A construction of the regular Polygon of 34 sides.

By

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In continuation of an earlier article (Math. Ann. 71, p. 592) by the writer, the following shorter construction is now submitted.

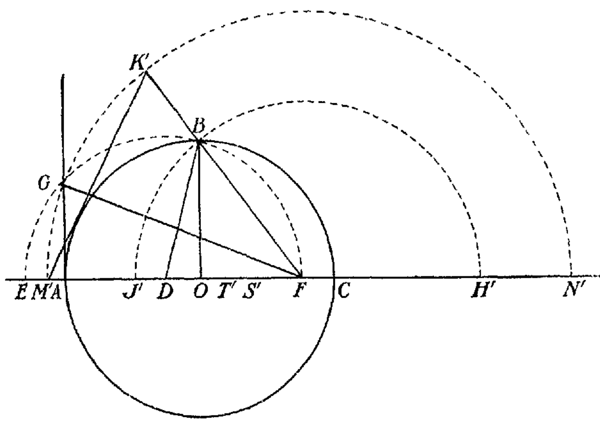

Fig. 1.

Let ABC be a given circle, centre O, radius r (Fig. 1). Draw the diameter AC, and GA tangent at A, and BO radius parallel to GA. Cut off $OD\left(=\frac{1}{4}r\right)$. Join DB; and with centre D, radius DB, describe a circle cutting AC in E and F, and AG in G*). Join FG, and with centre F, radius FG, describe a circle cutting AC in M' and N'. Join FB, and produce to cut this circle in K'. With centre F, radius FB, describe a circle cutting AC in J', and H'. Join $M'K'$, and lay off $H'S'(=M'K')$. Bisect OS' in T'. Then OT' equals the side of 34 gon.

For, by the method shown in § 4 of the earlier paper, it can be proved that $OT'\,(=\alpha)$ is connected with r by the system

*) In drawing it may be useful to note that $AG=\frac{1}{2}AB$.

$$r^2 - \alpha^2 = r(\beta - r),$$
$$\beta^2 - r^2 = r(\gamma + r),$$
$$\gamma^2 - r^2 = r(\delta + r),$$
$$\delta^2 - r^2 = r(r + \alpha).$$

And (Fig. 2) having laid off OT' $(= \alpha)$, and having described the circle (centre Q', radius r) passing through O and T', on proceeding with the figure we find successively

$$Q'Y' = \beta,$$
$$OZ' = \gamma,$$
$$Q'U' = \delta = AQ',$$

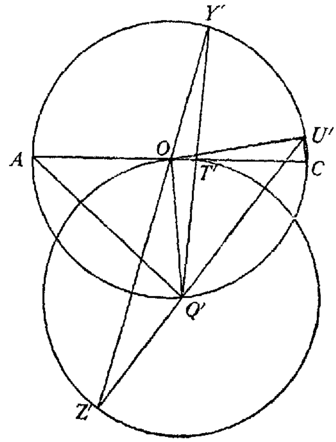

Fig. 2.

from which the proof follows immediately that angle $OU'C = 8$ times the angle COU'.

This construction will be seen to depend on the cosines of the period, $+ 2\cos\frac{2\pi}{17} + 2\cos\frac{4\pi}{17} + 2\cos\frac{8\pi}{17} + 2\cos\frac{16\pi}{17}$, that given in the earlier article depending on those of the other period, which was selected as seeming to show more clearly the geometry of the general case under discussion. The present construction, which lays off the actual side of the 34-gon, has distinct advantage in quickness, and more especially in practical accuracy.