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XII. *On the Figure of the Earth.* *By* Colonel A. R. CLARKE, *C.B., F.R.S.**

THE fraction $\frac{1}{300}$, which, in round numbers, is taken to express the ellipticity of the earth, has apparently a tendency, as far as it is deduced from the measurement of terrestrial arcs, to increase as the data of the problem are added to. The $\frac{1}{298}$, obtained by Airy and Bessel from the very imperfect data of forty years back, was replaced, on the completion of the Russian and English arcs in 1858, by $\frac{1}{294}$; and the geodetic work recently completed in India indicates a further increase of the fraction, and so an assimilation to that obtained from pendulum observations. The data of the Indian arc of 21°, as used in 1858, were vitiated by a serious uncertainty as to the unit of length used by Colonel Lambton in the measurement of the southern half of that arc. It appears from the Annual Reports of Colonel Walker, C.B., F.R.S., Surveyor-General of India, who has been for many years Superintendent of the Great Trigonometrical Survey of India (reports which are replete with scientific interest), that this southern portion of "the Great Arc," as Colonel Everest delighted to call it, has been completely remeasured and the latitudes of a great number of stations in it determined. A complete meridian chain of triangles has also been carried from Mangalore on the west coast, in latitude 12° 52′ and longitude 75° E., to a point in latitude 32°. As this triangulation is rigidly connected with the arc from Cape Comorin to Kaliana, in 78° E. longitude, it may be considered that the Indian Arc is now 24° in length.

* Communicated by the Author.

Colonel Walker's last Report contains the details of eleven determinations of difference of longitude by electro-telegraphy, with the corresponding geodetic differences. The differences of longitude are between Mangalore and Bombay on the west coast, Vizagapatam and Madras on the east coast, and Hydrabad, Bangalore, and Bellary in the interior. Bellary holds a somewhat central position in the polygon formed by the other points; Mangalore and Madras are very nearly in the same latitude; and Bangalore is midway between them. These differences of longitude have been determined with every refinement of modern science, and, taking into account the uncertainty of local attraction, may be considered little, if at all, inferior to latitude-determinations. "When the operations were commenced," says Colonel Walker, "I determined that they should be carried on with great caution and in such a manner as to be self-verificatory, in order that some more satisfactory estimate might be formed of the magnitudes of the errors to which they are liable than would be afforded by the theoretical probable errors of the observations. . . . . . The simplest arrangement appeared to be to select three trigonometrical stations A, B, C, at nearly equal distances apart on a telegraphic line forming a circuit, and, after having measured the longitudinal arcs corresponding to A B and B C, to measure A C independently as a check on the other two arcs."

The eleven determinations of difference of longitude between the seven points named above give thus five equations of condition among themselves, which enable us to assign a system of minimum corrections to the several determinations. The following Table contains the observed differences, together with the computed corrections, half-weight being given to the first two determinations, which were the earliest made, and which are affected with some slight defects, then undiscovered, in one of the transit-instruments:—

Year.	Arc.	Electro-telegraphic difference of longitude.	Corrections.
1872-73.	Madras—Bangalore.	° 2 39 45·63	+2·010
„	Bangalore—Mangalore.	2 44 11·54	+1·690
1875-76.	Hydrabad—Bombay.	5 42 12·74	−0·452
„	Bellary—Bombay.	4 6 44·39	−0·393
„	Hydrabad—Bellary.	1 35 28·25	+0·040
„	Madras—Hydrabad.	1 43 40·38	−0·412
„	Madras—Bellary.	3 19 8·45	−0·192
„	Bangalore—Bellary.	0 39 20·46	+0·160
1876-77.	Vizagapatam—Madras.	3 2 26·78	+0·401
„	Vizagapatam—Bellary.	6 21 35·84	−0·401
„	Mangalore—Bombay.	2 1 50·54	+0·845

The following Table contains in the second column the geodetic longitudes as given by Colonel Walker, computed on Everest's elements; viz. equatorial semiaxis $a'=20922932$, polar semiaxis $c'=20853375$:—

Names.	Geodetic longitudes on Everest's spheroid.	Geodetic longitudes on undetermined spheroid.
Vizagapatam	83° 19′ 47″·00	43″·05 − 2·320 u − 2·401 v
Hydrabad	78 33 38·50	37·51 − 0·580 u − 0·600 v
Bombay	72 51 16·23	18·80 + 1·511 u + 1·563 v
Mangalore	74 53 10·18	11·44 + 0·742 u + 0·768 v
Bangalore	77 37 27·72	27·32 − 0·234 u − 0·242 v
Madras	80 17 21·87	19·85 − 1·184 u − 1·226 v
Bellary	76 58 6·97	

The third column contains (omitting degrees and minutes) the same longitudes on the supposition that the elements are

$$c=20855500\left(1+\frac{u}{10000}\right),$$

$$\frac{a-c}{a+c}=n=\frac{1}{590}+v\sin 10'',$$

which I take for the undetermined elements of the spheroid most nearly representing the mean figure of the earth. The terms in u and v added to the longitudes in the above Table are thus obtained:—Let A be the central point Bellary, B one of the other stations, Q the point in which the normal at A meets the axis of revolution; let θ be the angle subtended at Q by the curve distance A B, this curve being the intersection of the spheroid with the vertical plane at A passing through B; then, if A B $=s$, and ϕ_1 be the latitude of A, and α the azimuth of B at A,

$$\theta=\frac{s}{c}\,\frac{1-n}{(1+n)^2}(1+2n\cos 2\phi_1+n^2)^{\frac{1}{2}}(1+\tfrac{2}{3}n\theta^2\cos^2\phi_1\cos^2\alpha).$$

Thus for any variations of n and c a determinate variation arises for θ which may be expressed in the above u and v.

Again, the variation of θ gives rise to a variation of ω, the longitude of B computed from A, viz. $\delta\omega=\sin B\sec\psi\,.\,\delta\theta$, where B is the azimuth of the curve A B at B, and ψ is the inclination of the line Q B to the equator.

Let us suppose the easterly component of the local attraction at A is y_1; then, longitudes being measured positively towards the east, the observed or astronomical longitude of A must receive the correction $y_1 \sec \phi_1$; so that of B receives a corresponding correction $y_2 \sec \phi_2$; and the difference of longitude as observed (*i. e.* of B east of A) must receive the correction $y_2 \sec \phi_2 - y_1 \sec \phi_1$. The astronomical difference of longitude thus corrected is to be equated to the corresponding difference of geodetic longitudes as expressed in terms of u and v; then multiplying by $\cos \phi_2$, we get an equation of the form

$$y_2 = y_1 \sec \phi_1 \cos \phi_2 + \alpha u + \beta v + \gamma.$$

The above data afford six such equations. Properly speaking, as a direct check upon these equations, we should add to them the six equations of a similar character which would result from a comparison of the observed directions of the meridian at the seven stations we are considering. I have not, however, the quantities for forming these equations.

Besides the data contained in his last annual report, Colonel Walker has kindly given me provisional results for his great arcs (or arc, for we may consider them as one)—not final results, but yet not likely to be materially altered. The Indian Triangulation contains a vast number of astronomical stations; but in the problem of the figure of the earth it is not desirable that the latitude-points in one of the arcs should be very much more numerous than in the others. The Russian arc of 25° has thirteen astronomical stations; the English has thirty-four; but only fifteen are used in this investigation, this number including those of the French arc: the length of the conjoined English and French arcs is 22°.

Taking fourteen evenly distributed latitudes in the Indian arc, they require the corrections shown in the following Table (the column on the left gives the approximate latitude of each station):—

32° 2′	$-4{\cdot}14-8{\cdot}562\,u+5{\cdot}102\,v+0{\cdot}997\,x$
30 22	$-0{\cdot}25-7{\cdot}969\,u+4{\cdot}988\,v+0{\cdot}998\,x$
29 30	$+3{\cdot}37-7{\cdot}662\,u+4{\cdot}912\,v+0{\cdot}998\,x$
27 55	$-1{\cdot}98-7{\cdot}092\,u+4{\cdot}742\,v+0{\cdot}998\,x$
24 25	$+2{\cdot}22-6{\cdot}312\,u+4{\cdot}446\,v+0{\cdot}998\,x$
24 7	$-0{\cdot}93-3{\cdot}725\,u+4{\cdot}179\,v+0{\cdot}999\,x$
22 1	$-2{\cdot}17-4{\cdot}970\,u+3{\cdot}783\,v+0{\cdot}999\,x$
20 44	$+5{\cdot}51-4{\cdot}511\,u+3{\cdot}514\,v+0{\cdot}999\,x$
18 3	$+2{\cdot}65-3{\cdot}545\,u+2{\cdot}886\,v+0{\cdot}999\,x$
16 10	$+6{\cdot}09-2{\cdot}865\,u+2{\cdot}397\,v+0{\cdot}999\,x$
14 55	$-1{\cdot}75-2{\cdot}418\,u+2{\cdot}057\,v+1{\cdot}000\,x$
12 59	$+5{\cdot}18-1{\cdot}726\,u+1{\cdot}502\,v+1{\cdot}000\,x$
10 59	$+0{\cdot}43-1{\cdot}005\,u+0{\cdot}893\,v+1{\cdot}000\,x$
8 12	$0{\cdot}00 \quad 0{\cdot}000\,u \quad 0{\cdot}000\,v+1{\cdot}000\,x$

It is interesting to consider the influence of each of the three great arcs in determining the semiaxes of the earth. The northern ten degrees from 60° to 70° of the Russian arc determine $\frac{3}{2}a-\frac{1}{2}c$; the ten degrees in England from 50° to 60° determine a; the ten degrees in France from 40° to 50° determine $\frac{1}{2}a+\frac{1}{2}c$; the ten degrees in India from 10° to 20° determine $-\frac{11}{14}a+\frac{25}{14}c$. Or, more precisely, taking the arcs in combination, suppose each arc to have six astronomical stations, equidistant, 5° apart in the Russian and 4° apart in the two other arcs; and let these arcs be combined by the method of least squares to determine the mean figure of the earth. Let $\theta_1 \ldots \theta_6$ be the latitudes of the stations in the Russian arc, numbered from north to south; $\phi_1 \ldots \phi_6$ those of the Anglo-French; $\psi_1 \ldots \psi_6$ those of the Indian; then, supposing these expressed in seconds, a involves, in feet,

Russian.	Anglo-French.	Indian.
$-117{\cdot}6\,\theta_1$	$-76{\cdot}2\,\phi_1$	$-5{\cdot}4\,\psi_1$
$-\ 63{\cdot}7\,\theta_2$	$-40{\cdot}7\,\phi_2$	$+0{\cdot}3\,\psi_2$
$-\ 14{\cdot}5\,\theta_3$	$-\ 8{\cdot}8\,\phi_3$	$+3{\cdot}0\,\psi_3$
$+\ 29{\cdot}3\,\theta_4$	$+19{\cdot}1\,\phi_4$	$+3{\cdot}2\,\psi_4$
$+\ 67{\cdot}3\,\theta_5$	$+43{\cdot}2\,\phi_5$	$+1{\cdot}2\,\psi_5$
$+\ 99{\cdot}3\,\theta_6$	$+63{\cdot}4\,\phi_6$	$-2{\cdot}3\,\psi_6$

and c involves

Russian.	Anglo-French.	Indian.
$-26{\cdot}5\,\theta_1$	$-39{\cdot}6\,\phi_1$	$-112{\cdot}5\,\psi_1$
$-23{\cdot}0\,\theta_2$	$-28{\cdot}9\,\phi_2$	$-\ 71{\cdot}1\,\psi_2$
$-14{\cdot}5\,\theta_3$	$-14{\cdot}6\,\phi_3$	$-\ 26{\cdot}7\,\psi_3$
$-\ 0{\cdot}6\,\theta_4$	$+\ 3{\cdot}9\,\phi_4$	$+\ 20{\cdot}3\,\psi_4$
$+19{\cdot}3\,\theta_5$	$+26{\cdot}3\,\phi_5$	$+\ 69{\cdot}5\,\psi_5$
$+45{\cdot}4\,\theta_6$	$+52{\cdot}8\,\phi_6$	$+120{\cdot}3\,\psi_6$

From which we see at a glance the effect that would result from an alteration of any one of the latitudes.

It seems unnecessary to give here the expressions for the corrections to the stations of the English, the Russian, Cape-of-Good-Hope, and Peruvian arcs, which are to be found in a paper on the Figure of the Earth, in the Memoirs of the Royal Astronomical Society for 1860, pp. 34, 35. It is only necessary to remark that the sign of u in those expressions is to be changed, and that I have now added three points to the Anglo-French arc as there used. Making now the sum of the squares of the corrections, or local attractions, at the forty-nine latitude stations and the seven longitude stations a minimum, the resulting equations in u and v are

$$0 = +56{\cdot}6615 + 301{\cdot}7624\,u + 126{\cdot}9252\,v,$$
$$0 = -16{\cdot}9677 + 126{\cdot}9252\,u + 221{\cdot}4307\,v;$$
$$\therefore\ u = -0{\cdot}2899;\quad v = +0{\cdot}2428.$$

From these we have, in feet of the standard yard,

$$\left.\begin{aligned} a &= 20926202, \\ c &= 20854895, \\ \frac{c}{a} &= \frac{292{\cdot}465}{293{\cdot}465}. \end{aligned}\right\} \quad \ldots\ldots \quad \text{(E)}$$

And this is the spheroid most nearly representing the mean figure of the earth.

But the Indian observations are not well represented by this figure. The southern station of the arc requires a large negative correction of $-3''{\cdot}14$, and the northern station a still larger negative correction of $-3''{\cdot}55$. Among the longitude stations, there is left at Bombay a westerly deflection of $4''{\cdot}05$, and at Madras an easterly deflection of $4''{\cdot}50$. The longitudes, in fact, require a larger value of a and a larger value of the ellipticity; while the form of the meridian-arc requires a smaller equatorial radius and a smaller ellipticity. In other words, so far as the observations we have at present to consider indicate, the surface of India does not seem to belong to a spheroid of revolution: if it does, we must admit large deflections towards the sea at Cape Comorin, at Bombay, and at Madras.

But we may obtain more strictly the form of the Indian arc from the sixty-six latitude stations it contains. Not to confine the arc to an elliptic form, let it be such that its radius of curvature in latitude ϕ is expressed by the equation

$$\rho = \mathrm{A}' + 2\mathrm{B}' \cos 2\phi + 2\mathrm{C}' \cos 4\phi,$$

a curve which includes the ellipse as a particular case. In order to determine A, B, C, we must apply symbolical corrections to the observed latitudes, and make the sum of the squares of these corrections a minimum. As the result of a very long calculation, the actual equation is found to be

$$\rho = 20932184{\cdot}1 - 167963{\cdot}6 \cos 2\phi + 28153{\cdot}2 \cos 4\phi. \quad . \quad \text{(E}'\text{)}$$

The correction to the latitude of the southern point is $+1''{\cdot}61$, and to the northern $-0''{\cdot}81$; and, generally, the residual corrections or apparent local attractions are free from any appearance of law, so that the above equation may be taken as very closely representing the form of the sea-level along the meridian of India. The geodetic operations give us the *form* of the curve in the shape of its intrinsic equation; and the absolute direction of the curve with reference to the polar axis is given by observing that, at its southern extremity, the actual direction of the surface of the sea makes an angle of $1''{\cdot}61$ with the curve. Now, the observed latitude of the southern point being $8^\circ\ 12'\ 10''{\cdot}44$, the direction of the normal to our curve (E′) at the same point makes the angle $8^\circ\ 12'\ 12''{\cdot}05$ with the plane of the equator, which determines the curve as to its absolute direction. So also, on referring the Indian meridian to the ellipse (E), determined above as representing the mean figure of the earth, this ellipse at the southern point of the Indian arc has its normal inclined to the equator at an angle equal to observed latitude $-3''{\cdot}14$, or $8^\circ\ 12'\ 7''{\cdot}30$. We can now trace the difference of the forms of the curves (E, E′) by making them coincide at the southern point of the arc. The selection for this purpose of the southern point is quite arbitrary; any other station would have done equally well. Multiply the expression for ρ, the radius of curvature, by $-\sin\phi\, d\phi$ and then by $\cos\phi\, d\phi$, and integrate; thus we get the following values of the coordinates of the curve (E′) in the meridian plane, parallel and perpendicular to the equator:—

$$\left.\begin{aligned} x' &= (A'-B')\cos\phi + \tfrac{1}{3}(B'-C')\cos 3\phi + \tfrac{1}{5}C'\cos 5\phi + H, \\ y' &= (A'+B')\sin\phi + \tfrac{1}{3}(B'+C')\sin 3\phi + \tfrac{1}{5}C'\sin 5\phi + K, \end{aligned}\right\} (E')$$

when H and K are disposable constants. The corresponding coordinates of the ellipse (E) may also be written in the form

$$\left.\begin{aligned} x &= (A-B)\cos\phi + \tfrac{1}{3}(B-C)\cos 3\phi + \tfrac{1}{5}C\ \cos 5\phi, \\ y &= (A+B)\sin\phi + \tfrac{1}{3}(B+C)\sin 3\phi + \tfrac{1}{5}C'\sin 5\phi. \end{aligned}\right\} \quad . \quad (E)$$

The values of H and K are now to be determined by putting $\phi = 8^\circ\ 12'\ 12''{\cdot}05$ in the expressions for x' and y', and $\phi = 8^\circ\ 12'\ 7''{\cdot}30$ in those for x and y; then putting $x = x'$, $y = y'$.

The normal distance between the curves (E, E′) in latitude ϕ is $\zeta = (x'-x)\cos\phi + (y'-y)\sin\phi$: this expresses the distance by which a point in (E′) is further from the centre of the earth than the corresponding point of (E). Put $A'-A = E$, $B'-B = F$, $C'-C = G$; then

$$\zeta = E - \tfrac{2}{3}F\cos 2\phi - \tfrac{2}{15}G\cos 4\phi + H\cos\phi + K\sin\phi.$$

The following Table shows, according to this formula, the departure of the curve best representing the Indian meridian from that best representing the earth as a whole. I add also similar quantities for the Russian and Anglo-French arcs; the only difference is that, in the case of these last arcs, the local curve is simply that elliptic curve which best represents the observations.

Indian.		Anglo-French.		Russian.	
Lat.	ζ.	Lat.	ζ.	Lat.	ζ.
°	ft.	°	ft.	°	ft.
10	−11·8	40	+ 8·1	48	−2·7
12	−18·5	42	+15·7	50	−3·7
14	−19·6	44	+18·9	52	−4·0
16	−16·7	46	+18·8	54	−3·7
18	−11·1	48	+16·1	56	−2·9
20	− 4·3	50	+11·8	58	−1·8
22	+ 2·1	52	+ 6·8	60	−0·5
24	+ 6·9	54	+ 1·9	62	+0·8
26	+ 9·3	56	− 1·8	64	+2·0
28	+ 8·3	58	− 3·6	66	+3·1
30	+ 3·8	60	− 2·7	68	+3·8
32	− 4·2			70	+4·1

Here we see the local form of the meridian sea-level in India with reference to the mean figure of the earth. Supposing that there is no disturbance of the sea-level at Cape Comorin, then from that point northwards a depression sets in, attaining a maximum of nearly 20 feet at about 14° latitude; thence it diminishes, disappearing at about 21°. An elevation then commences, attaining at 26° about nine feet; then this elevation diminishes, and becomes a small depression at 32°. This deformation may or may not be due to Himalayan attraction; at any rate we have here an indication that that vast tableland does not produce the disturbance that might *à priori* have been anticipated. This is in accordance with the fact that there is an attraction *seaward* at Mangalore and Madras, and slightly also at Bombay: and I think we have here a corroboration of Archdeacon Pratt's theory, that where the crust of the earth is thickest there it is least dense; and where thinnest, as in ocean-beds, there it is most dense.

The Anglo-French arc shows a deformation nearly as great as the Indian—though, after all, the linear magnitude in either case is certainly as small as could be expected. One cannot help remarking here, that the remeasurement of the French meridian-arc, with all modern refinements of observation and calculation, with a considerable increase in the number of latitude stations, would be a vast service to science.

With the elements of the earth's spheroidal figure at which we have arrived above (E) the following results are obtained. The radii of curvature in and perpendicular to the meridian in latitude ϕ being ρ, ρ', their values in standard feet are,

$$\rho = 20890564 - 106960 \cos 2\phi + 228 \cos 4\phi,$$

$$\rho' = 20961932 - 35775 \cos 2\phi + 46 \cos 4\phi.$$

The lengths of one degree in and perpendicular to the meridian, viz. δ, δ', are

$$\delta = 364609{\cdot}12 - 1866{\cdot}72 \cos 2\phi + 3{\cdot}98 \cos 4\phi,$$

$$\delta' = 365854{\cdot}72 - \ \ 624{\cdot}40 \cos 2\phi + 0{\cdot}80 \cos 4\phi.$$

Also the following:—

$$\log \frac{1}{\rho \sin 1''}$$

$$= 7{\cdot}994477820 + {\cdot}002223606 \cos 2\phi - {\cdot}000001897 \cos 4\phi,$$

$$\log \frac{1}{\rho' \sin 1''}$$

$$= 7{\cdot}992994150 + {\cdot}000741202 \cos 2\phi - {\cdot}000000632 \cos 4\phi.$$

Having seen that the surface of India cannot be represented by a spheroid of revolution, it is necessary now to inquire what ellipsoid best represents all the observations as the figure of the earth. On this hypothesis, the equator being no longer a circle, the ellipticity of a meridian is not a constant, but is a function of the longitude—say l, from Greenwich. We have consequently to replace our previous v by $v + w \cos 2l + z \sin 2l$; and the longitude l' of the greater semiaxis of the equator will be given by the equation $w \sin 2l' - z \cos 2l' = 0$. But this substitution cannot be made in the longitude-equations—they no longer hold good, having been formed on the distinct supposition of the earth being a surface of revolution, and they must now be put aside. If the earth should be found to be really ellipsoidal, this circumstance will involve a considerable increase of the labours of the geodetic computer. The "meridian" on an ellipsoid is somewhat vague. If it be taken as the locus of points of constant longitude ω, its equation in combination with that of the ellipsoid is

$$b^2 x \sin \omega - a^2 y \cos \omega = 0. \quad . \quad . \quad . \quad . \quad (1)$$

But it may also be defined as a line on the ellipsoid whose direction is always north and south. Suppose that a point on the surface of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ moves always towards a given fixed point $x'y'z'$, and let it be required to determine the nature of the curve traced by the moving point. Two consecutive points on the curve having coordinates x, y, z, $x + dx$, $y + dy$, $z + dz$ give the condition

$$\frac{x}{a^2} dx + -\frac{y}{b^2} dy + \frac{z}{c^2} dz = 0. \quad . \quad . \quad . \quad . \quad (2)$$

The equation of a plane passing through x, y, z and x', y', z' is

$$\mathrm{A}(x' - x) + \mathrm{B}(y' - y) + \mathrm{C}(z' - z) = 0.$$

This plane is to contain the normal at x, y, z, and the point $x+dx, y+dy, z+dz$, which conditions give two other equations in A, B, C; and eliminating these symbols we have the differential equation of the required curve expressed by the determinant

$$\begin{vmatrix} x'-x, & y'-y, & z'-z, \\ dx, & dy, & dz, \\ \frac{x}{a^2}, & \frac{y}{b^2}, & \frac{z}{c^2}. \end{vmatrix}$$

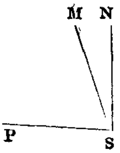
A north-and-south line is a particular case of this curve, viz. when $x'=0$, $y'=0$, $z'=\infty$; then the equation becomes

$$a^2 y\, dx - b^2 x\, dy = 0, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

of which the integral is

$$x^{a^2} = c y^{b^2}.$$

This is not a plane curve; and at each point its direction makes a definite angle with the meridian as expressed by (1). Let S be any point on the surface of the ellipsoid, say in that octant where x, y, z are all positive; let M be a point indefinitely near S on the same meridian (1), N a point on the north line (3), P a point on the parallel of latitude through S, of which, ϕ being the latitude, the equation is



$$\frac{x^2}{a^4} + \frac{y^2}{b^4} - \frac{z^2}{c^4}\cot^2\phi = 0. \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

The differential equation of the meridian is

$$-b^2 \sin\omega\, dx + a^2 \cos\omega\, dy = 0. \quad . \quad . \quad . \quad . \quad . \quad (5)$$

And if from this equation, with (2), we determine the ratios of dx, dy, dz, they are found to be proportional to

$$-a^2 z \cos\omega \quad : \quad -b^2 z \sin\omega \quad : \quad c^2(x\cos\omega + y\sin\omega) \quad . \quad (6)$$

And these are proportional to the direction-cosines of SM. So also, getting the ratios of dx, dy, dz from (2) and (3), we find the direction-cosines of SN to be proportional to

$$-b^2 xz \quad : \quad -a^2 yz \quad : \quad c^2\left(\frac{a^2}{b^2}y^2 + \frac{b^2}{a^2}x^2\right), \quad . \quad . \quad . \quad . \quad (7)$$

Similarly for the direction-cosines of SP; they are as

$$-a^2 yz\left(\frac{1}{b^2} + \frac{\cot^2\phi}{c^2}\right) : b^2 xz\left(\frac{1}{a^2} + \frac{\cot^2\phi}{c^2}\right) : c^2 xy\left(\frac{1}{b^2} - \frac{1}{a^2}\right). \ (8)$$

These enable us to determine the angles between the lines SM, SN, and SP. Let the semiaxes of the equator be expressed by the relations

$$a^2=k^2(1+i)\,; \quad b^2=k^2(1-i),$$

where i is a very small quantity whose square is to be neglected. Then the coordinates x, y, z of any point, as S, are proportional to

$$(1+i)\cos\omega : (1-i)\sin\omega : \frac{c^2}{k^2}\tan\phi.$$

Substitute these in (6), (7), (8), and we get finally the following results for the angles between the lines in question:—

$$\text{MSP}=\frac{\pi}{2}-i\sin\phi\sin 2\omega\left(1+\frac{c^2}{c^2\sin^2\phi+k^2\cos^2\phi}\right),$$

$$\text{NSP}=\frac{\pi}{2}-i\frac{c^2\sin\phi\sin 2\omega}{c^2\sin^2\phi+k^2\cos^2\phi},$$

$$\text{MSN}=i\sin\phi\sin 2\omega.$$

In the figure of the earth, as determined in the paper in the 'Memoirs of the Royal Astronomical Society' for 1860, there is a difference of a mile between the greatest and least radii of the equator. Although this seems but a small departure from the form of a circle, yet $i=52''\cdot33$ (in parts of radius unity), and the angles expressed above become somewhat large quantities. Supposing S to be on a meridian midway between the greatest and least radii of the equator, the angle between the "meridian" and the "north line" is $52''\cdot33\sin\phi$; and the defect of MSP from a right angle is about double this quantity. So large an angle as this should be detected by firstrate geodetic observations, though it would require a somewhat long measurement of meridian and parallel. It is to be remembered that, SM, SN being directed towards the north, and SP towards the minor axis of the equator, SM lies between SP and SN.

And in an ellipsoidal earth the direction of the principal sections of the surface (that is, of maximum and minimum curvature) are no longer coincident with meridians, north lines, or parallels. Supposing that S is not in a very high latitude, one of the lines of curvature, as SR through S, will lie somewhere in the direction of SP, and the second line of curvature will be perpendicular to SR. It may be shown that the angle

$$\text{RSN}=\frac{\pi}{2}-i\sin 2\omega\sin\phi\sec^2\phi\,\frac{c^2}{k^2-c^2},$$

an expression which does not hold in high latitudes; for in the vicinity of the umbilics, the lines of curvature are approximately confocal conics having the umbilics as foci. The defect of RSN from a right angle might, with the value of i we have been supposing, amount to some degrees without going to any high latitudes.

It appears, then, that it would not do to take the longitude-equations which we have used for the determination of a spheroidal figure for the earth also for the determination of an ellipsoidal figure. The only thing that can be done under the circumstances is to take simply the longitude-arc between Bombay and Vizagapatam, as these points are nearly in the same latitude, and to reduce it according to the expression for the length of an arc of parallel on the surface of an ellipsoid, given in the before-mentioned paper on the Figure of the Earth, page 43.

Then, with fifty-one equations I get the following:—

$$u = -0{\cdot}4903\,;$$
$$v = +0{\cdot}2842\,;$$
$$w = +0{\cdot}3599\,;$$
$$z = -0{\cdot}1067.$$

From these quantities the following values finally result:—

$$a = 20926629\,;$$
$$b = 20925105\,;$$
$$c = 20854477.$$

If by the word "ellipticity" of an ellipse we mean the ratio of the difference of the semiaxes to half the sum of the same, the ellipticities of the two principal meridians of the earth are

$$\frac{1}{289{\cdot}54},\ \frac{1}{295{\cdot}77}.$$

The longitude of the greater axis of the equator is 8° 15′ west of Greenwich—a meridian passing through Ireland and Portugal and cutting off a portion of the north-west corner of Africa; in the opposite hemisphere this meridian cuts off the north-eastern corner of Asia and passes through the southern island of New Zealand. The meridian containing the smaller diameter of the equator passes through Ceylon on the one side of the earth and bisects North America on the other. This position of the axis, brought out by a very lengthened calculation, certainly agrees very remarkably with the physical features of the globe—the distribution of land and water on its surface. On the ellipsoidal theory of the earth's figure, small as is the difference between the two diameters of the equator, only 3000 feet, the Indian longitudes are better represented than on the spheroidal; but there is still left at Madras and Mangalore an attraction or disturbance of the plumb-line seawards.

As to the relative evidence for the two figures presented in this paper, the sum of the squares of the residual corrections to the astronomical observations is, of course, less in the ellipsoid than in the spheroid; but the difference is certainly small. The radius of curvature perpendicular to the meridian in India, in latitude 15° say, is, on the spheroid, 20930972 feet, whereas on the ellipsoid it is 20932877; and this last is distinctly more in harmony with the Indian Longitude Observations.

Ordnance Survey Office, Southampton,
June 15, 1878.

XIII. *On Telephony.* *By* W. Siemens *.

THE surprising performances of the telephones of Bell and Edison rightly claim in a high degree the interest of natural philosophers. The solution (facilitated by it) of the problem of the conveyance of tones and the sounds of speech to distant places promises to give mankind a new means of intercourse and culture which will essentially affect their social relations and also render substantial service to science; and hence it seems fitting that the Academy should draw these exceedingly promising discoveries into the sphere of its contemplations.

The possibility of reproducing mechanically not merely tones, but also noises and spoken sounds, at great distances is given theoretically by Helmholtz's path-opening investigations, which elucidated the essential nature of shades of tone and the sounds of speech.

If, as he has demonstrated, noises and sounds are only distinguished from pure tones by the fact that the latter consist of simple, the former of a plurality of series of undulations, superposed to one another, of the sonorific medium, and if the noises of speech (*Sprachgeräusche*) may be conceived as irregular vibrations with which the vocal sounds begin or end, then it is also possible to reproduce mechanically a certain succession of such vibrations at distant localities. Indeed practical life has in this, as is frequently the case, outrun science. The hitherto too little regarded so-called "speaking telegraph," consisting of two membranes stretched by a strong and at the same time extremely light thread or fine wire which is fastened to their centres, effects a perfectly distinct transmission of speech to a distance of several hundred metres. The threads or wires can

* Translated from the *Monatsbericht der königlich preussischen Akademie der Wissenschaften zu Berlin*, January 1878, pp. 38–53.