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XXXI. *Investigations in Optics, with special reference to the Spectroscope.* By LORD RAYLEIGH, *F.R.S.**

[Plate VII.]

§ 1. *Resolving, or Separating, Power of Optical Instruments.*

ACCORDING to the principles of common optics, there is no limit to resolving-power, nor any reason why an object, sufficiently well lighted, should be better seen with a large telescope than with a small one. In order to explain the peculiar advantage of large instruments, it is necessary to discard what may be looked upon as the fundamental principle of common optics, viz. the assumed infinitesimal character of the wave-length of light. It is probably for this reason that the subject of the present section is so little understood outside the circles of practical astronomers and mathematical physicists.

It is a simple consequence of Huyghens's principle, that the direction of a beam of limited width is to a certain extent indefinite. Consider the case of parallel light incident perpendicularly upon an infinite screen, in which is cut a circular aperture. According to the principle, the various points of the aperture may be regarded as secondary sources emitting synchronous vibrations. In the direction of original propagation the secondary vibrations are all in the same phase, and hence the intensity is as great as possible. In other direc-

* Communicated by the Author.

tions the intensity is less; but there will be no sensible discrepancy of phase, and therefore no sensible diminution of intensity, until the obliquity is such that the (greatest) projection of the diameter of the aperture upon the direction in question amounts to a sensible fraction of the wave-length of the light. So long as the extreme difference of phase is less than a quarter of a period, the resultant cannot differ much from the maximum; and thus there is little to choose between directions making with the principal direction less angles than that expressed in circular measure by dividing the quarter wave-length by the diameter of the aperture. Direct antagonism of phase commences when the projection amounts to half a wave-length. When the projection is twice as great, the phases range over a complete period, and it might be supposed at first sight that the secondary waves would neutralize one another. In consequence, however, of the preponderance of the middle parts of the aperture, complete neutralization does not occur until a higher obliquity is reached.

This indefiniteness of direction is sometimes said to be due to "diffraction" by the edge of the aperture—a mode of expression which I think misleading. From the point of view of the wave-theory, it is not the indefiniteness that requires explanation, but rather the smallness of its amount.

If the circular beam be received upon a perfect lens, an image is formed in the focal plane, in which *directions* are represented by *points*. The image accordingly consists of a central disk of light, surrounded by luminous rings of rapidly diminishing brightness. It was under this form that the problem was originally investigated by Airy*. The angular radius θ of the central disk is given by

$$\theta = 1{\cdot}2197 \frac{\lambda}{2R}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

in which λ represents the wave-length of light, and $2R$ the (diameter of the) aperture.

In estimating theoretically the resolving-power of a telescope on a double star, we have to consider the illumination of the field due to the superposition of the two independent images. If the angular interval between the components of the star were equal to 2θ, the central disks would be just in contact. Under these conditions there can be no doubt that the star would appear to be fairly resolved, since the brightness of the external ring-systems is too small to produce any

* Camb. Phil. Trans. 1834.

material confusion, unless indeed the components are of very unequal magnitude.

The diminution of star-disks with increasing aperture was observed by W. Herschel; and in 1823 Frauenhofer formulated the law of inverse proportionality. In investigations extending over a long series of years, the advantage of a large aperture in separating the components of close double stars was fully examined by Dawes*. In a few instances it happened that a small companion was obscured by the first bright luminous ring in the image of a powerful neighbour. A diminution of aperture had then the effect of bringing the smaller star into a more favourable position for detection; but in general the advantage of increased aperture was very apparent even when attended by considerable aberration.

The resolving-power of telescopes was investigated also by Foucault†, who employed a scale of equal bright and dark alternate parts: it was found to be proportional to the aperture and independent of the focal length. In telescopes of the best construction the performance is not sensibly prejudiced by outstanding aberration, and the limit imposed by the finiteness of the waves of light is practically reached. Verdet‡ has compared Foucault's results with theory, and has drawn the conclusion that the radius of the visible part of the image of a luminous point was nearly equal to the half of the radius of the first dark ring.

Near the margin of the theoretical central disk the illumination is relatively very small, and consequently the observed diameter of a star-disk is sensibly less than that indicated in equation (1), how much less depending in some measure upon the brightness of the star. That bright stars give larger disks than faint stars is well known to practical observers.

With a high power, say 100 for each inch of aperture, the sharpness of an image given by a telescope is *necessarily* deteriorated, the apparent breadth of a point of light being at least $8\frac{1}{2}$ minutes. In this case the effective aperture of the eye is $\frac{1}{100}$ inch. In his paper on the limit of microscopic vision§, Helmholtz has shown that the aperture of the eye cannot be much contracted without impairing definition—from which it follows that the limit of the resolving-power of telescopes is attained with a very moderate magnification, probably about 20 for each inch in the aperture of the object-glass or mirror.

* Mem. Astron. Soc. vol. xxxv.

† *Ann. de l'Observ. de Paris,* t. v. 1858.

‡ *Leçons d'Optique Physique,* t. i. p. 309.

§ Pogg. *Ann.* Jubelband 1874.

We have seen that a certain width of beam is necessary to obtain a given resolving-power; but it does not follow that the whole of an available area of aperture ought to be used in order to get the best result. As the obliquity to the principal direction increases, the first antagonism of phase which sets in is between secondary waves issuing from marginal parts of the aperture; and thus the operation of the central parts is to retard the formation of the first dark ring. This unfavourable influence of the central rays upon resolving-power was well known to Herschel, who was in the habit of blocking them off by a cardboard stop. The image due to an annular aperture was calculated by Airy; and his results showed the contraction of the central disk and the augmented brightness of the surrounding rings*. More recently this subject has been ably treated by M. Ch. André†, who has especially considered the case in which the diameter of the central stop is half the full aperture. How far it would be advantageous to carry the operation of blocking out the central rays would doubtless depend upon the nature of the object under examination. Near the limit of the power of an instrument a variety of stops ought to be tried. Possibly the best rays to block out are those not quite at the centre (see § 2).

The fact that the action of the central rays may be disadvantageous shows that in the case of full aperture the best effect is not necessarily obtained when all the secondary waves arrive in the same phase at the focal point. If by a retardation of half a wave-length the phase of any particular ray is reversed, the result is of the same character as if that ray were stopped. Hence an exactly parabolic figure is not certainly the best for mirrors.

The character of the image of a luminous *line* cannot be immediately deduced from that of a luminous point. It has, however, been investigated by M. André, who finds that the first minimum of illumination occurs at a somewhat lower obliquity than in the case of a point. A double *line* is therefore probably more easily resolvable than a double *point*; but the difference is not great. In the case of a line the minima are not absolute zeros of illumination.

§ 2. *Rectangular Sections.*

The diffraction phenomena presented by beams of rectangular section are simpler in theory than when the section is circular; and they have a practical application in the spectro-

* See also Astron. Month. Notices, xxxiii. 1872.

† "Étude de la Diffraction dans les Instruments d'Optique," *Ann. de l'École Norm.* v. 1876.

scope, when the beam is limited by prisms or gratings rather than by the object-glasses of the telescopes.

Supposing, for convenience, that the sides of the rectangle are horizontal and vertical, let the horizontal aperture be a and the vertical aperture be b. As in § 1, there will be no direct antagonism among the phases of the secondary waves issuing in an oblique horizontal direction, until the obliquity is such that the projection of the horizontal aperture a is equal to $\frac{1}{2}\lambda$. At an obliquity twice as great the phases range over a complete period; and, *since all parts of the horizontal aperture have an equal importance*, there is in this direction a complete absence of illumination. In like manner, a zero of illumination occurs in every horizontal direction upon which the projection of a amounts to an exact multiple of λ.

The complete solution of the present problem, applicable to all oblique directions, is given in Airy's 'Tracts,' 4th edition, p. 316, and in Verdet's *Leçons*, t. i. p. 265. If the focal length of the lens which receives the beam be f, the illumination I^2 at a point in the focal plane whose horizontal and vertical coordinates (measured from the focal point) are ξ, η, is given by

$$I^2 = \frac{a^2b^2}{\lambda^2 f^2} \frac{\sin^2 \frac{\pi a \xi}{\lambda f}}{\frac{\pi^2 a^2 \xi^2}{\lambda^2 f^2}} \frac{\sin^2 \frac{\pi b \eta}{\lambda f}}{\frac{\pi^2 b^2 \eta^2}{\lambda^2 f^2}}, \quad . \quad . \quad . \quad . \quad (1)$$

the intensity of the incident light being unity. The image is traversed by *straight* vertical and horizontal lines of darkness, whose equations are respectively

$$\sin \frac{\pi a \xi}{\lambda f} = 0, \quad \sin \frac{\pi b \eta}{\lambda f} = 0. \quad . \quad . \quad . \quad . \quad (2)$$

The calculation of the image due to a luminous line (of uniform intensity) is facilitated in the present case by the fact that the law of distribution of brightness, as one coordinate varies, is independent of the value of the other coordinate. Thus the distribution of brightness in the image of a vertical line is given by

$$\int_{-\infty}^{+\infty} I^2 d\eta = \frac{a^2 b}{\lambda f} \frac{\sin^2 \frac{\pi a \xi}{\lambda f}}{\frac{\pi^2 a^2 \xi^2}{\lambda^2 f^2}}, \quad . \quad . \quad . \quad . \quad (3)$$

the same law as obtains for a luminous point when horizontal directions are alone considered. It follows from (3) that in the spectroscope *the definition is independent of the vertical aperture.*

In order to obtain a more precise idea of the character of the image of a luminous line, we must study the march of the function $u^{-2}\sin^2 u$. The roots occur when u is any multiple of π, except zero. The maximum value of the function is unity, and occurs when $u=0$. Other maxima of rapidly diminishing magnitude occur in positions not far removed from those lying midway between the roots. The image thus consists of a central band of half width corresponding to $u=\pi$, accompanied by lateral bands of width π, and of rapidly diminishing brightness. The accompanying Table and diagram (Plate VII. fig. 1) will give a sufficient idea of the distribution of brightness for our purpose.

TABLE I.

u.	$u^{-2}\sin^2 u$.	u.	$u^{-2}\sin^2 u$.
0	1·0000	π	·0000
$\frac{1}{6}\pi$	·9119	$\frac{5}{4}\pi$	·0324
$\frac{1}{4}\pi$	·8106	$\frac{4}{3}\pi$	·0427
$\frac{1}{3}\pi$	·6839	$\frac{3}{2}\pi$	·0450
$\frac{1}{2}\pi$	·4053	$\frac{7}{4}\pi$	·0165
$\frac{2}{3}\pi$	·1710	2π	·0000
$\frac{3}{4}\pi$	·0901	$\frac{5}{2}\pi$	·0162
$\frac{5}{6}\pi$	·0365	3π	·0000

The curve A B C D represents the values of $u^{-2}\sin^2 u$ from $u=0$ to $u=3\pi$. The part corresponding to negative values of u is similar, O A being a line of symmetry.

Let us now consider the distribution of brightness in the image of a double line whose components are of equal strength and at such an angular interval that the central line in the image of one coincides with the first zero of brightness in the image of the other. In fig. 1 the curve of brightness for one component is A B C D, and for the other O A′ C′; and the curve representing half the combined brightnesses is E′ B E F. The brightness (corresponding to B) midway between the two central points A, A′ is ·8106 of the brightness at the central points themselves. We may consider this to be about the limit of closeness at which there could be any decided appearance of resolution. The obliquity corresponding to $u=\pi$ is such that the phases of the secondary waves range over a complete period, *i. e.* such that the projection of the horizontal aperture upon this direction is one wave-length. We conclude that *a double line cannot be fairly resolved unless its components subtend an angle exceeding that subtended by the wave-length of light at a distance equal to the horizontal aperture**.

* In the spectroscope the angular width of the slit should not exceed a moderate fraction of the angle defined in the text, if full resolving-power be wanted.

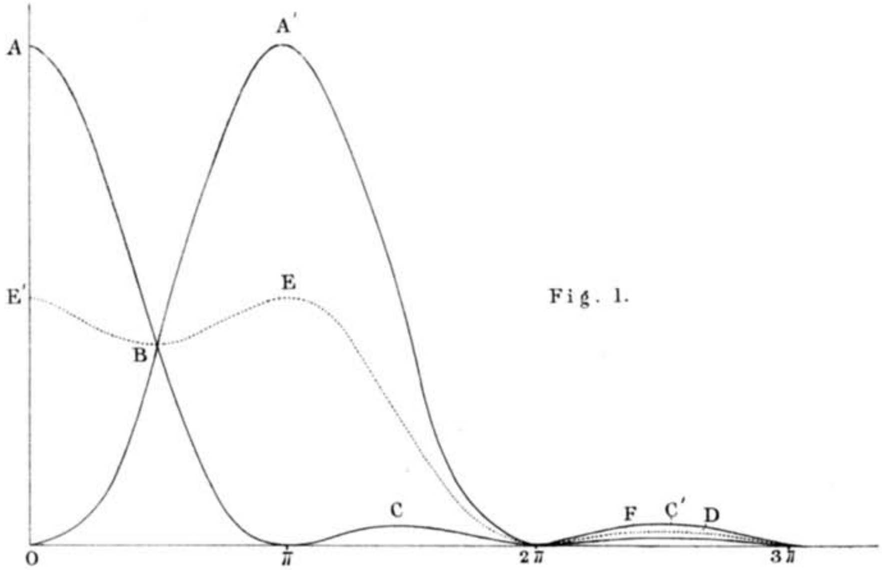


Fig. 1.

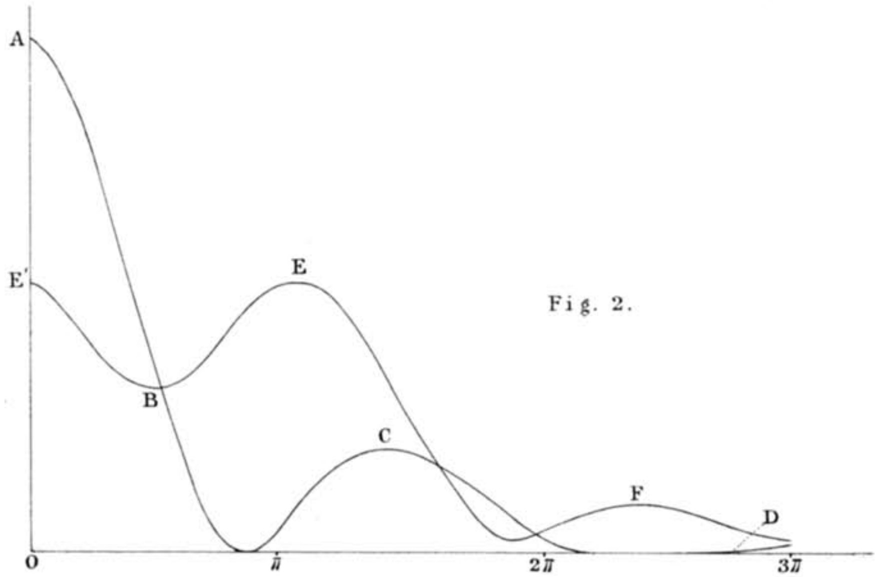


Fig. 2.

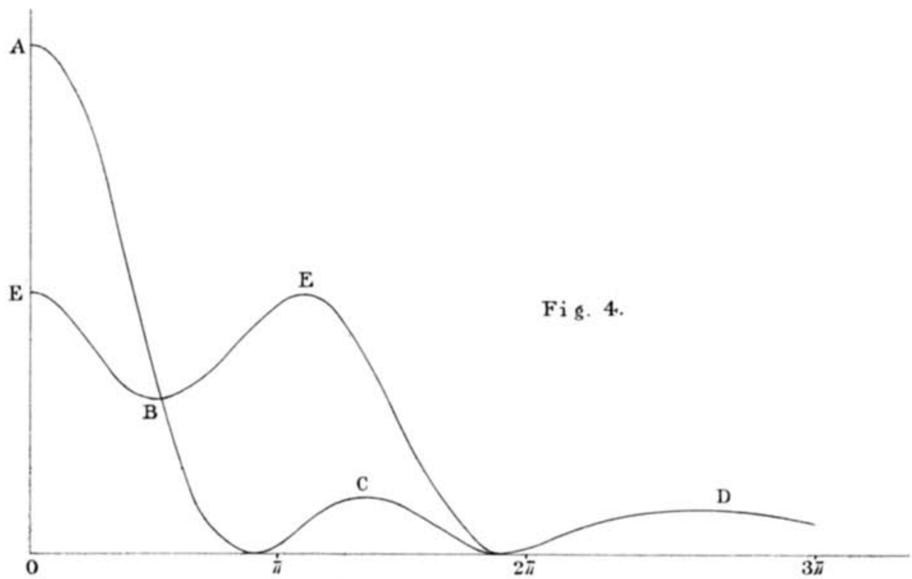


Fig. 4.

This rule is convenient on account of its simplicity; and it is sufficiently accurate in view of the necessary uncertainty as to what exactly is meant by resolution. Perhaps in practice somewhat more favourable conditions are necessary to secure a resolution that would be thought satisfactory.

If the angular interval between the components of the double line be half as great again as that supposed above, the brightness in the middle is ·1802 (2 × ·0901) as against 1·0450 (1 + ·0450) at the central line. Such a falling off in the middle must be more than sufficient for resolution. If the angle subtended by the components of the double line be twice that subtended by the wave-length at a distance equal to the horizontal aperture, the central bands are just clear of one another, and there is a line of absolute blackness in the middle of the combined images.

On the supposition that a certain horizontal aperture is available, a question (similar to that considered in § 1) arises, as to whether the whole of it ought to be used in order to obtain the highest possible resolving-power. From fig. 1 we see that our object must be to depress the curve A B C D at the point B. Now the phase of the resultant is that of the waves coming from the centre; and at the obliquity corresponding to B the phases of the secondary waves range over half a period. It is not difficult to see that the removal of some of the central waves will depress the intensity-curve at B, not only absolutely, but relatively to the depression produced at A. In order to illustrate this question, I have calculated the illumination in the various directions on the supposition that one sixth of the horizontal aperture is blocked off by a central screen. In this case the amplitude is represented by the function f, where

$$f=u^{-1}\left(\sin u-\sin\frac{u}{6}\right), \quad . \quad . \quad . \quad . \quad (4)$$

and, as usual, the intensity is represented by f^2.

TABLE II.

u.	f.	$f^2 \div f_0^2$.	u.	f.	$f^2 \div f_0^2$.
0	+·8333	1·0000	$\frac{7}{4}\pi$	−·2729	·1072
$\frac{1}{4}\pi$	·7342	·7763	2π	·1378	·0274
$\frac{1}{2}\pi$	·4717	·3205	$\frac{9}{4}\pi$	−·0307	·0014
$\frac{3}{4}\pi$	+·1377	·0273	$\frac{12}{5}\pi$	·0000	·0000
$\frac{6}{7}\pi$	·0000	·0000	$\frac{5}{2}\pi$	+·0043	·0000
π	−·1592	·0365	$\frac{18}{7}\pi$	·0000	·0000
$\frac{5}{4}\pi$	·3351	·1617	$\frac{11}{4}\pi$	−·0329	·0016
$\frac{3}{2}\pi$	·3622	·1889	3π	·1061	·0162

The third and sixth columns show the intensity in various directions relatively to the intensity in the principal direction ($u=0$); and the curve A B C D (fig. 2) exhibits the same results to the eye. A comparison with Table I. shows that a considerable advantage has been gained, the relative illumination at B being reduced from ·4053 to ·3205. On the other hand, the augmented brightness of the first lateral band (towards C) may be unfavourable to good definition. The second bright lateral band (towards D) is nearly obliterated. The curve E′ B E F represents the resultant illumination due to a double line whose components are of the same strength, and at the same angular interval as before. The relatively much more decided drop at B indicates a considerable improvement in resolving-power, at least on a double line of this degree of closeness.

The increased importance of the first lateral band is a necessary consequence of the stoppage of the central rays; for in this direction the resultant has a phase *opposite* to that of the rays stopped. The defect may be avoided in great measure by blocking out rays somewhat removed from the centre on the two sides, and allowing the central rays themselves to pass. As an example, I have taken the case in which the two parts stopped have each a width of one eighth of the whole aperture, with centres situated at the points of trisection (fig. 3).

Fig. 3.

The function f suitable to this case is readily proved to be

$$f=u^{-1}\left(\sin u-2\sin\frac{u}{8}\cos\frac{u}{3}\right). \quad . \quad . \quad . \quad (5)$$

The values of f and $f^2 \div f_0^2$ are given in Table III.; and the intensity-curve A B C D is shown in fig. 4.

TABLE III.

u.	f.	$f^2 \div f_0^2$.	u.	f.	$f^2 \div f_0^2$.
0	+·75	1·0000	$\frac{3}{2}\pi$	−·2122	·0801
$\frac{1}{4}\pi$	·6594	·7727	$\frac{7}{4}\pi$	−·0689	·0084
$\frac{1}{2}\pi$	·4215	·3158	2π	+·1125	·0225
$\frac{3}{4}\pi$	+·1259	·0282	$\frac{5}{2}\pi$	·2189	·0852
π	−·1218	·0264	3π	·1960	·0684
$\frac{5}{4}\pi$	·2422	·1043			

The depression at B is even greater than in fig. 2, while the rise at C is much less. Probably this arrangement is about as efficient as any.

I have endeavoured to test these conclusions experimentally with the spectroscope, using the double soda-line. The horizontal aperture of a single-prism instrument was narrowed by gradually advancing cardboard screens until there was scarcely any appearance of resolution. The interior rays were blocked out with vertical wires or needles, adjusted until they occupied the desired positions when seen through the telescope with eyepiece removed. With the arrangements either of fig. 2 or of fig. 4 a very decided improvement on the full aperture was observed; but there was no distinct difference between these two arrangements themselves. Indeed, no such difference was to be expected, since the brightness of the first lateral band has no bad effect on the combined images, as appears from the curve E′ B E F (fig. 2). Under other circumstances the influence of the bright lateral band might be more unfavourable.

In powerful spectroscopes the beam is often rendered unsymmetrical in brightness by absorption. In such cases an improvement would probably be effected by stopping some of the rays on the preponderating side, for which purpose a sloping screen might be used giving a variable *vertical* aperture. It should be noticed, however, that it is only when the vertical aperture is constant that the image of a luminous line is immediately deducible from that of a luminous point.

§ 3. *Optical Power of Spectroscopes.*

As the power of a telescope is measured by the closeness of the double stars which it can resolve, so the power of a spectroscope ought to be measured by the closeness of the closest double lines in the spectrum which it is competent to resolve. In this sense it is possible for one instrument to be more powerful than a second in one part of the spectrum, while in another part of the spectrum the second instrument is more powerful than the first. The most striking cases of this inversion occur when one instrument is a diffraction-spectroscope and the other a dispersion-spectroscope. If the instruments are of equal power in the yellow region of the spectrum, the former will be the more powerful in the red, and the latter will be the more powerful in the green. In the present section I suppose the material and the workmanship to be perfect, and omit from consideration the effects of unsymmetrical absorption. Loss of light by reflection or by uniform absorption has no effect on resolving-power. Afterwards I propose to examine the effect of some of the errors most likely to occur in practice.

So far as relates to the diffraction-spectroscope, the problem of the present section was solved in the Philosophical Magazine for March 1874. I there showed that if n denote the number of lines on a grating and m the order of the spectrum observed, a double line of wave-lengths λ and $\lambda+\delta\lambda$ will be just resolved (according to the standard of resolution defined in the previous section), provided

$$\frac{\delta\lambda}{\lambda}=\frac{1}{mn}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

which shows that the resolving-power varies directly as m and n. When the ruling is very close, m is always small (not exceeding 3 or 4); and even when a considerable number of spectra are formed, the use of an order higher than the third or fourth is often inconvenient in consequence of the overlapping. But if the difficulty of ruling a grating may be measured by the total number of lines (n), it would seem that the intervals ought not to be so small as to preclude the convenient use of at least the third and fourth spectra.

In the case of the soda- double line the difference of wave-lengths is a very little more than $\frac{1}{1000}$; so that, according to (1), about 1000 lines are necessary for resolution in the first spectrum. By experiment I found 1130*.

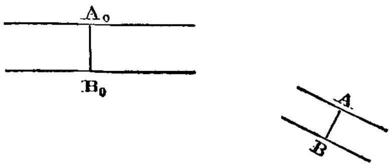
"Since a grating resolves in proportion to the total number of its grooves, it might be supposed that the defining-power depends on different principles in the case of gratings and prisms; but the distinction is not fundamental. The limit to definition arises in both cases from the impossibility of representing a line of light otherwise than by a band of finite though narrow width, the width in both cases depending on the horizontal aperture (for a given λ). If a grating and a prism have the same horizontal aperture and dispersion, they will have equal resolving-powers on the spectrum."

At the time the above paragraph was written, I was under the impression that the dispersion in a prismatic instrument depended on so many variable elements that no simple theory of its resolving-power was to be expected. Last autumn, while engaged upon some experiments with prisms, I was much struck with the inferiority of their spectra in comparison with those which I was in the habit of obtaining from gratings, and was led to calculate the resolving-power. I then found that the theory of the resolving-power of prisms is almost as simple as that of gratings.

* In my former paper this number is given as 1200. On reference to my notebook, I find that I then took the full width of the grating as an English inch. The 3000 lines cover a *Paris* inch, whence the above correction. From the nature of the case, however, the experiment does not admit of much accuracy.

Let $A_0 B_0$ (fig. 5) be a plane wave-surface of the light before it falls upon the prisms, A B the corresponding wave-surface

Fig. 5.



for a particular part of the spectrum after the light has passed the prism or after it has passed the eyepiece of the observing-telescope. The path of a ray from the wave-surface $A_0 B_0$ to A or B is determined by the condition that the optical distance, represented by $\int \mu\, ds$, is a minimum; and as A B is by supposition a wave-surface, this optical distance is the same for both points. Thus

$$\int \mu\, ds \text{ (for A)} = \int \mu\, ds \text{ (for B)}. \quad . \quad . \quad . \quad . \quad (2)$$

We have now to consider the behaviour of light belonging to a neighbouring part of the spectrum. The path of a ray from the wave-surface $A_0 B_0$ to A is changed; but in virtue of the minimum property the change may be neglected in calculating the optical distance, as it influences the result by quantities of the second order only in the change of refrangibility. Accordingly the optical distance from $A_0 B_0$ to A is represented by $\int (\mu + \delta\mu) ds$, the integration being along the path $A_0 \ldots A$; and, similarly, the optical distance between $A_0 B_0$ and B is represented by $\int (\mu + \delta\mu) ds$, where the integration is along the path $B_0 \ldots B$. In virtue of (2) the *difference* of the optical distances is

$$\int \delta\mu\, ds \text{ (along } B_0 \ldots B) - \int \delta\mu\, ds \text{ (along } A_0 \ldots A). \quad . \quad (3)$$

The new wave-surface is formed in such a position that the optical; distance is constant, and therefore the *dispersion*, or the angle through which the wave-surface is turned by the change in refrangibility, is found simply by dividing (3) by the distance A B. If, as in common flint-glass spectroscopes, there is only one dispersing substance, $\int \delta\mu\, ds = \delta\mu \,.\, s$, where s is simply the thickness traversed by the ray. If we call the width of the emergent beam a, the dispersion is represented by $\delta\mu \frac{s_2 - s_1}{a}$, s_1 and s_2 being the thicknesses traversed by the extreme rays. In a properly constructed instrument s_1 is negligible, and s_2 is the aggregate thickness of the prisms at their thick ends, which we will call t; so that the dispersion (θ) is given by

$$\theta = \frac{t\,\delta\mu}{a}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

By § 2 the condition of resolution of a double line whose components subtend an angle θ is that θ must exceed $\lambda \div a$. Hence from (4), in order that a double line may be resolved whose components have indices μ and $\mu + \delta\mu$, it is necessary that t should exceed the value given by the following equation,

$$t = \frac{\lambda}{\delta\mu}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

which expresses that the relative retardation of the extreme rays due to the change of refrangibility is the same (λ) as that incurred without a change of refrangibility when we pass from the principal direction to that corresponding to the first minimum of illumination.

That the resolving-power of a prismatic spectroscope of given dispersive material is proportional to the total thickness used, without regard to the number, angles, or setting of the prisms, is a most important, perhaps the most important, proposition in connexion with this subject. Hitherto in descriptions of spectroscopes far too much stress has been laid upon the amount of dispersion produced by the prisms; but this element by itself tells nothing as to the power of an instrument. It is well known that by a sufficiently close approach to a grazing emergence the dispersion of a prism of given thickness may be increased without limit; but there is no corresponding gain in resolving-power. So far as resolving-power is concerned, it is a matter of indifference whether dispersion be effected by the prisms or by the telescope. Two things only are necessary:—first, to use a thickness exceeding that prescribed by (5); secondly, to narrow the beam until it can be received by the pupil of the eye, or rather, since with full aperture the eye is not a perfect instrument, until its width is not more than one-third or one-fourth of the diameter of the pupil.

The value of expression (3) on which resolving-power depends is readily calculable in all cases of practical interest. For a compound prism of flint and crown, $\delta\mu \,.\, t$ is replaced by

$$\delta\mu \,.\, t - \delta\mu' \,.\, t', \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

where t and t' denote the respective thicknesses traversed, and $\delta\mu$, $\delta\mu'$ the corresponding variations of refractive index.

The relation between $\delta\mu$ and $\delta\lambda$ may generally be obtained with sufficient approximation from Cauchy's formula

$$\mu = A + B\lambda^{-2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Thus

$$\delta\mu = -2B\lambda^{-3}\delta\lambda. \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

The value of B varies of course according to the material of the prisms. As an example I will take Chance's "extra-dense flint." The indices for C and the more refrangible D are*

$$\mu_D = 1{\cdot}650388, \qquad \mu_C = 1{\cdot}644866;$$

so that

$$\mu_D - \mu_C = {\cdot}005522 \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

Also

$$\lambda_D = 5{\cdot}889 \times 10^{-5}, \qquad \lambda_C = 6{\cdot}562 \times 10^{-5},$$

the unit of length being the centimetre; whence by (7),

$$B = {\cdot}984 \times 10^{-10}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

Thus by (5) and (8),

$$t = \frac{\lambda^4}{2B\delta\lambda} = \frac{10^{10}\lambda^4}{1{\cdot}968\,\delta\lambda}. \quad . \quad . \quad . \quad . \quad . \quad (11)$$

For the soda-line,

$$\lambda = 5{\cdot}889 \times 10^{-5}, \quad \delta\lambda = {\cdot}006 \times 10^{-5};$$

and thus the thickness necessary to resolve this line is given by

$$t = 1{\cdot}02 \text{ centimetres}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

The number of times the power of a spectroscope exceeds that necessary to resolve the soda-lines might conveniently be taken as its practical measure. We learn from (12) that, according to this definition, the power of an instrument with simple prisms of "extra-dense glass" is expressed approximately by the number of centimetres of available thickness.

In order to confirm this theory, I have made some observations on the thickness necessary to resolve the soda-lines. The prism was of extra-dense glass of refractive index very nearly agreeing with that above specified, and had a refracting angle of 60°. Along one face sliding screens of cardboard were adapted, by which the horizontal aperture could be adjusted until, in the judgment of the observer, the line was

* Hopkinson, Proc. Roy. Soc. June 1877.

barely resolved. A soda-flame was generally used, though similar observations have been made upon the D line of the solar spectrum. When the adjustment was complete, the aperture along the face of the prism was measured, and gave at once the equivalent thickness, *i. e.* the *difference* of thicknesses traversed by the extreme rays, since the prism was in the position of minimum deviation. Care, of course, was taken that no ordinary optical imperfections of the apparatus interfered with the experiment.

One observer, familiar with astronomical work, fixed the point of resolution when the thickness amounted to from 1·00 to 1·20 centimetre. I was myself less easily satisfied, requiring from 1·35 to 1·40 centimetre. But even with a less thickness than 1 centimetre, it was evident that the object under examination was not a single line. With the same prism I found the thickness necessary to resolve $b_3 b_4$ in the solar spectrum to be about 2·5 centimetres. According to (11), the thickness required for $b_3 b_4$ should be 2·2 times that required for $D_1 D_2$. Probably something depends upon the relative intensities of the component lines.

From (1) and (11) we see that if a diffraction and a dispersion instrument have equal resolving-powers,

$$t=\frac{mn\lambda^3}{2B}; \quad \ldots \ldots \ldots \quad (13)$$

so that the power of a dispersion instrument relatively to that of a diffraction instrument varies inversely as the third power of the wave-length.

For the kind of glass considered in (10), and for the region of the D lines,

$$t=1{\cdot}037\,\frac{mn}{1000}. \quad \ldots \ldots \ldots \quad (14)$$

To find what thickness is necessary to rival the fourth spectrum of a grating of 3000 lines, we have merely to put $m=4$, $n=3000$; so that the necessary thickness is about $12\frac{1}{2}$ centimetres—a result which abundantly explains the observations which led me to calculate the power of prisms.

Terling Place,
August 12, 1879.

[To be continued.]