much greater than that of the jets from other orifices; in some cases the sensitiveness of a simple flame jet would approximate to that of the ear itself.

The flaring appears to depend, for a certain range of diameters of orifices, almost simply upon the linear rate of flow at the orifice. For diameters above this range, flaring occurs at much lower pressures.

The high temperature in ignited jets leads to increased viscosity, and this tends to explain the higher pressures then admissible. For a given pressure and orifice, the rate of flow is greater for an unignited than for an ignited jet.

In conclusion we wish to thank Professor Wilberforce for the keen interest which he has shown in these experiments.

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XXXIII. *Non-Newtonian Mechanics, The Mass of a Moving Body.* *By* RICHARD C. TOLMAN, *Ph.D., Assistant Professor of Physical Chemistry at the University of Cincinnati* *.

AN acceptance of the Einstein theory of relativity necessitates a revision of the Newtonian system of mechanics. In making such a revision it is desirable to retain as many as possible of the simpler principles of Newtonian mechanics. Some of the consequences have already been presented † of a system of mechanics which retains the conservation laws of mass, energy, and momentum, and defines force as the rate of increase of momentum; but to agree with the theory of relativity introduces an idea foreign to Newtonian mechanics by considering that both the mass and velocity of a body are variable.

From the theory of relativity, Einstein has calculated both the transverse and the longitudinal accelerations experienced by a charged body moving in an electromagnetic field. On the basis of these accelerations, it has been usual to place the "transverse mass" of a body moving with the velocity u as equal to $m_0/\sqrt{1-u^2/c^2}$, and its "longitudinal" mass as equal to $m_0/(1-u^2/c^2)^{\frac{3}{2}}$, where m_0 is the mass of the body at rest and c is the velocity of light. If, however, mass is a quantity to which a conservation law applies, the mass of a body cannot well be different in different directions; and

* Communicated by the Author. Contribution from the Chemical Laboratory of the University of Cincinnati.

† Lewis, Phil. Mag. xvi. p. 705 (1908). Lewis & Tolman, Phil. Mag. xvii. p. 510 (1909). Tolman, Phil. Mag. xxi. p. 296 (1911); xxii. p. 458 (1911).

it has been believed by Professor Lewis and the writer, that in general, without respect to direction, the expression $m_0/\sqrt{1-u^2/c^2}$ is best suited for THE mass of a moving body. They have already shown (*loc. cit.*), from the theory of relativity and the principles of non-Newtonian mechanics outlined above, that the consideration of a "transverse collision" between two moving bodies does lead to this expression for the mass of a moving body; and the purpose of the present article is to show that the consideration of any type of collision also leads to the same expression.

The immediate occasion of the present article is a recent attempt made by Mr. Norman Campbell * to show that the consideration of a "longitudinal collision" does not lead to the expression $m_0/\sqrt{1-u^2/c^2}$ for the mass of a moving body. There appears, however, to be an obvious error in his reasoning. Mr. Campbell wishes to find a relation between the mass of a body and its velocity and yet assumes that the mass of each of his bodies is the same after collision as before, although the velocities of course have changed (see equation (A) p. 627). Thus, although endeavouring to determine how the mass of a body depends on the velocity, he assumes in formulating his fundamental equation that it does not depend on the velocity at all †.

Longitudinal Collision.

Consider a system of Cartesian coordinates and two bodies moving in the X direction with the velocities $+u$ and $-u$ in such a way that a "longitudinal collision" will take place. Suppose the bodies are elastic and perfectly similar, each having the mass m_0 when at rest. On collision the bodies will evidently come gradually to rest, and then under the action of the elastic forces developed start up and move back on their original paths with the respective velocities $-u$ and $+u$ of the same magnitude as before.

Let us now consider how the collision will appear to an observer who is moving past the above system of coordinates with the velocity v in the X direction. Let u_1 and u_2 be the velocities of the two bodies as they appear before collision to this new observer. From Einstein's formulæ for the

* Phil. Mag. xxi. p. 626 (1911).

† In the same article, Mr. Campbell has also criticised the writer for referring the acceleration of a body under consideration to moving axes which have at the moment in question the same velocity as the body itself. As this is a procedure which has long been familiar to students of theoretical mechanics, has not in the past led to erroneous results, and in the cases under consideration leads to self-consistent conclusions, the writer cannot agree with Mr. Campbell's criticism.

composition of velocities we find for these velocities the relations $u_1 = \frac{u-v}{1-uv/c^2}$ and $u_2 = \frac{-u-v}{1+uv/c^2}$. Since these velocities are not of the same magnitude, the two bodies which have the same mass when at rest do not now have the same mass to this observer. Let us call these masses before collision m_1 and m_2. During collision, the velocities of the bodies will all the time be changing; from the principle of the conservation of mass, however, the sum of the two masses will always equal m_1+m_2 *. When in the course of the collision the bodies have come to relative rest and are both moving past our observer with the velocity $-v$, their momentum will be $-(m_1+m_2)v$, and from the principle of the conservation of momentum this must be equal to the original momentum before collision, giving us the equation,—

$$-(m_1+m_2)v = m_1u_1 + m_2u_2 = m_1\frac{u-v}{1-\frac{uv}{c^2}} + m_2\frac{-u-v}{1+\frac{uv}{c^2}}. \quad . \quad (1)$$

Simplifying, we have,—

$$\frac{m_1}{m_2} = \frac{1-\frac{uv}{c^2}}{1+\frac{uv}{c^2}}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

which by direct algebraic transformations may be shown to be identical with

$$\frac{m_1}{m_2} = \frac{\sqrt{1-\left(\frac{-u-v}{1+\frac{uv}{c^2}}\right)^2\Big/c^2}}{\sqrt{1-\left(\frac{u-v}{1-\frac{uv}{c^2}}\right)^2\Big/c^2}} = \frac{\sqrt{1-\frac{u_2^2}{c^2}}}{\sqrt{1-\frac{u_1^2}{c^2}}}. \quad . \quad (3)$$

* In this connexion an interesting fact has been pointed out to the writer by Professor Lewis. As stated above, the sum of the two masses is throughout collision always equal to m_1+m_2, and hence also at the time in the collision when the masses have come to relative rest their sum is m_1+m_2. Since at this time both bodies are moving with the velocity $-v$ we might suppose that m_1+m_2 equals $2m_0/\sqrt{1-v^2/c^2}$. This is not the case, however, since the bodies now possess additional elastic energy beyond that which they possess when at rest and not in contact. A relation between mass and energy has already been developed (*locc. citt.*), and the mass of this elastic energy must also be taken into account in calculating m_1+m_2. In fact the consideration of a collision of this type leads to a simple proof of the relation between mass and energy, a proof presented by Professor Lewis in a series of lectures on the Theory of Relativity given at Harvard University in the Spring of 1911.

Remembering that these were bodies which had the same mass when at rest, we see that the mass of a body is inversely proportional to $\sqrt{1-\frac{u^2}{c^2}}$, where u is its velocity, and have thus derived the desired relation,—

$$m=\frac{m_0}{\sqrt{1-\frac{u^2}{c^2}}}.$$

Collision of Any Type.

A treatment of the general case of any type of collision between any two bodies elastic or otherwise is also possible, and leads to the same conclusion as to the desirability of using the expression $m_0\Big/\sqrt{1-\frac{u^2}{c^2}}$ for the mass of a moving body.

For the mass m of a body moving with the velocity u let us write the equation $m=m_0 f(u^2)$ where $f(\ )$ is the function whose form we wish to determine. The mass is written as a function of the square of the velocity, since from the homogeneity of space the mass will be independent of the direction of the velocity, and the mass is made proportional to the mass at rest since a moving body may evidently be divided into parts without change in mass.

Let us now consider two bodies having the masses m_0 and n_0 when at rest, moving with the velocities u and v before collision and with the velocities U and V after a collision has taken place.

From the principle of the conservation of mass we have,—

$$m_0 f(u_x^2+u_y^2+u_z^2)+n_0 f(v_x^2+v_y^2+v_z^2)$$
$$=m_0 f(\mathrm{U}_x^2+\mathrm{U}_y^2+\mathrm{U}_z^2)+n_0 f(\mathrm{V}_x^2+\mathrm{V}_y^2+\mathrm{V}_z^2), \quad . \qquad (1)$$

and from the principle of the conservation of momentum,

$$m_0 f(u_x^2+u_y^2+u_z^2)u_x+n_0 f(v_x^2+v_y^2+v_z^2)v_x$$
$$=m_0 f(\mathrm{U}_x^2+\mathrm{U}_y^2+\mathrm{U}_z^2)\mathrm{U}_x+n_0 f(\mathrm{V}_x^2+\mathrm{V}_y^2+\mathrm{V}_z^2)\mathrm{V}_x, \quad (2)$$

$$m_0 f(u_x^2+u_y^2+u_z^2)u_y+n_0 f(v_x^2+v_y^2+v_z^2)v_y$$
$$=m_0 f(\mathrm{U}_x^2+\mathrm{U}_y^2+\mathrm{U}_z^2)\mathrm{U}_y+n_0 f(\mathrm{V}_x^2+\mathrm{V}_y^2+\mathrm{V}_z^2)\mathrm{V}_y, \quad (3)$$

$$m_0 f(u_x^2+u_y^2+u_z^2)u_z+n_0 f(v_x^2+v_y^2+v_z^2)v_z$$
$$=m_0 f(\mathrm{U}_x^2+\mathrm{U}_y^2+\mathrm{U}_z^2)\mathrm{U}_z+n_0 f(\mathrm{V}_x^2+\mathrm{V}_y^2+\mathrm{V}_z^2)\mathrm{V}_z. \quad (4)$$

These velocities u_x, u_y, u_z, v_x, v_y, v_z, U_x, &c., are measured with respect to some definite system of "space time" coordinates. An observer moving past this system of coordinates with the velocity ϕ in the X direction would find for the corresponding component velocities the values

$$\frac{u_x-\phi}{1-\frac{u_x\phi}{c^2}},\quad \frac{\sqrt{1-\phi^2/c^2}\,u_y}{1-\frac{u_x\phi}{c^2}},\quad \frac{\sqrt{1-\phi^2/c^2}\,u_z}{1-\frac{u_z\phi}{c^2}},\quad \frac{v_x-\phi}{1-\frac{v_x\phi}{c^2}},\ \&c.,$$

given by Einstein's transformation equations.

Since the laws of the conservation of mass and momentum must also hold for the measurements of this new observer, we may write the following new relations corresponding to equations 1 to 4:—

$$m_0f\left\{\left(\frac{u_x-\phi}{1-\frac{u_x\phi}{c^2}}\right)^2+\left(\frac{\sqrt{1-\phi^2/c^2}\,u_y}{1-\frac{u_x\phi}{c^2}}\right)^2+\left(\frac{\sqrt{1-\phi^2/c^2}\,u_z}{1-\frac{u_x\phi}{c^2}}\right)^2\right\}$$
$$+n_0f\left\{\left(\frac{v_x-\phi}{1-\frac{v_x\phi}{c^2}}\right)^2+\left(\frac{\sqrt{1-\phi^2/c^2}\,v_y}{1-\frac{v_x\phi}{c^2}}\right)^2+\left(\frac{\sqrt{1-\phi^2/c^2}\,v_z}{1-\frac{v_x\phi}{c^2}}\right)^2\right\}$$
$$=m_0f\left\{\left(\frac{U_x-\phi}{1-\frac{U_x\phi}{c^2}}\right)^2+\left(\frac{\sqrt{1-\phi^2/c^2}\,U_y}{1-\frac{U_x\phi}{c^2}}\right)^2+\left(\frac{\sqrt{1-\phi^2/c^2}\,U_z}{1-\frac{U_x\phi}{c^2}}\right)^2\right\}$$
$$+n_0f\left\{\left(\frac{V_x-\phi}{1-\frac{V_x\phi}{c^2}}\right)^2+\left(\frac{\sqrt{1-\phi^2/c^2}\,V_y}{1-\frac{V_x\phi}{c^2}}\right)^2+\left(\frac{\sqrt{1-\phi^2/c^2}\,V_z}{1-\frac{V_x\phi}{c^2}}\right)^2\right\}. \quad (1a)$$

$$m_0f\left\{u_x\ldots\right\}\frac{u_x-\phi}{1-\frac{u_x\phi}{c^2}}+n_0f\left\{v_x\ldots\right\}\frac{v_x-\phi}{1-\frac{v_x\phi}{c^2}}$$
$$=m_0f\left\{U_x\ldots\right\}\frac{U_x-\phi}{1-\frac{U_x\phi}{c^2}}+n_0f\left\{V_x\ldots\right\}\frac{V_x-\phi}{1-\frac{V_x\phi}{c^2}}. \quad (2a)$$

$$m_0f\left\{u_x\ldots\right\}\frac{\sqrt{1-\phi^2/c^2}\,u_y}{1-\frac{u_x\phi}{c^2}}+n_0f\left\{v_x\ldots\right\}\frac{\sqrt{1-\phi^2/c^2}\,v_y}{1-\frac{v_x\phi}{c^2}}$$
$$=m_0f\left\{U_x\ldots\right\}\frac{\sqrt{1-\phi^2/c^2}\,U_y}{1-\frac{U_x\phi}{c^2}}+n_0f\left\{V_x\ldots\right\}\frac{\sqrt{1-\phi^2/c^2}\,V_y}{1-\frac{V_x\phi}{c^2}}. \quad (3a)$$

$$m_0f\left\{u_x\ldots\right\}\frac{\sqrt{1-\phi^2/c^2}\,u_z}{1-\frac{u_x\phi}{c^2}}+n_0f\left\{v_x\ldots\right\}\frac{\sqrt{1-\phi^2/c^2}\,v_z}{1-\frac{v_x\phi}{c^2}}$$
$$=m_0f\left\{U_x\ldots\right\}\frac{\sqrt{1-\phi^2/c^2}\,U_z}{1-\frac{U_x\phi}{c^2}}+n_0f\left\{V_x\ldots\right\}\frac{\sqrt{1-\phi^2/c^2}\,V_z}{1-\frac{V_x\phi}{c^2}}. \quad (4a)$$

It is evident that these equations (1*a*–4 *a*) must be true no matter what the velocity between the original system of coordinates and the new observer, that is they are true for all values of ϕ. The velocities u_x, u_y, u_z, v_x, &c, are, however, perfectly definite quantities, measured with reference to a definite set of axes and entirely independent of ϕ. If these equations are to be true for perfectly definite values of u_x, u_y, u_z, v_x, &c., and for all values of ϕ, it is evident that the function $f(\ )$ must be of such a form that the equations are identities in ϕ. As a matter of fact ϕ can be cancelled from all the equations if we make $f(\ )$ of the form $\dfrac{1}{\sqrt{1-\dfrac{(\ )}{c^2}}}$; and we see that the expected relation is a solution of the equations. Although this does not exclude the possibility that there may be other solutions of these functional equations, nevertheless from a consideration of the complexity of the equations it appears doubtful if any other simple function would satisfy the necessary requirements.

In conclusion it is to be noted that in these derivations no reference has been made to any electrical charge which might be carried by the body whose mass is to be determined. Hence, if these considerations are correct, we may reject the possibility of explaining the Kaufmann-Bucherer experiment by assuming that the charge of a body decreases with its velocity*, since the increase in mass is alone sufficient to account for the results of the measurements.

Cincinnati, Ohio.
October 31, 1911.

XXXIV. *Theory of the Behaviour of the Quadrant Electrometer.* *By* Prof. A. ANDERSON †.

THE following presentation of the theory of the quadrant electrometer has, it seems to me, the merit of simplicity, and consequently may be of use to those who are engaged in working with the instrument. The theory given by Mr. G. W. Walker (Phil. Mag. Aug. 1903) is perhaps not elementary enough for the general reader, and the incidental reference to the subject by Prof. Sir J. J. Thomson in a paper on the Charge of Electricity carried by the Ions

* The possibility of explaining the Kaufmann-Bucherer experiment by assuming that the electrons have less charge at higher velocity was suggested by Professor More of this University: Phil. Mag. xxi. p. 196 (1911).

† Communicated by the Author.