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XC. *A Theory of the Absorption and Scattering of the α Rays.* *By* C. G. DARWIN, *B.A., Lecturer in Mathematical Physics, Manchester University* *.

PART I.—ABSORPTION.

THE α particles from radioactive substances have been very thoroughly investigated, so that the main features of their motion are now well known. On account of their great mass they go straight or nearly straight through matter until their energy is exhausted. It is for this reason principally that an experimental formulation of their law of motion is comparatively so much easier than for the β rays. Bragg† showed that the number of α rays remained constant after traversing matter and that the absorptive effect was to be attributed to changes of velocity; and Geiger ‡ found the form of the velocity curve §, which for mica he empirically represented by the equation $v^3 = V^3 (1 - x/R)$, where V is the initial velocity and R is the "range." The present paper is concerned with the theoretical reason for this curve and the deductions which may be made from it. It also considers, but less completely, the scattering of the α particles.

§ 1. *The Mechanism of Absorption.*

It is known that the ionization produced by an α particle is proportional to the rate at which it loses its energy. It is thus necessary to adopt a structure for matter such that the α particle pulls electrons out of the atoms containing them and in so doing loses velocity. I have taken the atomic structure proposed by Prof. Rutherford ‖. This supposes the atom to consist of a cluster of electrons held by an unknown field of forces round a central charge, which is of such amount as to neutralize them and which is supposed to be the seat of the mass of the atom. This structure presents fewer analytical difficulties in the present problem than any other, and there is strong experimental evidence for it in the large scattering of the α rays ¶. The adoption of this

* Communicated by Prof. E. Rutherford, F.R.S.

† W. H. Bragg, Phil. Mag. vol. x. p. 318 (1905).

‡ H. Geiger, Proc. Roy. Soc. A. vol. lxxxiii. p. 505 (1910).

§ I call by the name "velocity curve" the curve whose ordinate is the velocity of the α rays and whose abscissa is the distance they have travelled from their source.

‖ E. Rutherford, Phil. Mag. vol. xxi. p. 669 (1911).

¶ Rutherford, *loc. cit.*, and H. Geiger, Manc. Lit. & Phil. Soc. 1911.

system necessarily involves that the helium atom, which is formed from an α ray, has only two electrons, an assumption which appears reasonable when it is remembered that the α particle is expelled with enormous velocity from one atom and is perpetually undergoing violent collisions with others, so that it must be of a very simple nature. I proceed parallel with the alternative assumptions that the electrons are distributed in the atom (1) throughout the volume of a sphere round the centre and (2) over its surface.

In passing through an atom an α ray will exert forces on the centre and on all the electrons. It will set any charges it approaches in motion and may succeed in pulling an electron out of the atom. In doing so it will lose velocity, while the ejection of the electron will be equivalent to ionization. Now in the atom there is a complicated field of forces acting between the electron and the rest of the system. This field is quite unknown, but the ease of the occurrence of ionization by collision suggests that it is not very great. That is to say, while an α particle is passing an electron their mutual forces are very much greater than the forces on either of the rest of the atom. On account of its high velocity the α particle will spend a very short time near an electron, and *in considering the motion of the α particle* we shall commit no great error in neglecting the effects of the perturbation of the electron by the rest of the atom. With regard to the motion of the electron this need not at all be true. As soon as the α particle has passed, the predominating factor is the atomic field which may prevent the electron from escaping or may greatly reduce its velocity.

We therefore suppose that the α particle loses its velocity by setting in motion a cluster of electrons whose interactions are negligible. In actual atoms the electrons will probably be already in motion, but it is possible to show that such motion only affects the result by the ratio of the squares of the velocities of electron and α particle, and this may presumably be regarded as a small quantity.

In traversing matter some α particles encounter more atoms than others and go deeper into them. Thus after going a given distance the α particles will have straggled out, and some will be moving faster than others. I have not succeeded in finding the amount of this straggling in the present problem; but it is possible to prove with considerable generality that the mean loss of velocity of the particles after a given number of collisions is equal to the sum of the mean losses in each collision, a proposition which is not self-evident. It is this fact which makes the problem

of absorption much easier than that of transverse scattering, which is really more akin to straggling. In the scattering there is a considerable effect due to the inequalities and irregularities of distribution of the electrons. In the longitudinal motion such irregularities only affect the straggling and produce no change in the mean. The reason for this lies in the fact that here the composition of the effects of successive atoms is by simple addition, while for scattering it is by addition of squares.

§ 2. *The Velocity Curve**.

If E, M, e, m be the charges and masses of α particle and electron ; if $k=m/\mathrm{M}$ and $\lambda=(1+k)\mathrm{E}e/m$, and if v be the velocity with which an α particle approaches an electron on a line at distance p from it, then a simple calculation of their orbits shows that after the two have separated the velocity of the α particle is $v+\Delta v$ where

$$(v+\Delta v)^2=v^2(1+2k\cos 2\mu+k^2)/(1+k)^2 \quad . \quad . \quad (1)$$

if $\tan\mu=\lambda/pv^2$. The α particle is deflected through an angle ψ where

$$\tan\psi=k\sin 2\mu/(1+k\cos 2\mu). \quad . \quad . \quad . \quad . \quad (2)$$

These formulæ are accurate, whatever the values of m and M. But m is the mass of an electron so that k is small and (1) may be rewritten as

$$\Delta v=-kv(1-\cos 2\mu)=-2kv\frac{\lambda^2/v^4}{p^2+\lambda^2/v^4}. \quad . \quad . \quad (3)$$

This is the velocity which the α particle loses in passing a single electron. In summing the effects of all the electrons of the atom, we encounter the difficulty that the forces exerted between an electron at one side of an atom and the particle at the other are not greater than the internal forces of the atom. In such a case, however, (3) gives a very small value, so that practically it is only the electrons very near the path of the α ray which contribute sensibly to the reduction of its velocity. It seems that the inclusion of all the electrons in the atom is less objectionable than the fixing of an arbitrary limit beyond which no forces shall be allowed to count†. Let n be the number of electrons in the atom.

* For a definition of this term see p. 901, note §.

† I originally worked out the whole theory, supposing that only electrons in a cylinder near the path of the particle exerted any force on it. The results are almost identical with the present.

We shall first find the result supposing the electrons distributed uniformly inside a sphere of radius σ_1 and afterwards supposing them arranged over the surface of a sphere of radius σ_2. The numbers 1 and 2 subscribed will refer throughout to these alternative hypotheses.

Let P denote the distance from the centre of the atom of the initial line of motion of the α particle. In passing through the atom this line is slightly changed and the velocity is reduced, but the effects are small, and no error is produced by supposing the velocity on approaching each electron equal to the velocity of approach to the atom, and by supposing the line of motion through the atom to be straight. Let the position of an electron be denoted by cylindrical polar coordinates r, ϕ, z. The loss of velocity due to an electron at r, ϕ, z is

$$2kv\frac{\lambda^2}{v^4}\Big/\frac{\lambda^2}{v^4}+\mathrm{P}^2+r^2-2\mathrm{P}r\cos\phi,$$

and the chance of there being an electron at this point is

$$n_1\Big/\frac{4}{3}\pi\sigma_1^3.\,r\,dr\,d\phi\,dz.$$

The whole loss of velocity on the average thus is:—

$$2kv\frac{\lambda^2}{v^4}\frac{n_1}{4/3\pi\sigma_1^3}\int_0^{\sigma_1}rdr\int_{-\sqrt{\sigma_1^2-r^2}}^{\sqrt{\sigma_1^2-r^2}}dz\int_0^{2\pi}d\phi\Big/\frac{\lambda^2}{v^4}+\mathrm{P}^2+r^2-2\mathrm{P}r\cos\phi.$$

This expression is to be averaged for all values of P. The result gives ρ_1, the mean loss of velocity of a particle in traversing an atom. Then:—

$$\rho_1=2kv\frac{\lambda^2}{v^4}\frac{n_1}{4/3\pi\sigma_1^3}\int_0^{\sigma_1}\frac{2\mathrm{P}d\mathrm{P}}{\sigma_1^2}\int_0^{\sigma_1}rdr\int_{-\sqrt{\sigma_1^2-r^2}}^{\sqrt{\sigma_1^2-r^2}}dz\cdot$$
$$\int_0^{2\pi}d\phi\Big/\Big(\frac{\lambda^2}{v^4}+\mathrm{P}^2+r^2-2\mathrm{P}r\cos\phi\Big).$$

Three of the integrations can be performed, and if $\sigma_1^2v^4/\lambda^2=w_1$, the expression may be reduced to

$$\rho_1=2k\frac{v}{w_1}n_1\left\{\log(1+w_1)-f_1(w_1)\right\} \quad . \quad . \quad (4)$$

where

$$f_1(w)=\int_0^1\frac{x^{3/2}dx}{\sqrt{\left(x-\frac{1}{w}\right)^2+\frac{4}{w}}}\cdot\frac{2}{2+\frac{1}{w}-x+\sqrt{\left(x-\frac{1}{w}\right)^2+\frac{4}{w}}}.$$

The logarithmic term in ρ_1 is the more important. f_1 is the sum of elliptic functions of all three kinds.

In the case of a surface distribution of electrons the corresponding integral reduces to

$$\rho_2 = 2k\frac{v}{w_2}n_2 \left\{ \log(1+w_2) - f_2(w_2) \right\} \quad . \quad . \quad (5)$$

where $w_2 = \sigma_2^2 v^4/\lambda^2$ and

$$f_2(w) = \int_0^1 \frac{x^{1/2}dx}{\sqrt{\left(x-\frac{1}{w}\right)^2+\frac{4}{w}}} \cdot \frac{2}{2+\frac{1}{w}-x+\sqrt{\left(x-\frac{1}{w}\right)^2+\frac{4}{w}}}.$$

This is an elliptic function similar to f_1. For large values of w, f_1 and f_2 have limits 2/3 and 2 respectively and may be expanded in inverse powers of $w^{1/4}$, that is in the inverse velocity. For smaller velocities it is necessary to evaluate them by means of Jacobi's θ functions.

Before finding the equation of motion of the α particle we must estimate the direct effect on it of the central charge. The accurate equation (1) is applicable for this, with modified values of e and m. If these modifications are denoted by accents, the loss of velocity due to the central charge is, when small, given by

$$\Delta v = -\frac{2k'v}{(1+k')^2}\frac{\lambda'^2}{\lambda'^2+P^2v^4}.$$

The cases of large deflexion and large change of velocity are so rare that they may be disregarded, as they do not affect the mean. The average loss of velocity from the central charge thus is

$$\rho' = \frac{2k'v}{(1+k')^2}\frac{\lambda'^2}{\sigma^2v^4}\log\left(1+\frac{\sigma^2v^4}{\lambda'^2}\right).$$

Now the electrical charge e' is equal to ne and hence

$$\lambda' = n\frac{k}{k'}(1+k')\lambda,$$

so that

$$\rho' = \frac{2kv}{w}n \cdot \frac{nk}{k'} \cdot \log\left\{1+w\Big/\left(\frac{nk}{k'}\right)^2(1+k')^2\right\}.$$

Now nk/k' is the ratio of the mass of all the electrons to that of the central charge, and this is quite a small quantity unless n is very large indeed; and we shall subsequently prove that this is not so. It is thus justifiable to neglect ρ' and take ρ as the mean loss of velocity in the atom.

If the number of atoms in a cubic centimetre is N the mean free path for a fast moving particle is $1/N\pi\sigma^2$. If v is the velocity at a distance x from the source we have

$$\frac{1}{N\pi\sigma^2}\frac{dv}{dx}=-\rho,$$

so that if V is the initial velocity

$$N\pi\sigma^2 x=\int_v^V \frac{dv}{\rho}.$$

Putting in the value of ρ found for either volume or surface distribution of electrons, this gives:

$$\begin{aligned}8\pi kNn\sigma^2 x&=\int_v^V 4\,\frac{w}{v}\frac{dv}{\log(1+w)-f(w)}\\&=\int_{\sigma^2v^4/\lambda^2}^{\sigma^2V^4/\lambda^2}\frac{dw}{\log(1+w)-f(w)}\\&=\int_{\log\left(1+\frac{\sigma^2v^4}{\lambda^2}\right)}^{\log\left(1+\frac{\sigma^2V^4}{\lambda^2}\right)}\frac{e^u du}{u-f(e^u-1)}.\end{aligned}$$

For either subscript we shall put

$$\int_1^z \frac{e^u du}{u-f_{1,2}(e^u-1)}=E_{1,2}(z).$$

Except for a constant the functions E_1, E_2 are very similar to the integral-exponential function Ei; indeed, for large values of z they only differ by this constant from $e^z Ei\,(z-a)$ where $a_1=2/3$ and $a_2=2$.

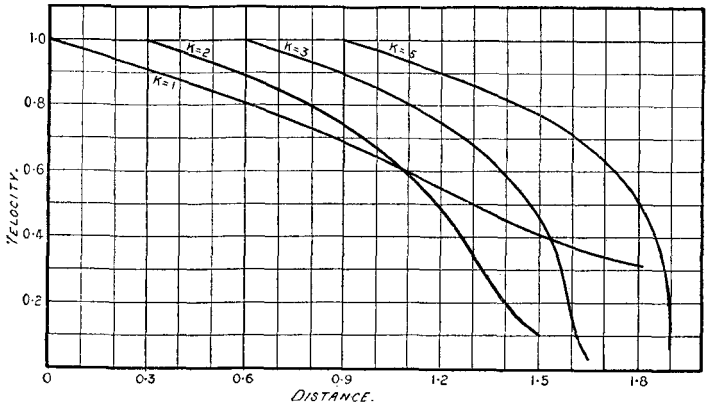
The velocity curve of the α particles thus is:—

$$8\pi kNn\sigma^2 x=E\log\left(1+\frac{\sigma^2V^4}{\lambda^2}\right)-E\log\left(1+\frac{\sigma^2v^4}{\lambda^2}\right)\quad\ldots\quad(6)$$

It is interesting that the form of the curve should depend on the whole number of electrons in a cubic centimetre, and on the radius of the atom, but not at all on the number of atoms or the number of electrons in each atom. For very high velocities the curve is of the form $1-x=Av^4/\log v$. With different substances σ will be different and will make the shape of the curve vary. To consider this we write it in the form

$$1-x=E\log(1+10^\kappa/f^4)/E\log(1+10^\kappa)$$

and call κ the parameter of the curve. It depends on σ and V and is taken in this form to facilitate the use of logarithms. The functions E_1, E_2 were evaluated by quadrature. The velocity curve obtained from E_1 is shown in the figure for



To avoid confusion each curve has been started 0·3 from the last.

parameters 1, 2, 3, 5. Those for E_2 are almost indistinguishable on the scale of the drawing. The curve for $\kappa=3$ is within $\frac{1}{2}$ per cent. of the curve $y^3=1-\cdot99x$ from $x=0$ to $x=\cdot95$, which is well within the experimental errors. That for $\kappa=2$ is nearly as good but diverges rather more at the end. For parameters greater than 2 it will be seen that the curve of velocity falls very steeply near $x=1$, and this corresponds to the existence of a range, which is so important a feature of the motion of the α rays. As the velocity diminishes there comes a point of inflexion and the final loss of velocity is exponential. Actually this part probably does not exist, as the α particles, which are straggling considerably, will undergo large deviations frequently, and it is very likely will pick up charges and become common gas molecules.

Equation (6) is thus capable of qualitatively representing the motion of the α rays. A great danger of basing a theory to account for experiments on a given law of force of the elementary parts, is that the supposed mechanism should not be the only one capable of accounting for the phenomena. This has caused trouble in such matters as viscosity, surface tension, etc. It would be hard, in the present case, to show that no other law of force could produce the same velocity curve, but that the curve cannot be obtained by merely supposing the α particles to surge through the atom with any

law of force may be proved by taking the only other simply soluble case, the inverse cube. Analytical difficulties enter a little earlier here, but when the velocity is very high it may be proved that the velocity curve is a common parabola, whereas with the inverse square it is $1-x=Av^4/\log v$. The difference is quite marked, and we may say that the results of experiment could not be deduced from any arbitrary law of force.

§ 3. *Application to Air.*

We next examine the velocity curve numerically. If the range of the α particles is R we have:—

$$8\pi N n k \sigma^2 R = E \log\left(1+\frac{\sigma^2 V^4}{\lambda^2}\right). \quad . \quad . \quad . \quad (7)$$

provided that $\log_{10}\sigma^2V^4/\lambda^2$, the parameter of the curve, is great enough to admit the existence of a range. We shall apply (7) to the case of RaC in air, as it is with this substance that most of the experiments have been made. The *mean* range is 6·8 cm. and the initial velocity $2{\cdot}0\times10^9$ cm. per sec. The quantities $N=5{\cdot}44\times10^{19}$, $k=1{\cdot}37\times10^{-4}$, $E=9{\cdot}3\times10^{-10}$(ES), and $e/m=5{\cdot}31\times10^{17}$(ES) are all directly known from various experiments. σ, the radius of the atom, is a more difficult matter. A value can be assigned from the kinetic theory of gases, but this need not be at all the same as the value required here. For the kinetic theory determines the mean distance of closest approach of two molecules in collision. Here we require the mean radius of the atom at all times. It is possible to suppose the atom highly compressible, so that the electrons are usually at a considerable distance from the centre, but are driven back on it by the approach of a second molecule. Or we may suppose them held very close to the centre, but capable of exerting outside their region large elastic forces on other atoms. In the first case σ will be larger, in the second smaller than the value given by the kinetic theory, which moreover refers to molecules, not atoms. If the atom of air is highly compressible its radius will be say 3×10^{-8} cm. This would give for RaC a parameter $\kappa=4{\cdot}8$, which is almost certainly too large. Moreover, it would result from compressibility that the mean radius would depend somewhat on the pressure; but the stopping of the α rays depends only on the number of atoms it encounters, and not at all on their pressure. This possibility may therefore be rejected, and to cover the other

I proceed by assuming for σ the three values 10^{-8}, $10^{-8\cdot5}$, 10^{-9} cm. The last is very small, the reason why it is included will appear in the discussion of the motion of the α particles through hydrogen. With these three values the parameters of the velocity curves are 3·82, 2·82, and 1·82, all values which would give a curve resembling fairly closely the experimental curve. There is now only one unknown in (7) and by solving we can find the value of n. In the three cases I find for volume distribution $n_1=7\cdot4$, 11·1, 19·2, and for surface $n_2=8\cdot7$, 13·2, 22·7. Thus, although σ is very uncertain, it appears that n is somewhere near the atomic weight. The slow variation of n_1 and n_2 with σ is due to the fact that for these parameters E_1 and E_2 are approximately

$$\frac{\sigma^2V^4}{\lambda^2}\Big/\log\frac{\sigma^2V^4}{\lambda^2},$$

so that σ only appears in (7) by its logarithm. These values of n are in good agreement with those determined by other methods; in particular, Rutherford * found the number of electrons as half the atomic weight from experiments on large scattering of the α rays. Some of the other determinations of n are on hypotheses contrary to the present, so that for these confirmation is meaningless.

§ 4. *Applications to other Substances.*

We must next examine the absorption under other conditions. Any mechanical hypothesis would give the correct law for variations of pressure and for compound substances. We need therefore only consider elements. In doing so it is not convenient to consider the whole range in each substance, as the end of the range can only be observed in a gas and also because it fails to convey part of the information which can be obtained. We take, as do experimenters, air as a standard, and compare the absorbing powers of thin layers of various elements with those of air. If we denote by accents the various quantities pertaining to the substance to be examined, the α particle in this substance will lose a velocity Δv in a length $\Delta x'$ where

$$\Delta v=N'\pi\sigma'^2\Delta x'.\,2k\frac{\lambda^2}{\sigma'^2v^3}\left\{\log(1+w')-f(w')\right\}$$

where as always $w'=\sigma'^2v^4/\lambda^2$. The same velocity is lost in a distance Δx of air, where

$$\Delta v=N\pi\sigma^2\Delta x\,.\,2k\frac{\lambda^2}{\sigma^2v^3}\left\{\log(1+w)-f(w)\right\}.$$

* Rutherford, *loc. cit.*

By division then :—

$$\frac{\Delta x}{\Delta x'} = \frac{N'n'}{Nn} \frac{\log(1+w') - f(w')}{\log(1+w) - f(w)}. \quad . \quad . \quad . \quad (8)$$

Then Δx is the "air equivalent" of a film of thickness $\Delta x'$ of the substance. (8) shows why equivalence should depend on the velocity of the α rays. By means of (8) we might theoretically solve and find all the atomic constants including the radius of the air atom. For if the equivalent of $\Delta x'$ is known at three different velocities, then two equations can be formed involving σ and σ', and can be solved. (7) then gives n and (8) n'. I have attempted such a solution for gold, but the values of σ and σ' which were obtained were certainly too small to be admissible. The data used were taken from some experiments by Taylor *, who records the equivalence of a certain gold foil at various velocities. He measured the velocity by finding the range which the α rays still had to run after passing the foil. This method of determining equivalence is open to the objection that it is not quite certain what is the quantity that is measured. The ionization falls off very rapidly at the end of the range and so very accurate measurements of change of range can be made. But the shape of the curve at the end depends largely on the amount of the straggling among the α rays, and this will vary systematically from one substance to another. Hence measurements at the end of the range need not give at all accurately the velocity at an earlier point, and we must abandon the hope of a complete direct solution.

By supposing σ known for air we need only use two measurements, and these may be taken fairly close together so as to minimize the effect of change of shape of the ionization curve. If Δx_1, Δx_2 are the distances in air equivalent to $\Delta x'$ at velocities v_1, v_2, then from (8) we have

$$\frac{\Delta x_1}{\Delta x_2} = \frac{\log(1+w_1') - f(w_1')}{\log(1+w_1) - f(w_1)} \frac{\log(1+w_2) - f(w_2)}{\log(1+w_2') - f(w_2')}. \quad . \quad (9)$$

To solve this for σ' we ought, strictly speaking, to determine the velocities v_1, v_2 from the recorded ranges by the velocity curve itself, but it is easy to see that the empirical formula $v^3 = V^3(1 - x/R)$ is quite accurate enough. The solution of (9) determines w_1' and w_2'. (8) then gives n'. The solutions all depend on the value assumed for σ the radius of the air atom. Table I. gives the solutions for the substances which Taylor † examined. The numbers in each column all depend on the value taken for the first row. For

* T. S. Taylor, Phil. Mag. vol. xviii. p. 604 (1909). The numbers used were extracted from Tables I. and II.

† Taylor, *loc. cit.*

TABLE I.

n is the number of electrons in the atom.
σ is the atomic radius.
κ is the parameter of the velocity curve.

$\Delta x'$ is the thickness of the film.
Δx_1 is the equivalent in air at range 5·7 cm.
Δx_2 „ „ „ 3·7 cm.
Δx_3 „ „ „ 2·1 cm.

For Al the change of equivalence is very slow and so the range 2·1 cm. was used for the solution and the range 3·7 cm. for comparison. For H, Δx_1 is at range 5·2 cm. and Δx_2 at range 4·0 cm.

Data used for the solutions.

		$\Delta x'$.	Δx_1.	Δx_2.	Δx_3 (obs.).	Δx_3 (calc.).
Au		$1{\cdot}27\times10^{-4}$	·719	·657	·600	·570
Pb		$2{\cdot}84\times10^{-4}$	1·104	·999	·888	·853
Sn		$3{\cdot}86\times10^{-4}$	1·011	·957	·882	·877
Al		$3{\cdot}30\times10^{-4}$	·597	·584	·597	·592
H		1·07	·231	·247	.....	.....
			Volume Distribution.		Surface Distribution.	
Air	σ.	10^{-8}	$10^{-8\cdot5}$	10^{-9}	10^{-8}	$10^{-8\cdot5}$	10^{-9}
	n.	7·4	11·1	19·2	8·7	13·2	22·7
	κ.	3·82	2·82	1·82	3·82	2·82	1·82
Au	σ.	$10{\cdot}0\times10^{-10}$	$6{\cdot}9\times10^{-10}$	$4{\cdot}0\times10^{-10}$	$9{\cdot}0\times10^{-10}$	$6{\cdot}1\times10^{-10}$	$3{\cdot}6\times10^{-10}$
	n.	89	117	182	113	152	230
	κ.	1·82	1·49	1·01	1·72	1·38	·93
Pb	σ.	$8{\cdot}8\times10^{-10}$	$6{\cdot}2\times10^{-10}$	$3{\cdot}7\times10^{-10}$	$7{\cdot}8\times10^{-10}$	$5{\cdot}4\times10^{-10}$	$3{\cdot}3\times10^{-10}$
	n.	106	140	215	137	182	275
	κ.	1·70	1·40	·95	1·61	1·28	·85
Sn	σ.	$17{\cdot}8\times10^{-10}$	$10{\cdot}7\times10^{-10}$	$5{\cdot}3\times10^{-10}$	$16{\cdot}6\times10^{-10}$	$9{\cdot}7\times10^{-10}$	$5{\cdot}0\times10^{-10}$
	n.	48	65	104	60	82	128
	κ.	2·32	1·88	1·27	2·26	1·79	1·21
Al	σ.	$6{\cdot}7\times10^{-9}$	$2{\cdot}5\times10^{-9}$	$0{\cdot}90\times10^{-9}$	$6{\cdot}6\times10^{-9}$	$2{\cdot}4\times10^{-9}$	$0{\cdot}88\times10^{-9}$
	n.	13·2	19·2	32·5	15·5	23·1	38·8
	κ.	3·47	2·62	1·72	3·46	2·60	1·70
H	σ.	.....	.....	$4{\cdot}3\times10^{-8}$	.....	.....	$6{\cdot}7\times10^{-8}$
	n.	.....	.....	·99	.....	.....	1·01
	κ.	.....	.....	5·1	.....	.....	5·5

each substance a row is devoted to the parameter (for particles from RaC). In some cases it appears very small, but this is implicit in the experiments, which show a great difference in the characters of the various velocity curves. In the case of hydrogen (9) is insoluble for the two larger values of σ. This means that with these values the relation between the air velocity curve and that of hydrogen is such that the latter is a more abrupt curve than would be one with an infinite parameter. The first part of the table gives the data used for the solutions. In addition to the atomic constants σ and n, I also calculated for comparison with experiment the equivalent of the film at a third velocity. Within narrow limits this came out the same for all six columns, which means that an enormous change must be made in σ before any appears in Δx_3. This shows that it would be impossible to get measurements of sufficient accuracy to determine the radius of the air atom in addition to the other constants, the theoretical possibility of which is indicated above.

The table shows that n is proportional to the atomic weight for the heavier substances. Hydrogen is in conformity with this, when the solution exists. It is clear, however, from the great difference between σ for surface and σ for volume distribution (which, when there is only one electron in the system, cannot correspond to any very great physical difference) that our analysis cannot be regarded as holding for systems containing a very small number of electrons. It is probably the assumption that the α particle exerts no force except when inside a sphere *round the centre*, which breaks down. Since the result with regard to σ is certainly unsatisfactory, it is very doubtful how much significance is to be attached to the value for n. The absence of the hydrogen solution for larger values of σ for air may also be due to the inapplicability of our analysis to the case of a very small number of electrons. If this larger value of σ for air is adopted, then for air and the metals n will be about half the atomic weight and hydrogen will be exceptional; if the small value of σ be taken, then n will be equal to the atomic weight and hydrogen (as far as the analysis is to be trusted for it) will be regular. In this case helium will be exceptional, for the whole hypothesis depends on the assumption that for it $n=2$. Great importance thus attaches to the case of helium, because there is only one adjustable constant σ instead of two, σ and n. Unfortunately no measurements of equivalence have been made, but only a determination of the whole range *. If the value found for this be put in (7)

* E. P. Adams, Physical Review, vol. xxiv. p. 108 (1907).

and the equation be solved for σ, a value is given several times greater than 10^{-8}. Thus here as for hydrogen σ has a value greater than that given by the kinetic theory, and this may probably be set down to the inaccuracy of meaning of σ in the case of a very small number of electrons in the atom.

From the table it appears that as the atomic weight grows σ becomes less and is very small for a heavy substance. This is rather surprising, but it does not seem unreasonable that a small cluster of a large number of electrons should exert elastic forces at as great a distance as a larger cluster composed of a smaller number. There is no great difference which should enable us to distinguish whether the distribution of electrons through the volume of the atom or that over its surface gives values most in agreement with the results of experiment.

The fact that stopping power depends on both n and σ prevents the deduction of any simple relation from Bragg's law of stopping power *. Table I. suggests that n is proportional to the atomic weight A, and if this is supposed accurately true it is possible to find a relation for σ; but it is not very intelligible because it contains the velocity, and it was only by neglecting the variation due to velocity that Bragg propounded his law. The "atomic stopping power" of Bragg is, from (8), proportional to $n'\{\log(1+w')-f(w')\}$ and this is proportional to $\sqrt{\mathrm{A}}$. Then approximately $\log w'$ varies as $1/\sqrt{\mathrm{A}}$, or making the velocity constant

$$\log \sigma' + \text{const.} \propto 1/\sqrt{\mathrm{A}},$$

which is the form that Bragg's law takes on the present theory.

The analysis of this paper is not sufficient to give a complete figure of the Bragg ionization curve. In the earlier part of the range the ionization is proportional to the mean rate of loss of energy of the α rays. But the interesting part of the curve is the end, and here measurement of loss of energy is useless, because the dominant factor is the straggling of the rays, and the problem of this straggling has not been discussed. In his investigation on the connexion between ionization and absorption, Geiger points out that the ionization is inversely proportional to the velocity of the rays. On the present hypothesis this is accidental. If the ionization is proportional to the rate of

* W. H. Bragg, *loc. cit.*

expenditure of energy, and if the velocity curve is approximately $v^3=V^3(1-x/R)$, then it follows directly that the ionization is approximately inversely proportional to the velocity ; and no further significance need be attached to this relation.

Part II.—Scattering.

The mechanism which accounts for absorption must also be capable of accounting for scattering. The problem of scattering is the more complicated of the two, as it deals not with a mean effect, but with the mean departure from such an effect, and in this way is analogous to the problem of straggling rather than to that of absorption. In consequence of this difference, difficulties enter which prevent the complete solution, and we must be satisfied with an approximation.

§ 5. *The Formulæ for the Scattering.*

The difficulty of the scattering lies in the fact that when an electron lies very close to the path of the particle it exerts a much greater effect than the average, and this effect is not counterbalanced by the absence of an electron in succeeding encounters. In the case of the absorption, any effect of such an electron does not alter the mean, but only tells on the amount of straggling. To consider the scattering it is thus convenient to divide the effect of the atom into two parts. The first is due to the regular average distribution of electrons together with the effect of the central charge. The combined effect of these is a deflexion in a plane through the centre. The second is due to the chance occurrence of electrons very near the path of the α particle. This deflexion will be in an arbitrary direction.

(i.) The deflexion by a single electron is from (2) equal to $2k\frac{\lambda pv^2}{\lambda^2+p^2v^4}$. The component of this expression in a plane through the centre of the atom is to be summed for all the electrons. In this summation the electrons very near the path of the particle will contribute equal amounts in all directions, so that there is no need to exclude them on the ground that they are to be counted later. The summation of all the electrons, and the averaging for all positions of the α particle, gives, as in the case of the absorption, a quadruple

integral. The mean deflexion from the electrons is

$$\psi_1' = \frac{2kn_1}{4/3\pi\sigma_1^3}\int_0^{\sigma_1}\frac{2\mathrm{P}d\mathrm{P}}{\sigma_1^2}\int_0^{\sigma_1} rdr\int_{-\sqrt{\sigma_1^2-r^2}}^{\sqrt{\sigma_1^2-r^2}} dz\int_0^{2\pi} d\phi\,\frac{\lambda/v^2(\mathrm{P}-r\cos\phi)}{\lambda^2/v^4+\mathrm{P}^2+r^2-2\mathrm{P}r\cos\phi}.$$

Only two of the integrations can be performed, and the expression then becomes elliptic and progress would be difficult. We shall at once evaluate it on the supposition that σ^2v^4/λ^2 is large, as the development of (ii.) later is only possible with large velocity. In this case

$$\psi_1' = 2k\frac{n_1\lambda}{\sigma_1 v^2}\left(2-\frac{3}{8}\pi\right).$$

In the case of a surface distribution a similar process gives

$$\psi_2' = 2k\frac{n_2\lambda}{\sigma_2 v^2}\left(2-\frac{1}{2}\pi\right).$$

The central charge plays a large part in scattering through small angles as well as through large. From (2) the deflexion is ψ'' where

$$\tan\psi'' = 2k'\lambda'\mathrm{P}v^2/\mathrm{P}^2v^4(k'+1)-\lambda'^2(k'-1).$$

When $k'>1$ this quantity can take any value positive, negative, or infinite. In experiments on small scattering when a large deflexion occurs, the α particle does not appear on the field of observation. Suppose that a particle which is deflected through an angle greater than δ is not observed. δ will be of the order of 2° or 3°. This excludes from the mean to be taken values of P less than a certain amount. For larger values of P, ψ'' may replace its tangent. When the mean is taken it appears that when the velocity is high the part dependent on δ is unimportant and that the mean is given by neglecting the second term of the denominator.

Thus $$\psi'' = 4\frac{\lambda'}{\sigma v^2}\frac{k'}{k'+1} = 4\frac{kn\lambda}{\sigma v^2}.$$

ψ'' is away from the centre, ψ' towards it. Hence their combined effect is

$$\psi''-\psi_1' = 2k\frac{n_1\lambda}{\sigma_1 v^2}\frac{3}{8}\pi$$

for volume, and

$$\psi''-\psi_2' = 2k\frac{n_2\lambda}{\sigma_2 v^2}\frac{1}{2}\pi$$

for surface distribution. Here for the first time there enters a distinct difference between the effects of the alternative distributions.

(ii.) Sir J. J. Thomson * has shown how to deal with the irregular deflexions due to electrons lying very close to the path of the α ray. The present is a modification of his method to suit the hypotheses of this paper. Suppose that any electron in a cylinder of radius l round the path is to be counted as very near it. The mean deflexion of such an electron is

$$2k\frac{\lambda}{v^2}\cdot\frac{2}{l}\left\{1-\frac{\lambda}{lv^2}\tan^{-1}\frac{lv^2}{\lambda}\right\}.$$

When v is large the second term is negligible, and unless this is so progress is prevented, since l will not disappear from the equation and it is impossible to know what is the right value to take for l. To estimate the whole effect of all the electrons in the cylinder we must multiply this expression by the square root of their number. For volume distribution their number is

$$n_1\pi l^2\,.\,2\sqrt{\sigma_1^2-\mathrm{P}^2}\Big/\frac{4}{3}\pi\sigma_1^3,$$

and the mean of the square root of this for all values of P is

$$\frac{l}{\sigma}\frac{4}{5}\sqrt{\frac{3}{2}}n.$$

Thus the whole deflexion due to these electrons is

$$\psi_1'''=\frac{4k\lambda}{\sigma_1 v^2}\sqrt{n_1}\,\frac{4}{5}\sqrt{\frac{3}{2}}.$$

The corresponding expression for surface distribution is

$$\psi_2'''=\frac{4k\lambda}{\sigma_2 v^2}\sqrt{n_2}\,\frac{4}{3}\sqrt{\frac{1}{2}}.$$

These deflexions are in arbitrary directions and must be compounded with those of (i.). The results are ψ_1, ψ_2 the mean deflexion of the whole atom.

$$\psi_1=\frac{2k\lambda}{\sigma_1 v^2}\left\{\left(\frac{3}{8}\pi n_1\right)^2+\left(\frac{8}{5}\sqrt{\frac{3}{2}}n_1\right)^2\right\}^{1/2}$$

$$=\frac{k\lambda n_1^{1/2}}{\sigma_1 v^2}\cdot\frac{3}{4}\pi\sqrt{n_1+2{\cdot}77}\,;\quad\ldots\ldots\quad(10)$$

$$\psi_2=\frac{2k\lambda}{\sigma_2 v^2}\left\{\left(\frac{\pi}{2}n_2\right)^2+\left(\frac{8}{3}\sqrt{\frac{1}{2}}n_2\right)^2\right\}^{1/2}$$

$$=\frac{k\lambda n_2^{1/2}}{\sigma_2 v^2}\cdot\pi\sqrt{n_2+1{\cdot}44}.\quad\ldots\ldots\quad(11)$$

* Sir J. J. Thomson, Proc. Camb. Phil. Soc. xv. pt. 5 (1910).

For high velocities the scattering should thus vary inversely as the square. The measurements of Geiger * on the dependence of scattering on velocity are not very accurate, but the inverse cube gave the best agreement. As this was determined with some quite low velocities, (10) and (11) may be regarded as reasonably satisfactory from this point of view.

The quantities ψ_1, ψ_2 are mean angles of deflexion. Geiger measured the most probable angle. Assuming (as has already been tacitly done) an error law of distribution, the latter is obtained by multiplying ψ by $\sqrt{2/\pi}$.

It is convenient to consider both ψ_1 and ψ_2 together by writing them

$$\frac{k\lambda n^{1/2}}{\sigma v^2}\pi\nu,$$

where

$$\nu_1=\tfrac{3}{4}\sqrt{n_1+2\cdot77} \quad \text{and} \quad \nu_2=\sqrt{n_2+1\cdot44}.$$

For a thin foil the whole scattering is obtained by multiplying ψ by the square root of the number of atoms encountered. In a foil of thickness Δx this number is $N\pi\sigma^2\Delta x$ and the most probable angle of scattering is

$$\chi=\frac{k\lambda}{v^2}\pi n^{1/2}\nu\sqrt{2N\Delta x}. \quad . \quad . \quad . \quad . \quad (12)$$

Experiments on scattering at high velocities should thus be able to determine n without σ.

Geiger performed experiments with both small and great thicknesses of gold, but for other metals he only had thick foils, and for such the change of velocity during transit has to be taken into account. Geiger † has shown how this explains the shape of the curve connecting scattering with thickness of the scattering foil. I follow his method, but use my own formulæ as the result is rather simple. When the variation of velocity is taken into account the most probable angle of scattering is

$$\chi=k\lambda\pi n^{1/2}\nu\left\{2N\,.\,\Sigma\frac{\Delta x}{v^4}\right\}^{1/2}.$$

Now

$$\Sigma N\pi\sigma^2\Delta x/v^4=\int_v^V dv/\rho v^4,$$

where V and v are the incident and emergent velocities

* H. Geiger, Proc. Roy. Soc. A. vol. lxxxiii. p. 492 (1910).
† H. Geiger, Proc. Roy. Soc. A. vol. lxxxvi. p. 236 (1912).

of the rays. Putting in the value of ρ from (4) or (5) we have

$$\chi=\sqrt{k\pi}\nu\cdot\frac{1}{2}\left\{\int_{\sigma^2v^4/\lambda^2}^{\sigma^2V^4/\lambda^2}\frac{dw}{w\{\log(1+w)-f(w)\}}\right\}^{1/2}$$

Now ψ was obtained on the assumption that σ^2v^4/λ^2 is large, and so here the second factor of the denominator may be replaced by $\log w-a$, a being the limiting value of f, 2/3 for f_1 and 2 for f_2. When this change is made the integration can be performed and gives

$$\chi=\tfrac{1}{2}\sqrt{k\pi}\nu\sqrt{\log\frac{\log\frac{\sigma^2V^4}{\lambda^2}-a}{\log\frac{\sigma^2v^4}{\lambda^2}-a}}\quad\ldots\quad(13)$$

§ 6. *Comparison with Experiment.*

Of the substances whose absorption has been discussed, Geiger* made experiments with gold, tin, and aluminium. I apply (13) to foils of these substances of thickness equivalent to 1 cm. of air, and I apply (12) to thin gold foils as well. The assumption that σ^2v^4/λ^2 is large is hardly justified for gold, especially for the values corresponding to a small radius of the air atom. Indeed for gold, even in the most favourable case an application of (13) to a thin foil gives a value differing by 10 per cent. from that deduced from (12), and (12) is only superior to (13) in that σ^2v^4/λ^2 has been supposed large once instead of twice.

Table II. gives, for comparison with the experimental

TABLE II.

Most probable Angles of Scattering.

	Observed.	Volume Distribution.			Surface Distribution.		
σ for air	.....	10^{-8}	$10^{-8\cdot5}$	10^{-9}	10^{-8}	$10^{-8\cdot5}$	10^{-9}
1 Au foil	10′	13′	17′	27′	22′	30′	45′
2 „ „	23′	19′	25′	38′	31′	42′	64′
Au equiv. to 1 cm. air ...	2°·1	1°·24	1°·63	2°·8	2°·63	4°·4	.....
Sn „ „ „ ...	1°·5	0°·79	1°·05	1°·74	1°·43	2°·14	5°·9
Al „ „ „ ...	0°·6	0°·35	0°·48	0°·80	0°·53	0°·79	1°·58

* H. Geiger, Proc. Roy. Soc. A. vol. lxxxiii. p. 492 (1910).

results, the values of the most probable angles of scattering deduced from the numbers in the six columns of Table I. For the thick foils (13) was used, for the thin (12). I have calculated the latter for both 1 and 2 foils, as Geiger's measurement with one foil seemed to be in not very good agreement with the next few.

Except for the last column the results are of the right order. The divergence of the last column is chiefly due to the fact that $\log w_2 - a_2$ does not then in the least approximate to $\log (1+w_2) - f_2(w_2)$. In the case of gold indeed $\log w_2 - a_2$ changes sign. In spite of this divergency for the surface distribution expressions, which masks their true value, it does seem possible now to discriminate definitely in favour of the volume distribution, and it seems that the smaller values of σ are best. Since the closeness of the approximation of χ to its true value is unknown, it does not seem profitable to fix the value with any greater accuracy.

The agreement between the observed and calculated values for the scattering is thus good enough entirely to confirm the hypothesis of this paper, but not so good as to help much in a more accurate specification of the atomic constants.

Summary.

An hypothesis is put forward whereby the α particles in passing through matter pull electrons out of the atoms they traverse, acting on them with the ordinary law of the inverse square.

An equation is deduced relating their velocity to the distance they have travelled from their source. This is the "velocity curve" and agrees closely with the experimental curve.

The equation involves two unknown constants: n the number of electrons in each atom, σ the radius of the atom. In the case of air, if σ be assumed known, n can be deduced from the range. Widely different values of σ give very similar values of n.

From comparison of the stopping powers of air and other substances, σ and n for these can be deduced, the values all depending on the original value assumed for the radius of the air atom.

The number of electrons in the atom appears to be intermediate between the atomic weight and its half. The atomic radii decrease with increasing atomic weight.

In the case of hydrogen it seems probable that the formula for σ does not hold on account of there being only very few

electrons in the atom. If it is regarded as holding, then $n=1$ almost exactly, but σ is very much larger than seems probable. σ is also too large for helium, for which $n=2$.

The numbers obtained from absorption are applied to scattering and give results in all cases in very good agreement with experimental measurements.

My thanks are due to Professor Rutherford for the interest he has taken in this paper and for the advice with which he has helped me.

XCI. *On the Deduction of Wien's Displacement Law.* *By* E. BUCKINGHAM *.

1. ALTHOUGH Wien's displacement law may be regarded as quite well established by experiment, its great importance seems to justify attempts to improve or simplify the reasoning by which it may be deduced *à priori* as a consequence of the general principles of thermodynamics and the electromagnetic theory of radiation. Any such deduction must, in substance, contain the following four elements :—

(*a*) The treatment, by Doppler's principle, of the change of wave-length produced when diffuse radiation is compressed or expanded within a perfectly reflecting shell, *i. e.* adiabatically.

(*b*) The evaluation, by means of the principle of the conservation of energy, of the change of the volume density of the radiant energy which occurs during the adiabatic change of volume and accompanies the change of wave-length. This step involves the use of the value of the pressure of diffuse radiation on a bounding surface, deduced from the electromagnetic theory and confirmed by experiment.

(*c*) The demonstration, by means of the second law of thermodynamics, that black radiation remains black when its density and temperature are changed by adiabatic change of volume.

(*d*) The use of the Stefan-Boltzmann law to correlate the results obtained by the steps (*a*), (*b*), and (*c*), so that the displacement law shall appear as a necessary consequence of those results.

These parts of the deduction need not be kept entirely separate, nor do they necessarily occur in the order given

* Communicated by the Author.