating-coil acted in the same direction (+), the displacement was 8 divisions.

In Table I. the first column gives the total resistance of the circuit in ohms, and the second gives the reciprocals of the first, numbers proportional to the current or magnetizing force. Repetitions of a cycle are shown on the same horizontal line, for greater convenience of comparison. Thus the first application of current +197 gave the reading 242; a second application, after the cycle +197, 0, −197, 0, gave 241½. After two of these cycles had been completed, the current +326 gave the reading 245. To the readings as entered a small correction to infinitely small arcs has been applied. The letters R, L in the first column indicate the alternative positions of the battery reversing-key. It will be seen that very nearly the same numbers are obtained on repetition of a cycle, and that even the first application of an increased force gives a normal result.

The first question which suggests itself is the law connecting the magnitude of a current with the alteration of magnetization caused by its reversal. The quantities under consideration are exhibited in Table II., where the first column gives the current (x) and the second column the displacement (y) due to reversal. The relation between x and y is well expressed by the formula

$$y = -\cdot0053x + 1\cdot072x^2, \quad . \quad . \quad . \quad . \quad . \quad (1)$$

of which the whole of the second member is shown in column 5, and the two parts separately in columns 3 and 4. Column 6 gives the differences between the observed displacements and those calculated from the formula; they do not much exceed the errors of observation.

It will of course be borne in mind that the magnetization exhibited here is additional to the part rendered latent by the compensating-coil, and that the existence of the small linear term may be attributed to a defective adjustment of that coil. The calculated value of y for the step from infinite resistance

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* For greater delicacy, recourse was had to the "method of multiplication," assisted by a pendulum, as already described.

TABLE I.—Jan. 4, 1887.

Resistance.	Current.	Corrected Readings.			
∞	0	240			
1007 R	+ 099	241			
∞	0	241			
1007 L	− 099	240			
∞	0	240			
507 R.........	+ 197	242	241½		
∞	0	241¼	241		
507 L...... ..	− 197	238½	238¼		
∞	0	239	239		
307 R.........	+ 326	245	245		
∞	0	......	243		
307 L.........	− 326	235	235		
∞	0	237	237		
207 R.........	+ 483	250½	250¼		
∞	0	246	246		
207 L.........	− 483	228	228		
∞	0	232½	232½		
107 R.........	+ 934	283¼	283¾	284	
∞	0	264½	264½	265	
107 L.........	− 934	195¼	195¼	195¼	
∞	0	214	214	213¼	
87 R	+1149	306½	307½	307	
∞	0	276¾	276¾	277	
87 L	−1149	172¼	171¼	171¼	
∞	0	201½	201	201	
177 R.........	+ 565	......	238½		
77 R.........	+1298	325¾	325¾		
177 R.........	+ 565	......	315½		
∞	0	286¼	286		
177 L	− 565	......	237½		
77 L.........	−1298	151½	150¾		
177 L.........	− 565	......	160		
∞	0	190½	188¾		
167 R.........	+ 599	......	......	232½	232¼
67 R.........	+1493	352¾	353	353½	353¾
167 R.........	+ 599	......	......	337½	338¾
∞	0	299	299½	300	301½
167 L.........	− 599	......	......	241½	
67 L.........	−1493	121¾	121¼	121¼	
167 L.........	− 599	......	......	136	
∞	0	174¾	174½	173½	

to 2007 ohms, which is one quarter of the first step in the table (from 1007R to 1007L), is

$$y = -\cdot 13 + \cdot 06 = -\cdot 07 \text{ division.}$$

This is the step for which the coil was adjusted ; and the difference between the calculated and observed (zero) value of y

is perhaps as small as could have been expected. It is fair to conclude that, if the compensating-coil could have been perfectly adjusted for a very small step (the actual step was scarcely small enough), the uncompensated effects visible with larger currents would have been expressible by a quadratic term simply.

The currents (x) given in the tables are reduced to C.G.S. measure when divided by 10^6. On the same system the magnetizing force is

$$8{\cdot}2 \times 10^{-4} \times x\,;$$

so that the force due to the strongest current referred to in the table is 1·2 C.G.S., or about 7H. When the current is reversed, the change of magnetic force is of course the double of this quantity.

In extending the definition of susceptibility to cases in which the force is not very small, we might proceed in more than one way. If we take the ratio of the change of magnetization to change of force when the force is reversed, we are following good authorities; and we get a definition which is at any rate consistent with the definition necessary when small forces are concerned. The values of k for different forces are not given by a direct comparison of the numbers in Table II., since the magnetometer-scale is arbitrary; but we may find for what force the susceptibility is (for example) the double of that applicable to infinitely small forces.

Table II.

Current, x.	Displacement, y.	$\cdot 0053x$.	$1{\cdot}072x^2$.	$-0{\cdot}0053x$. $+1{\cdot}072x^2$.	Diff.
99	1	0·52	1·05	0·5	+0·5
197	3¼	1·0	4·2	3·2	0
326	10	1·7	11·4	9·7	+0·3
483	22¼	2·6	25·0	22·4	−0·2
934	88½	4·9	93·7	88·8	−0·3
1149	136	6·1	141·5	135·4	+0·6
1298	174	6·9	180·6	173·7	+0·3
1493	231	7·9	238·9	231·0	0

For this purpose we must note that the conjoint effect of the magnetization due to current 50, simply applied or removed, and of the compensating-coil, was 8 divisions, of which half is due to each cause. The effect of the coil for a *reversal* of current 50 is thus 8 divisions, and being proportional to the current can be deduced for any other case. At the bottom of the table, where the current is 1493, the displacement rendered latent by the coil is thus about 240 divisions ; and since at this point the uncompensated displacement is nearly of the same amount, we see that the value of k (as above defined) is here doubled. Thus, if $\mathfrak{H}$ denote the magnetizing force in C.G.S. measure, we have

$$k = 6{\cdot}4\,(1 + {\cdot}8\mathfrak{H}).$$

The *form* of the relations of k to $\mathfrak{H}$ for small forces is pretty accurately demonstrated by the observations. On the other hand, the reduction to absolute measure is rather rough*—a point of less consequence, inasmuch as the constants may be expected to vary according to the sample and condition of the iron.

The observations in Table I. give a good deal more than the extreme range of magnetization due to the reversal of a force. In all cases the two residual magnetizations (when the force is zero) are recorded ; while in the two latter, where the range is greatest, further intermediate points are included. The results are plotted in Plate II. fig. 2, where it will be seen that the curves start backwards in a horizontal direction after a maximum or minimum of force. Special observations (not recorded in the table) were directed to this point. Neither at the maxima nor at the zeros of force was there any evidence of failure of compensation when a small backward movement was made.

The curves do not differ much from parabolas ; and in other cases, where the applied magnetic forces were all of one sign, I have found that after a large movement in one direction, the curve representing a backward movement coincides somewhat closely with a parabola whose magnitude is nearly the same under different circumstances, and which is placed so that its axis is vertical and vertex coincident with the point where the backward movement commences. The reader will not forget that to obtain the real curves fully expressing the relation between magnetization and force, we must add the effect, proportional to the force, rendered latent by the compensating-coil.

On the basis of this parabolic law we may calculate the influence of hysteresis in the magnetization of iron upon the apparent self-induction and resistance of the magnetizing-coil, when periodic currents of moderate power are allowed

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* In all probability the number 6·4, applicable when $\mathfrak{H}=0$, is too small.

to pass. If we reckon from the mean condition, we may express the relation between the extreme changes of magnetization and force by the formula

$$\mathfrak{I}'=\alpha\mathfrak{H}'+\beta\mathfrak{H}'^2, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where α and β are constants, corresponding with the 6·4 and 6·4 × ·8 of the example given above. But no such single formula can express the relation for the rest of the cycle. When $\mathfrak{H}$ is diminishing from $\mathfrak{H}=\mathfrak{H}'$ to $\mathfrak{H}=-\mathfrak{H}'$,

$$\mathfrak{I}=\alpha\mathfrak{H}+\beta\mathfrak{H}'^2\{1-\tfrac{1}{2}(1-\mathfrak{H}/\mathfrak{H}')^2\};$$

but when $\mathfrak{H}$ is increasing from $\mathfrak{H}=-\mathfrak{H}'$ to $\mathfrak{H}=\mathfrak{H}'$,

$$\mathfrak{I}=\alpha\mathfrak{H}+\beta\mathfrak{H}'^2\{-1+\tfrac{1}{2}(1+\mathfrak{H}/\mathfrak{H}')^2\}.$$

These expressions coincide at the limits $\mathfrak{H}=\pm\mathfrak{H}'$, but differ at intermediate points. Since the force is supposed to be periodic, we may conveniently write

$$\mathfrak{H}=\mathfrak{H}'\cos\theta\,;$$

whence, putting also for brevity α' in place of $\alpha\mathfrak{H}'$, β' in place of $\beta\mathfrak{H}'^2$, we get

$$\mathfrak{I}=\alpha'\cos\theta+\beta'\{\cos\theta+\tfrac{1}{2}\sin^2\theta\}$$

from $\theta=0$ to $\theta=\pi$,

$$\mathfrak{I}=\alpha'\cos\theta+\beta'\{\cos\theta-\tfrac{1}{2}\sin^2\theta\}$$

from $\theta=\pi$ to $\theta=2\pi$.

We have now to express $\mathfrak{I}$ for the complete cycle in Fourier's series proceeding by the sines and cosines of θ and its multiples. The part

$$\alpha'\cos\theta+\beta'\cos\theta,$$

being the same in the two expressions, is already of the required form. For the other part we get

$$\pm\tfrac{1}{2}\sin^2\theta=\mathrm{B}_1\sin\theta+\mathrm{B}_3\sin 3\theta+\mathrm{B}_5\sin 5\theta+\ldots., \quad . \quad (3)$$

where only odd terms appear, and B_n is given by

$$\mathrm{B}_n=\frac{-4}{\pi n(n^2-4)}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Thus

$$\mathfrak{I}=(\alpha'+\beta')\cos\theta+\beta'\left\{\frac{4}{3\pi}\sin\theta-\frac{4}{15\pi}\sin 3\theta-\frac{4}{105\pi}\sin 5\theta-\ldots\right\}. \quad (5)$$

If the range of magnetization be very small, β' vanishes, and the influence of the iron upon the enveloping coil is merely to increase its self-induction; but if β' be finite, the matter is less simple. The terms in $\sin 3\theta$, $\sin 5\theta$, &c. indicate that the response of the iron to a harmonic force is not even purely harmonic, but requires higher components for its expression. If we put these terms out of account as relatively small, we must still regard the phase of $\mathfrak{J}$ as different from that of $\mathfrak{H}$. The term in $\sin\theta$ will show itself as an apparent increase in the *resistance* of the coil, due to hysteresis, and independent of that which may be observed even with very small forces as a consequence of induced currents in the interior of the iron. The augmentation of resistance now under consideration may be expected to be insensible when the extreme range of magnetizing force does not exceed one tenth of the earth's horizontal force.

In the absolute determination (p. 235) of the susceptibility to very small forces of the hard Swedish iron wire (3·30 millim. diameter), the length (about 100 diameters) was scarcely sufficient for an accurate estimate. Similar experiments on a thinner wire (1·57 millim. diameter) of the same quality of iron gave $k=6\cdot85$, corresponding to $\mu=87$. This is in the hard-drawn condition. After annealing the same piece of wire gave a higher result, but in this case the observation is complicated by the assumption of the magnetic state occupying a sensible time. The susceptibility applicable to the final condition is as high as 22·0, more than three times as great as before annealing. But a lower number would better represent the facts, when the small magnetic force is rapidly periodic ; and it may even be that under forces of frequencies such as occur in telephonic experiments, most of the difference due to annealing would disappear. Such a conclusion is suggested by the slight influence of annealing in the experiment described in Part II.,[*] where is determined the increment of resistance of an iron wire due to the concentration of a variable current in the outer layers. But the matter is one requiring further examination under better experimental conditions.

The sensitiveness of the magnetometer-needle in the experiments directed to prove the constancy of susceptibility to small forces, suggests the inquiry whether iron should be used when the object is purely galvanometric. An attempt to produce a sensitive galvanometer by hanging a mirror and needle between the pointed pole-pieces of a large electromagnet, arranged as in diamagnetic experiments, was not very successful. A better result was obtained with an astatic needle system, and an electromagnet on a much smaller scale. This was of horseshoe form, the core being of hard Swedish

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[*] Phil. Mag. Dec. 1886, p. 488.

iron wire 3·3 millim. diameter. The insulated copper wire was in three layers, of resistance ·34 ohm, and the total weight of the electromagnet was 283 grams. It was held so as to embrace the upper needle system. When the time of swing from rest to rest was 4 seconds, the movement due to a current of about $\frac{1}{20,000}$ ampere was 100 divisions. The zero was steady enough to allow a displacement of half a division to be detected with tolerable certainty in each trial; so that, as actually used, the arrangement was sensitive to a current of $\frac{1}{4} \times 10^{-6}$ ampere. The addition of a similar electromagnet embracing the lower needle system, and connected in series, would double the sensitiveness, and raise the resistance to ·68 ohm. A galvanometer thus constructed, and of resistance equal to 1 ohm, would show a current of 10^{-7} ampere. Using finer wire, we might expect an instrument of 100 ohms to show a current of 10^{-8} ampere, and so on.

For comparison with the above I tried, in as nearly as possible the same way, the sensitiveness of a good Thomson astatic galvanometer of resistance 1·3 ohm. With an equal time of vibration, a current of $\frac{1}{20,000}$ ampere produced a movement of 300 divisions. The zero was perhaps a little steadier than before; but it will be seen that the sensitiveness was of the same order of magnitude. In both cases, by taking precautions and by using repetition, the delicacy might have been increased, probably tenfold.

The experiments show that there is no difficulty in constructing a galvanometer of high sensitiveness upon these lines. According to theory, with ideal iron of permeability 100, it should be possible to attain a much higher degree of sensitiveness than without iron. But the tendency to retain residual magnetism would certainly be troublesome, and probably neutralize in practice most of the advantage arising from the higher permeability, which allows of windings more distant from the needles being turned to good account. Another inconvenience may be mentioned. If the iron poles are brought at all close to the needles, there is a strong tendency to instability at moderate angles of displacement.

Experiments already described proved conclusively that the response of iron and steel to small periodic magnetic forces is not affected by the presence of a constant force, or of a residual magnetization, of moderate intensity. At the same time it appeared in the highest degree probable that the independence was not absolute, and that the response to a given small change of force would fall off as the condition of "saturation" is approached, even though we admit, in accordance with recent evidence, that saturation is attainable only in a very rough sense. The question was too important to be left undecided, but it was difficult to deal with by the magnetometric method. If the arrangement is sensitive enough to allow the effect of the small force to be measured with reasonable accuracy, it is violently disturbed by the occurrence of high degrees of magnetization. Moreover it is undesirable to depend so much, as in this method, upon what may happen near the free extremities of the iron rod, where the magnetic forces must vary rapidly. The "ballistic method," in which the changes of magnetization are indicated by the throw of a galvanometer-needle in connection with a secondary coil embracing the central parts of the rod, has the great advantage for this purpose, that the reading is independent of the stationary condition of the iron. In the first experiments by this method the magnetizing helix was similar to one already described (p. 234); and the small, as well as the large, alterations of force were effected by varying the resistance of the circuit. By suitably choosing the resistances from a box, the small alterations of current could be obtained with sufficient suddenness by the simple introduction or removal of a plug, and were taken of the same order of magnitude at different parts of the scale. A comparison of effects (with the aid of a table of reciprocals) proved that a pretty strong total force* or magnetization did not interfere much with the response of the iron to a given force of small magnitude.

This arrangement did not well allow of the investigation being pushed further so as to deal with stronger magnetizing forces. If, with the view of increasing the current, we cut down the german-silver resistance too closely, the estimate of total resistance depends too much upon the battery, and the current becomes uncertain. This difficulty is evaded by the use of a double wire—one conveying the strong current, of which the measurement does not require to be very exact; the other conveying the weak current, of which the effect at different parts of the scale is to be examined.

In order to obtain a satisfactory ratio of length to diameter, without the loss of sensitiveness that would accompany a diminution in the section of the iron, a helix was prepared of length 59·6 centim. It was wound upon a glass tube with a double wire in three layers, the whole number of turns of each wire being 1376. The magnetizing force due to unit current in one wire is therefore

$$4\pi \times 1376/59{\cdot}6 = 290{\cdot}1.$$

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* Up to about 6 C.G.S. The iron was unannealed Swedish, 3·3 millim. in diameter.

The resistance of each wire is 3·2 ohms; and thus when two Grove cells are used in connection with one of the wires, a current of about an ampere (·1 C.G.S.) can be commanded. Smaller currents were obtained by the insertion of resistances from a box.

Although the secondary coil, connected with a delicate galvanometer, contained a large number of convolutions, the sensitiveness was insufficient to allow of the small magnetizing force being taken as low as would otherwise have been desirable. It was obtained by means of the second wire of the helix, which was included in the circuit of a Daniell cell and 200 ohms from a resistance-box. When the circuit was completed (or broken) at a key, the force brought into operation, or removed, was

$$\frac{290{\cdot}1}{2040}={\cdot}14\ \text{C.G.S.}$$

In making a series of observations it was usual, after each alteration of the strong magnetizing force, to apply and remove the small magnetizing force several times before attempting to take readings.

The results obtained by this method were of a pretty definite character. The small force produced a constant effect upon a wire of unannealed Swedish iron, 3·3 millim. in diameter, until the large force was increased from 0 to about 5 C.G.S. At about 10 C.G.S. the effect of the small force fell off 5 per cent. The highest force used, about 29 C.G.S., reduced the effect to about 60 per cent. of its original amount. On complete removal of the force due to the Grove cells, there was but a partial recovery of effect, doubtless in consequence of residual magnetization. After the wire had been removed from the helix and well shaken, the small force was found to have recovered its full efficiency.

The wire was then annealed and submitted anew to a similar series of operations. The magnetization due to the alternate application and removal of the small force was found to be at first, *i. e.* in the absence of a constant force, *twice* as great as before[*].

The increase, however, is not long maintained, a steady force of 2 C.G.S. being already sufficient to cause a marked falling off (of about 20 per cent.). Under the operation of 29 C.G.S., the effect of the small force fell to about $\frac{1}{6}$ of its original

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[*] It should here be remembered that any part of the change of magnetization which lags behind for more than a second or two, fails to manifest itself fully in the indications of the galvanometer.

amount. Removal from the helix and shaking in a zero field sufficed to restore the wire to its initial condition.

Similar experiments upon an annealed wire of "best spring steel" showed no sensible change of effect when the steady force was varied from 0 to about 16 C.G.S. In this case the ratio of length to diameter was about 300.

We may now regard it as established :—

That in any condition of force and magnetization, the susceptibility to small periodic changes of force is a definite, and not very small, quantity, independent of the magnitude of the small change.

That the value of the susceptibility to small changes of force is approximately independent of the initial condition as regards force and magnetization, until the region of saturation is approached.

Terling Place, Witham, Essex,
Jan. 24, 1887.

XXVI. *The Permanent and Temporary Effects on some of the Physical Properties of Iron, produced by raising the Temperature to* 100° *C.* *By* HERBERT TOMLINSON, *B.A.* *

Introduction.

FOR many years I have been carrying on researches respecting the effects of stress on the physical properties of matter, and during this period I have become acquainted with certain phenomena, which, though pertaining more or less to most metals, are so conspicuous in iron as to render it worthy of special attention. As these phenomena bear importantly on what Sir William Thomson has designated the thermodynamic qualities of metals, the investigation of which seems to be attracting daily more and more attention, I propose to lay before the Physical Society, from time to time, such information concerning them as a patient study has enabled me to acquire.

On the present occasion I would invite attention to certain remarkable effects produced on some of the physical properties of iron, by merely raising the temperature to a degree not exceeding 100° C.

The Internal Friction of Iron.

If an iron wire be suspended vertically with its upper extremity clamped to a rigid support, and its lower one clamped or soldered to the centre of a horizontal bar of metal, attached

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* Communicated by the Physical Society: read January 22, 1887.