

THE
LONDON, EDINBURGH, AND DUBLIN
PHILOSOPHICAL MAGAZINE
AND
JOURNAL OF SCIENCE.

[SIXTH SERIES.]

FEBRUARY 1910.

XXII. *A New Modification of the Cloud Method of Determining the Elementary Electrical Charge and the most Probable Value of that Charge.* By Prof. R. A. MILLIKAN, University of Chicago*.

§1. *Introduction.*

AMONG all physical constants there are two which will be universally admitted to be of predominant importance; the one is the velocity of light, which now appears in many of the fundamental equations of theoretical physics, and the other is the ultimate, or elementary, electrical charge, a knowledge of which makes possible a determination of the absolute values of all atomic and molecular weights, the absolute number of molecules in a given weight of any substance, the kinetic energy of agitation of any molecule at a given temperature, and a considerable number of other important physical quantities.

While the velocity of light is now known with a precision of one part in twenty thousand, the value of the elementary electrical charge has until very recently been exceedingly uncertain. The results herewith presented seem to show that the method here used for its determination—a modification of the Thomson-Wilson cloud method—turnishes the value of e with a directness, certainty, and precision, easily comparable with that obtained by any of the methods which

* Communicated by the Author.

have thus far been used, the error in the final mean value being not more than 2 per cent. Furthermore, with the use of a chronograph in place of a stop watch for taking time intervals, the method is perhaps capable of a slightly greater accuracy than has as yet been given to it.

As is well-known, H. A. Wilson's modification* of Thomson's cloud method† of determining e consists in observing first, the rate of fall under gravity of a cloud produced in an ionized fog-chamber by a sudden expansion, and second, the rate of fall of a like cloud when a vertical electrical field is superposed upon gravity.

If v_1 is the velocity under gravity alone, v_2 the velocity when the field is on, e the charge on an ion, m the mass of a drop, a the radius of a drop, X the field strength in electrostatic units, d the density of the falling drop, and η the coefficient of viscosity of the medium through which the cloud falls, then the equations from which e is determined are:—

$$\frac{v_1}{v_2} = \frac{mg}{mg + Xe} \quad \dots \quad (1)$$

$$v_1 = \frac{2}{9} \frac{ga^2d}{\eta} \text{ (Stokes)}, \quad \dots \quad (2)$$

$$m = \frac{4}{3} \pi a^3. \quad \dots \quad (3)$$

The solution of these equations is

$$e = \left\{ \frac{4}{3} \pi \left(\frac{9\eta}{2g} \right)^{\frac{3}{2}} \right\} \frac{g}{X} \frac{(v_2 - v_1)v_1^{\frac{1}{2}}}{d^{\frac{3}{2}}} \quad \dots \quad (4)$$

Substituting in this equation the value of η which he considered appropriate, and placing the density of water-drops equal to unity, Wilson obtained

$$e = 3.1 \times 10^{-9} \times \frac{g}{X} (v_2 - v_1)v_1^{\frac{1}{2}} \quad \dots \quad (5)$$

As the mean of 11 determinations which varied from 2.0×10^{-10} to 4.4×10^{-10} , Wilson obtained $e = 3.1 \times 10^{-10}$.

In view of the importance of the constant and the wide variations in Wilson's individual determinations, which were perhaps due to the variability of the X-ray bulb which produced the ionization, Mr. Begeman and the writer

* H. A. Wilson, *Phil. Mag.* ser. 6, vol. v. p. 429 (1903).

† J. J. Thomson, *Phil. Mag.* ser. 5, vol. xlv. p. 528 (1898): vol. xlviii. p. 547 (1899); ser. 6, vol. v. p. 346 (1903).

attempted two years ago to perfect Wilson's method by using radium instead of X-rays for the ionizing agent, by employing stronger electrical fields for the sake of increasing the difference between v_2 and v_1 in equation (4), and by observing the fall of the cloud through smaller distances and shorter times in order to reduce the error due to the evaporation of the cloud during the time of observation.

We obtained at once much more consistent values than those reported by Wilson, and made a preliminary report* in which we gave as the mean of ten observations which varied from 3.66 to 4.37 the value $e = 4.06 \times 10^{-10}$. We stated at the time that although we had not eliminated altogether the error due to evaporation, we thought that we had rendered it relatively harmless, and that our final value, although considerably larger than either Wilson's or Thomson's (3.1 and 3.4 respectively) must be considered an approach at least toward the correct value. All of the determinations which have since been made have placed the value of e as high as 4×10^{-10} and some considerably higher.

In this former work Mr. Begeman and I used Wilson's constant 3.1×10^{-9} for the value of the quantity within the brackets in equation 4; for we assumed that any errors involved in this constant must be small in comparison with the other errors of the experiment, and confined our attention wholly to improving the consistency and accuracy of the determination of the other factors in the equation. As will presently appear this assumption was not justified.

The outstanding causes of uncertainty in Wilson's method, as used by ourselves, were as follows:—

1. There is an experimental difficulty involved in obtaining clouds which fall without any distortion of the upper surface because of air currents.

2. The upper surface of a cloud falling in an electrical field is exceedingly difficult to follow on account of the scattering of the cloud which is usually produced by throwing on the field.

3. The method necessitates the assumption that it is possible to obtain in successive expansions exactly identical drops, so that v_2 and v_1 can be used in equation (5) as though they applied to the same drop.

4. The assumption is made that the cloud falls uniformly, and that there is no appreciable evaporation during the time of observation.

5. The assumption is made that the temperature of the air

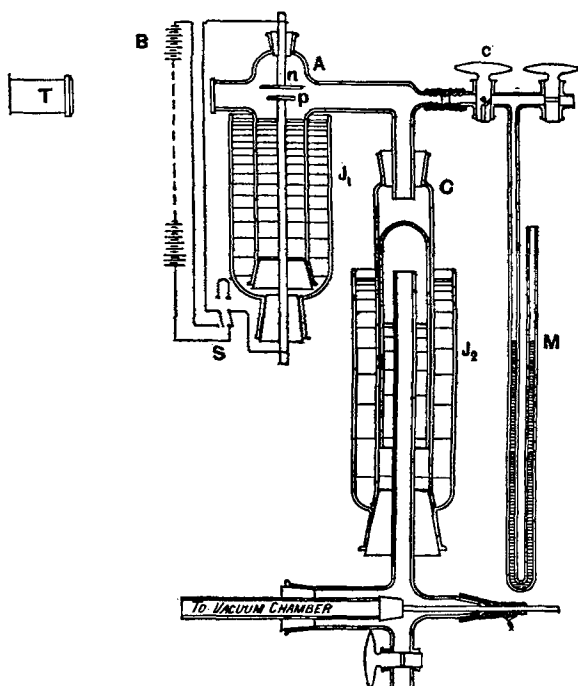
* Millikan and Begeman, *Phys. Rev.* vol. xxvi. p. 198 (1908).

through which the cloud falls is the equilibrium temperature after condensation—a quantity obtained from theoretical considerations relating to the adiabatic expansion of saturated vapours and the experimental curve expressing the relation between the temperature and density of a saturated vapour.

The results obtained by the method herewith presented are freed from all of these sources of uncertainty.

§2. *The Temperature of the Cloud Chamber after Expansion.*

In order to be free from all uncertainty as to the temperature of the cloud chamber I first attempted to measure it with an ordinary mercury manometer, which was attached to the chamber as shown in the figure. The diameter of the



glass vessels A and C was about 5 cm., the figure being drawn to scale but reduced to one-sixth actual size. The expansions used in these test experiments corresponded to a fall in pressure in the cloud chamber from about 75 cm. of

mercury to 55 cm. of mercury. Within a second after the expansion communication between the cloud chamber and the manometer was opened by means of the cock *c*, the height of the mercury in the manometer having been first adjusted by trial so that there was no motion upon opening the cock. The computed equilibrium temperature* after the formation of the cloud was in this case $14^{\circ}2$ C., or approximately 12° below the temperature of the room (26° C.). Hence, as the cloud chamber returned to the temperature of the room, there should have been a fall in the mercury in the left arm of the manometer of approximately 3 cm. Under no circumstances was there ever an actual change of more than half a millimetre, even though the communication between the manometer and the chamber was made within a second of the time of expansion. This indicated that the temperature of the cloud chamber, just as soon as one could possibly begin to observe the fall of a cloud, say two or three seconds after expansion, was not appreciably different from the temperature of the room.

The above method of observation gives only the mean temperature of the cloud chamber. To test directly the temperature of the air exactly at the point of observation; that is, midway between the plates, I made a thermo-couple out of 1 mil (0.25 mm.) iron wire and 3 mil platinum wire, and tested as follows its ability to register the instantaneous temperature of the gas in which it was immersed. I placed it in the centre of a carboy of 100 litres capacity and pumped air into this carboy with a common hand bicycle pump. Although the barrel of the pump had no more than $1/1000$ the volume of the carboy, every stroke of the pump could be easily seen by the movement of the galvanometer in series with the couple, the scale reading changing from 1 to 2 mm. at every stroke, even though the strokes followed one another at intervals of no more than a second. This indicated that the couple had a sufficiently small thermal capacity to respond with very little lag to changes in the temperature of the surrounding gas. This conclusion was further confirmed by comparing the observed and the computed values of the instantaneous fall in temperature produced by an adiabatic expansion of the air in the carboy. The computed fall was $1^{\circ}16$ C., the observed fall was $1^{\circ}12$ C.

The couple was next placed inside the fog chamber, midway between the plates *p* and *n* (see figure), and an expansion of 20 cm. of mercury produced. The galvanometer showed

* For the method of computation see C. T. R. Wilson, *Phil. Trans. A*, p. 299 (1907) or J. J. Thomson, *Phil. Mag. ser. 5*, vol. xlv. p. 538 (1898).

a sudden movement of 2 mm., and within ten seconds had crept back to its original zero. One degree of change in temperature was found by a separate experiment to produce a deflexion of 8.5 mm. Hence the maximum fall in temperature in the expansion as indicated by this couple was about $\frac{1}{4}^{\circ}\text{C}$.

This couple was then replaced by another which was made of 1 mil copper wire and 1 mil iron wire, and had therefore but about one-seventh the thermal capacity of the first. An expansion of 20 cm. of mercury produced in this case a sudden deflexion which corresponded to an apparent instantaneous fall in temperature of $0^{\circ}8\text{C}$. As before, within 9 or 10 seconds the original zero had been altogether regained, and within 6 or 7 seconds it had been regained to within less than half a degree. Both before and after this test the couple was placed inside a two-litre bottle containing water, and an expansion of 20 cm. of mercury produced by turning a stop-cock of about 4 sq. cm. opening, which connected this bottle with a 100-litre carboy within which the pressure was maintained at 55 cm. of mercury. In both cases the deflexion was about ten times as much as when the couple was between the plates p and n of the apparatus shown in the figure. Within a period of 15 seconds the initial temperature had been regained.

A more complete study of the relation between the volume and shape of the vessel and the temperature existing at given time intervals after the sudden expansion of a saturated vapour is now in progress. The above experiments, however, are sufficient to demonstrate, at least, that with the sort of fog chamber here used the temperature existing midway between the plates six seconds or more after expansion does not differ appreciably from that of the room. Since all measurements made by the method herein described are taken at times which begin between 6 and 15 seconds after an expansion, it is evident that the coefficient of viscosity of the medium at these times must be considered to be that corresponding to the temperature of the room. Furthermore, even when the observations are made by Wilson's method and are begun within two or three seconds of the time of an expansion, there can be little doubt that the mean temperature during the observed time of fall is, approximately at least, the temperature of the room.

Still a third reason for confidence in this conclusion is found in the following observations. In order to control the temperature of the chamber, water-jackets J_1 , J_2 were placed about its two branches in the manner indicated in the figure.

When the water in these jackets was two or three degrees above the temperature of the room, that is, when the lower electrode was but a degree or two warmer than the air above it, the cloud formed by a sudden expansion would always evaporate rapidly from the lower plate up, so that the appearance was as though it were rising instead of falling. It required not more than two seconds for the entire cloud between the plates to evaporate from the lower plate to the upper. If then the cloud could not last with a difference in temperature of not more than 2° between itself and the lower electrode, it seems certain that it would evaporate much more quickly if the difference in temperature were 12° or 15° .

§3. *The Viscosity of Air Saturated with the Vapours of Water and Alcohol, at 26° C.*

A large amount of work has been done within the last decade upon the exact determination of the coefficients of viscosity of dry air. According to three independent determinations, all made with the utmost of care, the value of this constant at 26° C., the temperature of the room in these experiments, is 0.0001863. This number is the mean of the values 0.0001863 obtained by Breiterbach*, 0.0001865 obtained by Schultze†, and 0.0001861 obtained by Fischer‡. Mr. Fred Allison made a careful determination for me in the Ryerson Laboratory of the ratio between the viscosity of dry air at 26° and that of air saturated with the vapours, first of water, and then of alcohol. He obtained for the times of outflow of given volumes through the same capillary the numbers contained in the following table:—

	Dry Air.	Saturated Air (Water).	Saturated Air (Alcohol).
1st Obs.	250.6 sec.	256.0 sec.	252.6 sec.
2nd Obs.	250.6 „	256.0 „	252.6 „
3rd Obs.	250.4 „	256.4 „	252.6 „
	<hr/>	<hr/>	<hr/>
Means	250.5 „	256.1 „	252.6 „
	<hr/>	<hr/>	<hr/>

Applying these corrections there results for the viscosity of air saturated with water vapour at 26° C., $\eta = 0.0001904$,

* P. Breiterbach, *Ann. der Physik*, v. p. 168 (1901).

† H. Schultze, *Ann. der Physik*, v. p. 157 (1901).

‡ W. J. Fischer, *Phys. Rev.* xxviii. p. 104 (1909). Fischer makes seven determinations at a mean temperature of 24.2° C. and obtains $\eta = 0.0001852$. This reduces to 0.0001861 at 26° C.

and for the viscosity of air saturated with the vapour of alcohol $\eta = \cdot 0001878$.

Under the conditions, then, which apply to the following experiments the value of the constant within the brackets in equation (4) is, for water, 3.422×10^{-9} in place of 3.1×10^{-9} as in Wilson's formula, and for alcohol, 3.353×10^{-9} . Furthermore if approximately the same temperature conditions apply, as they probably do, to Wilson's method as used in the preceding determination of e by Mr. Begeman and myself, which was also made at 26°C. , then the error in our former result due to the use of the constant 3.1×10^{-9} is somewhat more than 10 per cent. When the value 4.06×10^{-10} is corrected for this error it becomes 4.5×10^{-10} and when further corrected for a $1\frac{1}{2}$ per cent. error which was found in the calibration of the voltmeter with which the original potential-differences were measured, it becomes 4.57×10^{-10} .

§ 4. *The Balancing of Individual Charged Drops by an Electrostatic Field.*

My original plan for eliminating the evaporation error was to obtain, if possible, an electric field strong enough to exactly balance the force of gravity upon the cloud and by means of a sliding contact to vary the strength of this field so as to hold the cloud balanced throughout its entire life. In this way it was thought that the whole evaporation-history of the cloud might be recorded, and suitable allowances then made in the observations on the rate of fall to eliminate entirely the error due to evaporation. It was not found possible to balance the cloud as had been originally planned, but it was found possible to do something very much better; namely, to hold individual charged drops suspended by the field for periods varying from 30 to 60 seconds. I have never actually timed drops which lasted more than 45 seconds, although I have several times observed drops which in my judgment lasted considerably longer than this. The drops which it was found possible to balance by an electrical field always carried multiple charges, and the difficulty experienced in balancing such drops was less than had been anticipated.

The procedure is simply to form a cloud and throw on the field immediately thereafter. The drops which have charges of the same sign as that of the upper plate or too weak charges of the opposite sign, rapidly fall, while those which are charged with too many multiples of sign opposite to that

of the upper plate are jerked up against gravity to this plate. The result is that after a lapse of 7 or 8 seconds the field of view has become quite clear save for a relatively small number of drops which have just the right ratio of charge to mass to be held suspended by the electric field. These appear as perfectly distinct bright points. I have on several occasions obtained but one single such "star" in the whole field and held it there for nearly a minute. For the most part, however, the observations recorded below were made with a considerable number of such points in view. Thin, flocculent clouds, the production of which seemed to be facilitated by keeping the water-jackets J_1 and J_2 a degree or two above the temperature of the room, were found to be particularly favourable to observations of this kind.

Furthermore, it was found possible to so vary the mass of a drop by varying the expansion, or the charge carried by a drop by varying the ionization, that drops carrying in some cases two, in some three, in some four, in some five, and in some six, multiples could be held suspended by nearly the same field. The means of gradually varying the field which had been planned were therefore found to be unnecessary. If a given field would not hold any drops suspended it was varied by steps of 100 or 200 volts until drops were held stationary, or nearly stationary. When the P.D. was thrown off it was often possible to see different drops move down under gravity with greatly different speeds, thus showing that these drops had different masses and correspondingly different charges.

The life history of these drops is as follows. If they are a little too heavy to be held quite stationary by the field they begin to move slowly down under gravity. Since, however, they slowly evaporate, their downward motion presently ceases, and they become stationary for a considerable period of time; then the field gets the better of gravity and they move slowly upward. Toward the end of their life in the space between the plates, this upward motion becomes quite rapidly accelerated and they are drawn with considerable speed to the upper plate. This, taken in connexion with the fact that their whole life between plates only 4 or 5 mm. apart is from 35 to 60 seconds, will make it obvious that during a very considerable fraction of this time their motion must be exceedingly slow. I have often held drops through a period of from 10 to 15 seconds, during which it was impossible to see that they were moving at all. Shortly after

an expansion I have seen drops which at first seemed stationary, but which then began to move slowly down in the direction of gravity, then become stationary again, then finally began to move slowly up. This is probably due to the fact that large multiply-charged drops are not in equilibrium with smaller singly-charged drops near them, and hence, instead of evaporating, actually grow for a time at the expense of their small neighbours. Be this as it may, however, it is by utilizing the experimental fact that there is a considerable period during which the drops are essentially stationary that it becomes possible to make measurements upon the rate of fall in which the error due to evaporation is wholly negligible in comparison with the other errors of the experiment. Furthermore, in making measurements of this kind the observer is just as likely to time a drop which has not quite reached its stationary point as one which has just passed through that point, so that the mean of a considerable number of observations would, even from a theoretical standpoint, be quite free from an error due to evaporation.

§ 5. *The Method of Observation.*

The observations on the rate of fall were made with a short-focus telescope T (see figure p. 212) placed about 2 feet away from the plates. In the eyepiece of this telescope were placed three equally spaced cross-hairs, the distance between the extreme ones corresponding to about one third of the distance between the plates. A small section of the space between the plates was illuminated by a very narrow beam from an arc-light, the heat of the arc being absorbed by three water cells in series. The air between the plates was ionized by 200 mg. of radium, of activity 20,000, placed from 3 to 10 centimetres away from the plates. A second or so after expansion the radium was removed, or screened off with a lead screen, and the field thrown on by hand by means of a double-throw switch. If drops were not found to be held suspended by the field the P.D. was changed or the expansion varied until they were so held. The cross-hairs were set near the lower plate, and as soon as a stationary drop was found somewhere above the upper cross-hair, it was watched for a few seconds to make sure that it was not moving, and then the field was thrown off and the plates short-circuited by means of the double-throw switch, so as to make sure that they retained no charge. The drop was then timed by means of an accurate stop-watch as it passed across the three cross-hairs, one of the two hands of the watch being stopped at

the instant of passage across the middle cross-hair, the other at the instant of passage across the lower one. It will be seen that this method of observation furnishes a double check upon evaporation ; for if the drop is stationary at first, it is not evaporating sufficiently to influence the reading of the rate of fall, and if it begins to evaporate appreciably before the reading is completed, the time required to pass through the second space should be greater than that required to pass through the first space. It will be seen from the observations which follow that this was not, in general, the case.

It is an exceedingly interesting and instructive experiment to watch one of these drops start, and stop, or even reverse its direction of motion, as the field is thrown off and on. I have often caught a drop which was just too light to remain stationary and moved it back and forth in this way four or five times between the same two cross-hairs, watching it first fall under gravity when the field was thrown off, and then rise against gravity when the field was thrown on. The accuracy and certainty with which the instants of passage of the drops across the cross-hairs can be determined is precisely the same as that obtainable in timing the passage of a star across the cross-hairs of a transit instrument.

Furthermore, since the observations upon the quantities occurring in equation (4) are all made upon the same drop all uncertainties as to whether conditions can be exactly duplicated in the formation of successive clouds obviously disappear. There is no theoretical uncertainty whatever left in the method unless it be an uncertainty as to whether or not Stokes' law applies to the rate of fall of these drops under gravity. The experimental uncertainties are reduced to the uncertainty in a time determination of from three to five seconds, when the object being timed is a single moving bright point. This means that when the time interval is say 5 seconds, as it is in some of the observations given below, the error which a practised observer will make with an accurate stop-watch in any particular observation will never exceed 2 parts in 50. The error in the mean of a considerable number of concordant observations will obviously be very much less than this.

Since in this form of observation the v_2 of equation (4) is zero, and since X is negative in sign, equation (4) reduces to the simple form

$$e = 3.422 \times 10^{-9} \times \frac{X}{g} (v_1)^{\frac{2}{3}} \quad . \quad . \quad . \quad (6)$$

§ 6. *The Results.*

The results of all the observations taken by Mr. Begeman and myself, who worked together on all experiments which were made after this method had been worked out, are contained in the following tables. The observations marked with a triple star are those which were marked "best" in my notebook and represent those which were taken under what appeared to be perfect conditions. This means that we could watch the drop long enough to be very certain that it was altogether stationary ; that we could time its passages across the cross-hairs with perfect precision, and that it showed no apparent retardation in falling through the two equal spaces. The double-starred observations were marked in my notebook "very good." Those marked with single stars were marked "good" and the others "fair."

The only observations taken and not recorded in the tables are the following :—

First, I discarded three very good observations of my own, taken under conditions of potential and position of cross-hairs which made them uncertain in spite of the accurate timing. These observations belong to series No. 2 and would not affect appreciably the final result if they were included. Second, I have discarded three observations which I took on unbalanced drops, timing them as they rose against gravity under the influence of the field, and then again as they fell under gravity between the same cross-hairs, when the field was thrown off. Although all of these observations gave values of e within 2 per cent. of the final mean, the uncertainties of the observation were such that I would have discarded them had they not agreed with the results of the other observations, and consequently I felt obliged to discard them as it was. In the third place, I have discarded one uncertain and unduplicated observation apparently upon a singly charged drop, which gave a value of the charge on the drop some 30 per cent. lower than the final value of e . With these exceptions all of the data recorded in our note-books are given below. Furthermore none of the observations were worked out until after all of the observations had been taken, so that neither Mr. Begeman nor myself had any predispositions as to what should be the correct time of fall in the case of any drop.

Finally, in order to vary the conditions of the experiment

as much as possible, in the experiment recorded in series No. 3 we used alcohol drops instead of water, pushed the plates closer together, and changed the distance between the cross-hairs. All of these observations on alcohol were exceedingly satisfactory, although in this case we did not have the double check upon the evaporation, since we had but two cross-hairs instead of three.

Series No. 5 represents two single observations taken under conditions less favourable than the rest, and for this reason this series has been assigned a weight of but one in the final summary.

TABLE.

SERIES No. 1 (Bal. pos. water drops).				SERIES No. 2 (Bal. pos. water drops).			
Distance between plates .545 cm. Measured distance of fall .155 cm.				Distance between plates .545 cm. Measured distance of fall .155 cm.			
<i>Volts.</i>	<i>Time.</i> 1 space.	<i>Time.</i> 2 spaces.	<i>Observer.</i>	<i>Volts.</i>	<i>Time.</i> 1 space.	<i>Time.</i> 2 spaces.	<i>Observer.</i>
**2285	2.4 sec.	4.8 sec.	Millikan.	2365	1.8 sec.	4.0 sec.	Millikan.
2285	2.4 sec.	4.8 sec.	"	**2365	1.8 sec.	4.0 sec.	"
**2275	2.4 sec.	4.8 sec.	Begeman.	*2365	2.2 sec.	3.8 sec.	"
***2325	2.4 sec.	4.8 sec.	Millikan.	*2365	1.8 sec.	4.0 sec.	"
2325	2.6 sec.	4.8 sec.	"	*2395	2.0 sec.	4.0 sec.	Begeman.
*2325	2.2 sec.	4.8 sec.	"	*2395	2.0 sec.	4.0 sec.	"
2365	2.4 sec.	4.8 sec.	"	*2395	2.0 sec.	3.8 sec.	"
				2365	1.8 sec.	4.0 sec.	Millikan.
				2365	1.8 sec.	4.0 sec.	"
				2365	1.8 sec.	4.0 sec.	"
2312	2.4	4.8		2374	1.90	3.96	
Mean time for .155 cm. = 4.8 sec. $e_3 = 3.422 \times 10^{-9} \times \frac{980.3}{14.14} \times \left(\frac{.155}{4.8} \right)^{\frac{3}{2}}$ $= 13.77 \times 10^{-10}.$ $\therefore e = 13.85 \times 10^{-10} \div 3 = 4.59 \times 10^{-10}.$				Mean time for .155 cm. = 3.91 sec. $e_4 = 3.422 \times 10^{-9} \times \frac{980.3}{14.52} \times \left(\frac{.155}{3.91} \right)^{\frac{3}{2}}$ $= 18.25 \times 10^{-10}$ $\therefore e = 18.25 \div 4 = 4.56 \times 10^{-10}.$			

TABLE (continued).

SERIES No. 3 (Bal. pos. alcohol drops).				SERIES No. 4 (Bal. pos. water drops).			
Distance between plates .463 cm. Measured distance of fall .1 cm. Density of alcohol .805 cm. Volts applied to plates .2335.				Distance between plates .545 cm. Measured distance of fall .155 cm.			
Times of fall of balanced drop.							
	Observer.		Observer.	Volts.	Time.	Time.	Observer.
*3.8 sec.	Millikan.	*3.8 sec.	Begeman.		1 space.	2 spaces.	
*3.8 sec.	"	*3.8 sec.	"	2365	1.6 sec.	3.2 sec.	Millikan.
4.0 sec.	"	3.8 sec.	"	*2365	1.6 sec.	3.2 sec.	"
*3.8 sec.	"	3.6 sec.	"	**2365	1.6 sec.	3.4 sec.	"
**3.8 sec.	"	4.0 sec.	"	2365	1.6 sec.	3.4 sec.	"
3.6 sec.	"			**2365	1.6 sec.	3.2 sec.	"
3.8 sec.		3.8		2365	1.6	3.28	
$e_2 = 3.353 \times 10^{-9} \times \frac{980.3}{16.81} \times \left(\frac{1}{.805}\right)^{\frac{1}{2}} \times \left(\frac{.1}{3.8}\right)^{\frac{2}{3}} = 9.284 \times 10^{-10}.$ $\therefore e = 9.284 \times 10^{-10} \div 2 = 4.64 \times 10^{-10}.$				Mean time for .155 cm. = 3.25 sec. $e_5 = 3.422 \times 10^{-9} \times \frac{980.3}{14.46} \times \left(\frac{.155}{3.25}\right)^{\frac{2}{3}} = 24.14 \times 10^{-10}.$ $\therefore e = 24.14 \times 10^{-10} \div 5 = 4.83 \times 10^{-11}.$			
SERIES No. 5 (Bal. pos. water drops).				SERIES No. 6 (Bal. pos. water drops).			
Distance between plates .545 cm. Measured distance of fall .155 cm.				Distance between plates .545 cm. Measured distance of fall .155 cm.			
Volts.	Time.	Time.	Observer.	Volts.	Time.	Time.	Observer.
	1 space.	2 spaces.			1 space.	2 spaces.	
2295	3.0 sec.	6.2 sec.	Begeman.	**2295	1.4 sec.	2.8 sec.	Begeman.
2295	3.0 sec.	6.0 sec.	"	2395	1.5 sec.	3.0 sec.	"
				*2395	1.4 sec.	3.0 sec.	"
2295	3.0	6.1		2395	1.43	2.93	
Mean time for .155 cm. 6.07 sec. $e_2 = 3.422 \times 10^{-9} \times \frac{980.3}{14.04} \times \left(\frac{.155}{6.07}\right)^{\frac{2}{3}} = 9.742 \times 10^{-10}.$ $\therefore e = 4.87 \times 10^{-10}.$				Mean time for .155 cm. 2.91 sec. $e_6 = 3.422 \times 10^{-9} \times \frac{980.3}{14.64} \times \left(\frac{.155}{2.91}\right)^{\frac{2}{3}} = 28.16 \times 10^{-10}.$ $\therefore e = 4.69 \times 10^{-10}.$			

SUMMARY.

Series.	Charge.	Value of e .	Weight assigned.
No. 1	3 e	4.59	7
No. 2	4 e	4.56	7
No. 3	2 e	4.64	6
No. 4	5 e	4.83	4
No. 5	2 e	4.87	1
No. 6	6 e	4.69	3

Simple mean $e = 4.70 \times 10^{-10}$.

Weighted „ $e = 4.65 \times 10^{-10}$

It will be observed that we did not succeed in balancing any singly charged drop. This is because the field was not strong enough to hold singly charged drops unless they were exceedingly small. Such drops, both because of the smallness of their size and the smallness of their charge, are not in equilibrium with multiply-charged drops (see above) and consequently evaporate so rapidly that their life is relatively short. The single observation mentioned above was probably upon such a drop, but it was evaporating so rapidly that I obtained a poor value of e . Big drops and heavily charged drops are those which evaporate most slowly, and hence those which are most easy to hold stationary. On the other hand the error in the time determination becomes greater if the rate of fall under gravity is large. All of these considerations have been taken into account in assigning the weights given in the summary. In finding the mean time in a given series the observations through one space have been given one half the weight of the observations through two spaces.

The measurement of the potentials applied to the plates was made with a Braun electrometer reading to 3000 volts. I calibrated this instrument with an accuracy of about $\frac{1}{3}$ of 1 per cent., by comparing it with a Weston voltmeter which was sent to the Bureau of Standards for standardization. The distance between the plates was measured with a cathetometer after all of the observations upon the rates of fall had been made. The distance between the cross-hairs was also measured with the aid of the vertical cathetometer scale.

§ 7. *Discussion of Results.*

It will be observed that the only possible elementary charge of which the observed charges are multiples is 4.65×10^{-10} , and further that the measured charges represent all the possible multiples of this charge between 2 and 6. This shows that the elementary charge cannot possibly be the smallest charge which we observed: namely, 9.3×10^{-10} , since we obtained odd as well as even multiples of one half of this quantity. Furthermore, the elementary charge can scarcely be a sub-multiple (*e. g.* a half) of 4.65×10^{-10} , since the observed charges would then represent only even multiples of this elementary charge, and there is no reason why odd as well as even multiples of the elementary charge should not have been observed.

As indicated above the only theoretical assumption involved in this determination is the assumption of the validity of Stokes' law for these drops. Since this law has been shown by direct experiment to hold for large spheres falling slowly in a viscous fluid*, and since the spheres under observation in these experiments have diameters which vary from .00034 cm. to .00047 cm.—numbers which are from 35 to 50 times the mean free path of the air molecules—it is scarcely conceivable that Stokes' law fails to hold for them. Furthermore if Stokes' law did not hold for these drops, that is, if their actual velocity were the velocity given by Stokes' law multiplied by some constant factor, then the above series of values would simply be multiplied by this factor raised to the $3/2$ power. So far as these observations, taken by themselves, are concerned, there is no reason why this may not be the case. If, for example, the final multiplying factor were 2, then the elementary charge would be 9.3×10^{-10} , and our series would contain all multiples of this charge up to 6. If this were the case then the charge carried by the α particle, which Rutherford has found to be 9.3×10^{-10} , would be the elementary charge instead of twice the elementary charge, and the α particle could not then be helium. If it be considered as proven that the α particle is helium, and that the charge which it carries is 9.3×10^{-10} , then the above observations may be taken as experimental verification of the validity of Stokes' law for water drops of the size used in this experiment. In other words these observations become altogether irreconcilable with Rutherford's experiments if Stokes' law does not hold.

* H. S. Allen, *Phil. Mag.* ser. 5, vol. 1. p. 519 (1900).

§ 8. *The Most Probable Value of the Elementary Electrical Charge.*

There are four recent independent determinations of e in addition to the above, the results of which seem to be deserving in each case of much weight. They are:—

1. Planck's value* 4.69×10^{-10} based upon theoretical considerations, and Kurlbaum's experimental data.

2. Rutherford and Geiger's value †, 4.65×10^{-10} , obtained by counting the number of α particles emitted by a known quantity of radium and measuring the total electrical charge carried by these particles.

3. Regener's value ‡, 4.79×10^{-10} , obtained by remarkably careful and consistent work in the counting of the number of scintillations produced by the α particles emitted by a known amount of polonium and measuring the total charge carried by these same particles. (Regener estimates his error at not more than 3 per cent.)

4. Begeman's recent and as yet unpublished value of 4.67×10^{-10} , obtained as a mean of a very large number of consistent observations made in the Ryerson Laboratory by Wilson's method.

As already indicated the chief source of error in the preceding determination by Begeman and the author, using Wilson's method, was the wrong assumption as to the viscosity of the gas through which the cloud fell. In the light of the results obtained above both as to this matter and as to the rate of evaporation of multiply-charged drops, Wilson's method, although still involving theoretically more elements of uncertainty than the method herewith presented, may yet be made, as Begeman's results demonstrate, a consistent and apparently a reliable means of determining e , particularly if the observations in the electrical field are made upon multiply-charged layers. Begeman's observations so made show a most satisfactory concordance.

In addition to the five independent determinations above considered there are four other important and interesting pieces of work by Ehrenhaft, Broglie, Perrin, and Moreau, all of which lead to estimates as to the value of e . These estimates have not been included in the final mean given below for the reason that the methods employed by these

* Planck, *Wärme Strahlung* (Barth), p. 163 (1906).

† Rutherford and Geiger, *Proc. Roy. Soc. A*, vol. lxxxi. pp. 141 & 161 (1908).

‡ Regener, *Sitz.-Ber. der K. Preuss. Acad. der Wiss.* vol. xxxviii. p. 948 (1909).

observers appear to involve larger elements of uncertainty than are found in any of the foregoing methods.

Ehrenhaft's mean value *, obtained by a method similar to the one here presented save that it involves the measurement of the velocities produced first by the action of gravity, and second by the action of an electrical field upon the charged particles thrown off by a metallic arc, is 4.6×10^{-10} . Although this number is in very good agreement with the preceding values the method itself appears to involve the following uncertainties : first, an uncertainty as to the correctness of the assumption that Stokes' law is applicable to the motion of particles whose diameters are not negligible in comparison with the mean free path of gas molecules, and which are perhaps also of doubtful sphericity ; second, an uncertainty which arises from the fact that instead of making the two observations of velocity upon one and the same particle, as is done in the method herein described, Ehrenhaft is compelled to take the mean of the velocities of different particles although these velocities vary among themselves by 60 per cent. ; third, an experimental uncertainty involved in the determination of the mean radius of the particles. The value of this radius, computed by applying Stokes' law both to the velocity under gravity and the velocity in the electrical field, is checked to within about 7 per cent. by direct microscopic measurement, and it is this observed rather than the computed radius which is substituted in the Stokes' law equation as applied to the motion in the electrical field to obtain the final value $e = 4.6 \times 10^{-10}$. The observed radii of the different particles are found, however, to vary by more than 50 per cent., and since the mean value is but about .00003 cm., it is obvious that even a moderate degree of precision in this measurement must be very difficult to obtain. Finally there may be some question as to whether multiples of the elementary charge may not be carried by some of the particles.

Perrin's work †, while of the utmost importance from other points of view, involves so many assumptions of questionable rigour that it can scarcely be regarded as furnishing a determination of e with a certainty which is at all comparable with that found in any of the five methods first considered. Thus, it assumes that Stokes' law holds for particles showing Brownian movements in liquid emulsions. Second, it assumes that these same particles follow the Maxwell-Boltzmann law of distribution of velocities in gas mixtures. Third, it assumes

* Ehrenhaft, *Phys. Zeit.* 1 Mai, 1909.

† Perrin, *C. R.* vol. cxlvii. p. 594 and p. 530 ; also *C. R.* vol. cxlvi. p. 967.

that the density of the particles constituting an emulsion of gum gamboge is the same as the density of gum gamboge in mass. Fourth, from the experimental standpoint, the difficulty in measuring accurately the small differences in level which he is obliged to use (less than $\cdot 1$ mm.), and in counting correctly the number of particles in a given level, are such as to render easily intelligible such lack of consistency as is shown in Perrin's different determinations, which give values of N (the number of molecules in 1 gram-molecule) varying from $5\cdot 4 \times 10^{23}$ to $7\cdot 1 \times 10^{23}$.

Brogie's result* is $4\cdot 5 \times 10^{-10}$. It is obtained by measuring the velocities in an electrical field of the charged particles of tobacco smoke, the mean radius of these particles being obtained from kinetoscopic records of the mean displacement which they undergo in a given time because of their Brownian movements. The method involves the assumption of Perrin's value of N , and for both theoretical and experimental reasons will scarcely claim to represent an accurate determination of e .

Moreau's † measurement of the charge carried by the ions in flames depends upon Perrin's determination of e . Hence, Moreau's final result, viz. $4\cdot 3 \times 10^{-10}$ must involve any errors contained in Perrin's work.

If then, equal weights be assigned to all the recent determinations of e by methods which seem least open to question, the most probable value of e at present obtainable is that given below:—

Planck	$4\cdot 69 \times 10^{-10}$
Rutherford & Geiger	4·65 "
Rogener	4·79 "
Begeman	4·67 "
Millikan	4·65 "
Mean.....	4·69 "

§ 9. Conclusion.

1. The temperature of the fog chamber a very few seconds after expansion in the form of apparatus shown in the figure (p. 212) is essentially the temperature of the room.

2. The balanced drop method herewith presented for the determination of e involves an experimental error of not more than 2 per cent., and is entirely free from all theoretical uncertainties except such as are incident to the application of

* Brogie, *Le Radium*, vol. vi. p. 208, Juillet, 1909.

† Moreau, *C. R.* vol. cxlviii. p. 1255, 10 Mai, 1909.

Stokes' law to liquid spheres of diameters varying from 30 to 50 times the mean free path of air molecules.

3. The results obtained by this method taken in connexion with Rutherford's experiments seem to constitute experimental verification of Stokes' law for these drops.

4. Positively charged drops of water and alcohol are found by direct measurement to carry charges which are multiples of 4.65×10^{-10} , and all of the multiples from 2 to 6 inclusive have been obtained.

5. The mean of the five most reliable determinations of e is 4.69×10^{-10} . The corresponding value of n (the number of molecules in 1 cubic cm. of gas at 0° C., 76 cm. pressure) is 2.76×10^{19} : that of N (the number of molecules in a gram-molecule) is 6.18×10^{23} : that of ϵ ($= 3/2 \frac{RT}{N}$, the kinetic energy of agitation in ergs of a molecule at 0° C., 76 cm. pressure) is 2.01×10^{-16} ; that of m (the mass in grams of an atom of hydrogen) is 1.62×10^{-24} .

Ryerson Laboratory,
University of Chicago,
October 9, 1909.

XXIII. *The Asymptotic Expansions of Bessel Functions.* By J. W. NICHOLSON, M.A., D.Sc., Isaac Newton Student in the University of Cambridge*.

MANY physical problems depend, for their final solution, upon a knowledge of the approximate values of Legendre and Bessel functions for a large range of their argument and order. In the case of the Bessel functions, investigators† have almost entirely confined their attention to those special types in which the order n is small, though the argument z may be large or small.

A treatment of the more general problem presented when n is also large has been given by Lorenz‡, but only when n is half an odd integer. The immediate object of Lorenz was to obtain some expansions necessary for his investigation of the scattering of light by a glass sphere, in which, as in most problems of this type, only Bessel functions expressible in finite form are required. His results were first approximations

* Communicated by the Author. Read before the British Association, Dublin, 1908.

† Poisson, *Journal de l'Ecole*, 1823; Stokes, *Camb. Phil. Trans.* 1856; Hankel, *Math. Ann.* i. 1869; Lipschitz, *Crelle*, 1859; and others.

‡ *Œuvres Scientifiques*, vol. i. p. 435 et seq.