



XLIX. On quaternions; or on a new system of imaginaries in algebra

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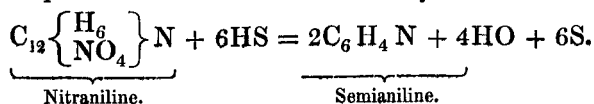


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will doubtless be procured by the further action of reducing agents upon nitraniline. We shall shortly ascertain this.



The small quantity of resinous matter which remains behind upon dissolving crude nitraniline in boiling water is probably Zinin's semianiline.

XLIX. On Quaternions; or on a New System of Imaginaries in Algebra. By Professor Sir WILLIAM ROWAN HAMILTON, LL.D., Corresponding Member of the Institute of France, and Royal Astronomer of Ireland.

[Continued from p. 122.]

28. **T**HE known and purely *graphic* property of the cone of the second degree which constitutes the theorem of Pascal, and which expresses the coplanarity of the three lines of meeting of opposite plane faces of an inscribed hexahedral angle, may be transformed into another known but purely *metric* property of the same cone of the second degree, which is a form of the theorem of M. Chasles, respecting the constancy of an anharmonic ratio. This transformation may be effected without difficulty, on the plan of the present paper; for if we multiply into $v. \gamma \gamma'$ both members of the equation (3.) of the 24th article, and then operate by the characteristic s ., attending to the general properties of scalars of products, we find, for *any six vectors* $\alpha \alpha' \beta \beta' \gamma \gamma'$, the formula

$$s(v. \alpha \alpha'. v. \beta \beta'. v. \gamma \gamma') = s. \alpha \gamma \gamma'. s. \alpha' \beta \beta' - s. \alpha' \gamma \gamma'. s. \alpha \beta \beta'; \quad (1.)$$

which gives, for *any five vectors* $\alpha \alpha' \alpha'' \gamma \gamma'$, this other:

$$s(v. \alpha \alpha'. v. \alpha' \alpha''. v. \gamma \gamma') = s. \alpha \alpha' \alpha''. s. \gamma \alpha' \gamma' \quad . \quad (2.)$$

If, then, we take six arbitrary vectors $\alpha \alpha' \alpha'' \alpha''' \alpha^{IV} \alpha^V$, and deduce nine other vectors from them by the expressions

$$\left. \begin{aligned} \alpha_0 &= v. \alpha \alpha', \alpha_1 = v. \alpha' \alpha'', \alpha_2 = v. \alpha'' \alpha''', \\ \alpha_3 &= v. \alpha''' \alpha^{IV}, \alpha_4 = v. \alpha^{IV} \alpha^V, \alpha_5 = v. \alpha^V \alpha, \\ \beta &= v. \alpha_0 \alpha_3, \beta' = v. \alpha_1 \alpha_4, \beta'' = v. \alpha_2 \alpha_5; \end{aligned} \right\} . \quad . \quad (3.)$$

we shall have, *generally*,

$$\left. \begin{aligned} s. \beta \beta' \beta'' &= s. \alpha_0 \alpha_3 \alpha_5 + s. \alpha_3 \alpha_1 \alpha_4 - s. \alpha_3 \alpha_2 \alpha_5 + s. \alpha_0 \alpha_1 \alpha_4 \\ &= s. \alpha_0 \alpha_1 \alpha_4 + s. \alpha_2 \alpha_3 \alpha_5 - s. \alpha_3 \alpha_4 \alpha_1 + s. \alpha_5 \alpha_0 \alpha_2 \\ &= s. \alpha \alpha' \alpha'', s. \alpha^{IV} \alpha' \alpha^V, s. \alpha'' \alpha''' \alpha^{IV}, s. \alpha^V \alpha''' \alpha \\ &\quad - s. \alpha''' \alpha^{IV} \alpha^V, s. \alpha' \alpha^{IV} \alpha'', s. \alpha^V \alpha \alpha', s. \alpha'' \alpha \alpha''' \\ &= s. \alpha \alpha' \alpha'', s. \alpha'' \alpha''' \alpha^{IV}, s. \alpha \alpha''' \alpha^V, s. \alpha^V \alpha' \alpha^{IV} \\ &\quad - s. \alpha \alpha''' \alpha'', s. \alpha'' \alpha' \alpha^{IV}, s. \alpha \alpha' \alpha^V, s. \alpha^V \alpha''' \alpha^{IV}. \end{aligned} \right\} . \quad . \quad (4.)$$

Thus if, in particular, the six vectors $\alpha \dots \alpha^v$ are such as to satisfy the condition

$$s. \beta \beta' \beta'' = 0, \quad . \quad . \quad . \quad . \quad . \quad . \quad (5.)$$

they will satisfy also this other condition, or this other form of the same condition :

$$\frac{s. \alpha \alpha' \alpha''}{s. \alpha \alpha''' \alpha'''} \cdot \frac{s. \alpha'' \alpha''' \alpha^{iv}}{s. \alpha'' \alpha' \alpha^{iv}} = \frac{s. \alpha \alpha' \alpha^v}{s. \alpha \alpha''' \alpha^v} \cdot \frac{s. \alpha^v \alpha''' \alpha^{iv}}{s. \alpha^v \alpha' \alpha^{iv}}; \quad . \quad . \quad (6.)$$

and reciprocally the former of these two conditions will be satisfied if the latter be so.

These two equations, (5.) and (6.), express, therefore, each in its own way, the existence of one and the same geometrical relation between the six vectors $\alpha \alpha' \alpha'' \alpha''' \alpha^{iv} \alpha^v$: and a slight study of the *forms* of these equations suffices to render evident that they both agree in expressing that these six vectors are *homoconic*, in the sense of the 25th article ; or in other words, that the six vectors are sides (or edges) of one common cone of the second degree. Indeed the equation (5.) of the present article, in virtue of the definitions (3.), coincides with the equation (2.) of the article just cited, the symbols β, β', β'' retaining in the one the significations which they had received in the other. The recent transformations show, therefore, that the *equation of homoconicism*, assigned in article 25, may be put under the form (6.) of the present article, which is different, and in *some* respects simpler. The former expresses a *graphic* property, or relation between *directions*, namely that the three lines β, β', β'' , which are the respective intersections of the three pairs of planes $(\alpha \alpha', \alpha''' \alpha^{iv})$, $(\alpha' \alpha'', \alpha^{iv} \alpha^v)$, $(\alpha'' \alpha''', \alpha^v \alpha)$, are all situated in one common plane, if the six homoconic vectors be supposed to diverge from one common origin ; the latter expresses the *metric* property, or relation between *magnitudes*, that the ratio compounded of the two ratios of the two pyramids $(\alpha \alpha' \alpha'') (\alpha'' \alpha''' \alpha^{iv})$ to the two other pyramids $(\alpha \alpha''' \alpha'')$ $(\alpha'' \alpha' \alpha^{iv})$, or that the product of the volumes of the first pair of pyramids divided by the product of the volumes of the second pair, does not vary, when the vector α'' , which is the common edge of these four pyramids, is changed to the new but homoconic vector α^v , as their new common edge, the four remaining homoconic and coinitial edges $\alpha \alpha' \alpha''' \alpha^{iv}$ of the pyramids being supposed to undergo no alteration. The one is an expression of the property of the *mystic hexagram* of Pascal ; the other is an expression of the constancy of the *anharmonic ratio* of Chasles*. The calculus of Quaternions (or the

* Although the foregoing process of calculation, and generally the method of treating geometrical problems by quaternions, which has been extended by the writer to questions of dynamics and thermology, appears to him to be

method of scalars and vectors) enables us, as we have seen, to pass, by a very short and simple symbolical transition, from either to the other of these two great and known properties of the cone of the second degree.

[To be continued.]

L. Intelligence and Miscellaneous Articles.

ON THE DETERMINATION OF CARBONIC ACID IN SALINE COMPOUNDS. BY C. BRUNNER, SEN.

THE estimation of carbonic acid in its combinations is generally effected by ignition, when the compound is one of those which part with the whole of the acid at a red heat. When water is disengaged at the same time, its quantity must either be determined by a separate experiment, and subtracted from the loss experienced on ignition, or collected in a suitable apparatus during the calcination, and so calculated. With those compounds where this method cannot be applied, it is customary to expel the carbonic acid by a stronger acid, for instance sulphuric acid, and to determine its amount from the loss, taking care in this case to retain the water accidentally carried over with it by some suitable substance. Apparatus for this mode of determination have been described by Rose, and recently by Fresenius.

It is readily seen that, according to these methods, the result is always obtained in a negative manner, that is to say, by a loss in weight. As we should certainly endeavour to exchange all such negative methods for positive, I will here communicate one which appears to me applicable in most cases. The substance to be examined is placed in the little flask *a*, and a suitable quantity, for instance an ounce, of water poured over it; upon which the flask is closed with a tight-fitting cork provided with three tubes, and when necessary coated with cement. The straight tube terminates above in a small funnel, through which the sulphuric acid is poured; the second, provided with two bulbs, is connected by its rectangular bend with a tube *bc*, from a third to half an inch wide; and, in order to gain space, likewise furnished with two expansions containing asbestos moistened with sulphuric acid; the third is bent

new, yet it is impossible for him, in mentioning here the name of Chasles, to abstain from acknowledging the deep intellectual obligations under which he feels himself to be, for the information, and still more for the impulse given to his mind by the perusal of that very interesting and excellent History of Geometrical Science, which is so widely known by its own modest title of *Aperçu Historique* (Brussels, 1837). He has also endeavoured to profit by a study of the Memoirs by M. Chasles, on Spherical Conics and Cones of the Second Degree, which have been translated, with Notes and an Appendix, by the Rev. Charles Graves (Dublin, 1841); and desires to take this opportunity of adding, that he conceives himself to have derived assistance, as well as encouragement, in his geometrical researches generally, from the frequent and familiar intercourse which he has enjoyed with the last-named gentleman.