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The complex neutrosophic soft expert set and its application in decision making

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Abstract. This paper presents a novel complex neutrosophic soft expert set (CNSES) concept. The range of values of CNSES is extended to the unit circle in the complex plane by adding an additional term called the phase term which describes CNSES's elements in terms of the time aspect. CNSES is a hybrid structure of soft sets and single-valued neutrosophic sets (SVNSs) defined in a complex setting where the experts' opinions are included, thus making it highly suitable for use in decision-making problems that involve uncertain and indeterminate data where the time factor plays a key role in determining the final decision. Based on this new concept we define some concepts related to this notion as well as some basic operations namely the complement, union, intersection, AND and OR. The basic properties and relevant laws pertaining to this concept such as the De Morgan's laws are also verified. Lastly, we propose an algorithm to solve complex neutrosophic soft expert decision-making problem by converting it from the complex state to the real state and subsequently provided the detailed decision steps. This study is supported by the comparison with other existing methods.

Keywords: Complex neutrosophic set, decision making, neutrosophic set, single-valued neutrosophic set, soft expert set

1. Introduction

Smarandache [1] firstly proposed the theory of neutrosophic set as a generalization of fuzzy set [2] and intuitionistic fuzzy set [3]. Neutrosophic set can deal with uncertain, indeterminate and incongruous information where the indeterminacy is quantified explicitly and truth membership, indeterminacy membership and falsity membership are completely independent. The neutrosophic set was introduced for the first time by Smarandache in his 1998 book [4] which is also mentioned by Howe in the free online dictionary of computing. In order to apply neutrosophic set in real- life problems, its operators need to be specified, therefore, the single-valued neutrosophic set and its basic operations were defined by Wang et al. [5] as a special case of neutrosophic set, since single value is an instance of set value. Subsequently, the works on SVNSs and their hybrid structures in theories and applications have been progressing rapidly [6–9]. Multi-criteria decision-making (MCDM) is an important branch of decision theory, which has been extensively studied in many research [10–13]. Due to the complexity of real decision-making problems, the decision information is often incomplete, indeterminate and inconsistent information, then the aforementioned uncertainty sets can offer useful tools to handle such decision-making problems. Therefore, the integration of these uncertainty sets in MCDM techniques has increasingly attracted the attention of many researchers. This lead to a productive output in relevant research literature [14–26]. Soft set theory, on the other hand, was initiated by Molodtsov [27] as a general mathematical tool used to handle uncertainties, imprecision and vagueness. Since its inception, a lot of extensions of soft set model have been developed such as fuzzy soft

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sets [28], vague soft sets [29], interval-valued vague soft sets [30–32], soft expert sets [33], soft multiset theory [34] and neutrosophic soft set [35–39]. At present, soft set has allured wide attention and made many achievements [40–42]. The development of the uncertainty sets that have been mentioned above are not limited to the real field but extended to the complex field. The introduction of fuzzy sets was followed by their extension to the complex fuzzy set [43]. In complex fuzzy set, the degree of membership function μ is traded by a complex-valued function of the form $r_{\tilde{s}}(x)e^{i\omega_{\tilde{s}}(x)}$ $\left(i=\sqrt{-1}\right)$, where $r_{\tilde{s}}(x)$ and $\omega_{\tilde{s}}(x)$ are both real-valued functions and $r_{\tilde{s}}(x)e^{i\omega_{\tilde{s}}(x)}$ has the range in complex unit circle. There is also an added additional term called the phase term to solve the enigma in translating some complex-valued functions on physical terms to human language and vice versa. Alkouri and Salleh [44] introduced the concept of complex intuitionistic fuzzy set to represent the information which is happening repeatedly over a period of time, while Selvachandran et al. [45] introduced the concept of complex vague soft sets which combine the key features of soft and complex fuzzy sets. To handle imprecise, indeterminate, inconsistent, and incomplete information that has periodic nature, Ali and Smarandache [46] introduced complex neutrosophic set. In complex neutrosophic set, each membership function associates with a phase term. This feature gives wave-like properties that could be used to describe constructive and destructive interference depending on the phase value of an element, as well as its ability to deal with indeterminacy.

Over the years, many techniques and methods have been proposed as tools to be used to find the solutions of problems that are nonlinear or vague in nature, with every method introduced superior to its predecessors. Following in this direction, our proposed model is an extension of soft expert set, fuzzy soft expert set [47], intutionistic fuzzy soft expert set (IFSES) [48], vague soft expert set [49] and single-valued neutrosophic soft expert set (SVNSES) [50]. Thus it will incorporate the advantages of all of these models. To facilitate our discussion, we first review some background on SVNS and complex neutrosophic set in Section 2. In Section 3, we give the motivation for this paper. In Section 4, we introduce the concept of CNSES and give its theoretic operations. In Section 5, we discuss an application of this concept in economy. In Section 6, the comparison analysis is conducted to verify the validity of the proposed approach. Finally, conclusions are pointed out in Section 7. Consequently, our proposed concept will enrich current studies in neutrosophic soft sets [51–55] and complex fuzzy sets [43, 56].

2. Preliminaries

In this section, we recapitulate the concepts of neutrosophic and complex neutrosophic sets and present an overview of the operations structures of the complex neutrosophic model that are relevant to the work in this paper. The complex neutrosophic soft set (CNSS) is also introduced.

Definition 2.1. (see [1]) Let U be a universe of discourse. A neutrosophic set N in U is defined as: $A = \{< u; T_N(u); I_N(u); F_N(u) >; u \in U\}$ where $T_N(u)$, $I_N(u)$ and $F_N(u)$ are the truth membership function, the indeterminacy membership function and the falsity membership function, respectively, such that $T; I; F : X \rightarrow]^-0; 1^+[$ and $^-0 \leq T_N(u) + I_N(u) + F_N(u) \leq 3^+$.

In order to apply neutrosophic set on the scientific fields, its parameters should have to be specified. Hence Wang et al. [5] provided the following definition.

Definition 2.2. (see [5]) Let U be a universe of discourse. A single-valued neutrosophic set (SVNS) S in U defined as: $S = \int_U \langle T(U), I(U), F(U)\rangle/u,\ u \in U$, when U is continuous and $S = \sum_{i=1}^{n} \langle T(U_i), I(U_i), F(U_i)\rangle/u_i,\ u_i \in U$, when U is discrete, where T_S, I_S and F_S are the truth membership function, the indeterminacy membership function and the falsity membership function, respectively and $T_S; I_S; F_S : U \rightarrow [0, 1]$.

Definition 2.3. (see [50]) Let $U = \{u_1, u_2, ..., u_n\}$ be a universal set of elements, $E = \{e_1, e_2, ..., e_m\}$ be a universal set of parameters, $X = \{x_1, x_2, ..., x_i\}$ be a set of experts (agents) and $O = \{1 = agree, 0 = disagree\}$ be a set of opinions. Let $Z = \{E \times X \times O\}$ and $A \subseteq Z$. Then the pair (U, Z) is called a soft universe. Let $F : Z \rightarrow SVN^U$, where SVN^U denotes the collection of all single-valued neutrosophic subsets of U. Suppose $F : Z \rightarrow SVN^U$ be a function defined as:

$$F(z) = F(z)(u_i),\ \forall u_i \in U.$$

Then $F(z)$ is called a single-valued neutrosophic soft expert value over the soft universe (U, Z).

Ali and Smarandache [46] conceptualized complex neutrosophic set and gave the basic operations in the following two definitions.

Definition 2.4. (see [46]) Let a universe of discourse U, a complex neutrosophic set S in U is characterized by a truth membership function $T_S(u)$, an indeterminacy membership function $I_S(u)$, and a falsity membership function $F_S(u)$ that assigns an element $u \in U$ a complex-valued grade of $T_S(u)$, $I_S(u)$, and $F_S(u)$ in S. By definition, the values $T_S(u)$, $I_S(u)$, $F_S(u)$ and their sum may all be within the unit circle in the complex plane and are of the form, $T_S(u) = p_S(u).e^{j\mu_S(u)}$, $I_S(u) = q_S(u).e^{j\nu_S(u)}$ and $F_S(u) = r_S(u).e^{j\omega_S(u)}$, each of $p_S(u), q_S(u), r_S(u)$ and $\mu_S(u), \nu_S(u), \omega_S(u)$ are, respectively, real valued and $p_S(u), q_S(u), r_S(u) \in [0, 1]$ such that $0^- \leq P_S(u) + q_S(u) + r_S(u) \leq 3^+$.

Definition 2.5. (see [46]) Let A and B be two complex neutrosophic sets on the universe U, where A is characterized by a truth membership function $T_A(u) = p_A(u).e^{j\mu_A(u)}$, an indeterminacy membership function $I_A(u) = q_A(u).e^{j\nu_A(u)}$ and a falsity membership function $F_A(u) = r_A(u).e^{j\omega_A(u)}$ and B is characterized by a truth membership function $T_B(u) = p_B(u).e^{j\mu_B(u)}$, an indeterminacy membership function $I_B(u) = q_B(u).e^{j\nu_B(u)}$ and a falsity membership function $F_B(u) = r_B(u).e^{j\omega_B(u)}$.

We define the the complement, subset, union and intersection operations as follows.

(1) The complement of A, denoted as $\tilde{c}(A)$ is specified by functions:

$$T_{\tilde{c}(A)}(u) = p_{\tilde{c}(A)}(u).e^{j\mu_{\tilde{c}(A)}(u)} = r_A(u).e^{j(2\pi-\mu_A(u))},$$

$$I_{\tilde{c}(A)}(u) = q_{\tilde{c}(A)}(u).e^{j\nu_{\tilde{c}(A)}(u)} = (1 - q_A(u)).e^{j(2\pi-\nu_A(u))},$$

and

$$F_{\tilde{c}(A)}(u) = r_{\tilde{c}(A)}(u).e^{j\omega_{\tilde{c}(A)}(u)} = p_A(u).e^{j(2\pi-\omega_A(u))}.$$

(2) A is said to be complex neutrosophic subset of B ($A \subseteq B$) if and only if the following conditions are satisfied:
   (a) $T_A(u) \leq T_B(u)$ such that $p_A(u) \leq p_B(u)$ and $\mu_A(u) \leq \mu_B(u)$.
   (b) $I_A(u) \geq I_B(u)$ such that $q_A(u) \geq q_B(u)$ and $\nu_A(u) \geq \nu_B(u)$.
   (c) $F_A(u) \geq F_B(u)$ such that $r_A(u) \geq r_B(u)$ and $\omega_A(u) \geq \omega_B(u)$.

(3) The union(intersection) of A and B, denoted as $A \cup (\cap) B$ and the truth membership function $T_{A\cup(\cap)B}(u)$, the indeterminacy membership function $I_{A\cup(\cap)B}(u)$, and the falsity membership function $F_{A\cup(\cap)B}(u)$ are defined as:

$$T_{A\cup(\cap)B}(u) = [(p_A(u) \vee (\wedge) p_B(u))] .e^{j(\mu_A(u)\vee(\wedge)\mu_B(u))},$$

$$I_{A\cup(\cap)B}(u) = [(q_A(u) \wedge (\vee) q_B(u))] .e^{j(\nu_A(u)\wedge(\vee)\nu_B(u))},$$

and

$$F_{A\cup(\cap)B}(u) = [(r_A(u) \wedge (\vee) r_B(u))] .e^{j(\omega_A(u)\wedge(\vee)\omega_B(u))},$$

where $\vee$ = max and $\wedge$ = min.

We will now introduce the concept of CNSS.

Definition 2.6. Let U be a universe, E be a set of parameters and $A \subseteq E$. Let $CNS(U)$ be a set of all complex neutrosophic subsets of U. A pair (H, A) is called a complex neutrosophic soft set (CNSS) over U where H is a mapping given by

$$H : A \to CNS(U).$$

In other words, the CNSS (H, A) is a parameterized family of all complex neutrosophic sets of U.

3. Motivation for complex neutrosophic soft expert set

Neutrosophic set deals with information or data which contain uncertainty, indeterminacy and falsity. Fuzzy set and intuitionistic fuzzy set do not handle indeterminacy, whereby the information might be true and false or neither true nor false at the same time. Thus, neutrosophic set can solve some problems where indeterminacy is deeply embedded in human thinking due to the imperfection of knowledge that human receives or observes from the external world. In reality, many phenomena and events happened periodically and all of the above models cannot address these situations. Therefore, many uncertainty approaches are developed such as complex fuzzy set which is characterized by a complex-valued membership function that handles information with uncertainty and periodicity simultaneously. Consequently, complex intuitionistic fuzzy set was thereafter developed by adding a complex-valued nonmembership function that handles the falsity and periodicity simultaneously. Nonetheless, these models cannot deal with indeterminate information which appear in a periodic manner in real life. To overcome this difficulty, complex neutrosophic set is introduced by adding a complex-valued indeterminacy membership function which tackles the indeterminacy and periodicity simultaneously. The complex neutrosophic set is superior to these models with three complex-valued membership functions which hold uncertainty, indeterminacy and falsity with periodicity. Further, the complex neutrosophic set is essentially neutrosophic set defined in a complex setting. Thus, it has the added advantages of the neutrosophic set by virtue of the complexity feature which has the ability to capture information that are periodic in nature, whereas neutrosophic set does not have this feature. The discussion above shows the ascendancy of complex neutrosophic set.

However, complex neutrosophic set lacks the adequate parameterization tool to facilitate the representation of parameters and it it does not have a mechanism to incorporate the opinion of all experts in one model. This decreases the validity of this model as most situations in the real-word are open to interpretations by different people. Thus, the CNSES is proposed to provide a more adequate parameterization tool that can represent the problem parameters in a more comprehensive and complete manner. It has also the added advantage of allowing the users to know the opinion of all the experts in a single model without the need for any additional cumbersome operations. The proposed CNSES model however, provides a more accurate representation of two-dimensional information i.e. information presented by the amplitude terms and information presented by the phase terms. The phase term represents the time factor that may interfere, constructively or destructively, with the associated amplitude term in the decision process. This makes it more valid and real in modeling real life problems where time factor and the judgments of human beings play a major role.

A novel adjustable approach to decision-making problems based on CNSES is also introduced. This approach converts the CNSES to a SVNSES using a practical and useful algorithm which highlights the role of the time factor in determining the final decision. The newly proposed approach efficiently captures the incomplete, indeterminate, and inconsistent information and extends existing decision-making methods to provide a more comprehensive outlook for decision-makers.

4. Complex neutrosophic soft expert set

In this section, we introduce the definition of complex neutrosophic soft expert set (CNSES) which is a combination of soft expert set and single-valued neutrosophic set defined in a complex setting. We define some operations on this concept, namely subset, equality, complement, union, intersection, AND and OR. We also show that De Morgan's law and other pertaining laws also hold in CNSES.

We begin by proposing the definition of CNSES, and give an illustrative example of it.

Let U be a universe, E a set of parameters, X a set of experts (agents), and $O = \{1 = agree, 0 = disagree\}$ a set of opinions. Let $Z = E \times X \times O$ and $A \subseteq Z$.

Definition 4.1. A pair (H, A) is called a complex neutrosophic soft expert set (CNSES) over U, where H is a mapping given by

$$H : A \to CN^U,$$

where CN^U denotes the power complex neutrosophic set of U.

It is to be noted that $\forall \alpha \in A,\ H(\alpha)$ represents the degree and the phase of belongingness, indeterminacy and non-belongingness of the elements of U in $H(\alpha)$.

The CNSES (H, A) can be written as:

$$(H, A) = \left\{ \left\langle \alpha, T_{H(\alpha)}(u), I_{H(\alpha)}(u), F_{H(\alpha)}(u) \right\rangle : \alpha \in A, u \in U \right\},$$

where $\forall u \in U$, $\forall \alpha \in A$, $T_{H(\alpha)}(u) = p_{H(\alpha)}(u) .e^{j\mu_{H(\alpha)}(u)}$, $I_{H(\alpha)}(u) = q_{H(\alpha)}(u).e^{j\nu_{H(\alpha)}(u)}$ and $F_{H(\alpha)}(u) = r_{H(\alpha)}(u).e^{j\omega_{H(\alpha)}(u)}$ with $T_{H(\alpha)}(u), I_{H(\alpha)}(u)$ and $F_{H(\alpha)}(u)$ representing the complex-valued truth membership function, complex-valued indeterminacy membership function and complex-valued falsity membership function, respectively $\forall u \in U$. The values $T_{H(\alpha)}(u), I_{H(\alpha)}(u), F_{H(\alpha)}(u)$ are within the unit circle in the complex plane and both the amplitude terms $p_{H(\alpha)}(u), q_{H(\alpha)}(u), r_{H(\alpha)}(u)$ and the phase terms $\mu_{H(\alpha)}(u), \nu_{H(\alpha)}(u), \omega_{H(\alpha)}(u)$ are real valued such that $p_{H(\alpha)}(u), q_{H(\alpha)}(u), r_{H(\alpha)}(u) \in [0, 1]$ and $0 \le p_{H(\alpha)}(u) + q_{H(\alpha)}(u) + r_{H(\alpha)}(u) \le 3$.

Example 4.2. Suppose that a pharmaceutical company develops two types of its medicine and wishes to take the opinion of some experts concerning these

$$
\begin{aligned}
(H, A) &= \Bigg\{ \left\{ (e_1, p, 1), \left\{ \frac{\langle 0.7e^{j2\Pi(0.3)},\ 0.2e^{j2\Pi(0.3)},\ 0.1e^{j2\Pi(0.2)} \rangle}{u_1}, \frac{\langle 0.9e^{j2\Pi(0.6)},\ 0.6e^{j2\Pi(0.4)},\ 0.3e^{j2\Pi(0)} \rangle}{u_2} \right\} \right\}, \\
&\left\{ (e_1, q, 1), \left\{ \frac{\langle 0.5e^{j2\Pi(0.3)},\ 0.2e^{j2\Pi(0.9)},\ 0.9e^{j2\Pi(0.8)} \rangle}{u_1}, \frac{\langle 0.9e^{j2\Pi(0.9)},\ 0.4e^{j2\Pi(0.5)},\ 0.5e^{j2\Pi(0.6)} \rangle}{u_2} \right\} \right\}, \\
&\left\{ (e_2, p, 1), \left\{ \frac{\langle 0.3e^{j2\Pi(0.9)},\ 0.2e^{j2\Pi(0.6)},\ 0.9e^{j2\Pi(0.1)} \rangle}{u_1}, \frac{\langle 0.9e^{j2\Pi(0.5)},\ 0.5e^{j2\Pi(0.7)},\ 0.4e^{j2\Pi(0.1)} \rangle}{u_2} \right\} \right\}, \\
&\left\{ (e_2, q, 1), \left\{ \frac{\langle 0.1e^{j2\Pi(0.8)},\ 0.2e^{j2\Pi(0.4)},\ 0.9e^{j2\Pi(0.8)} \rangle}{u_1}, \frac{\langle 0.9e^{j2\Pi(0.5)},\ 0.1e^{j2\Pi(0.1)},\ 0.2e^{j2\Pi(0)} \rangle}{u_2} \right\} \right\}, \\
&\left\{ (e_3, p, 1), \left\{ \frac{\langle 0.1e^{j2\Pi(0.4)},\ 0.2e^{j2\Pi(0.6)},\ 0.9e^{j2\Pi(0.2)} \rangle}{u_1}, \frac{\langle 0.9e^{j2\Pi(0.3)},\ 0.1e^{j2\Pi(0.5)},\ 0.3e^{j2\Pi(0.8)} \rangle}{u_2} \right\} \right\}, \\
&\left\{ (e_3, q, 1), \left\{ \frac{\langle 0.1e^{j2\Pi(0.3)},\ 0.2e^{j2\Pi(0.7)},\ 0.9e^{j2\Pi(0.9)} \rangle}{u_1}, \frac{\langle 0.9e^{j2\Pi(0.4)},\ 0.1e^{j2\Pi(0.6)},\ 0.5e^{j2\Pi(0)} \rangle}{u_2} \right\} \right\}, \\
&\left\{ (e_1, p, 0), \left\{ \frac{\langle 0.1e^{j2\Pi(0.7)},\ 0.8e^{j2\Pi(0.7)},\ 0.7e^{j2\Pi(0.8)} \rangle}{u_1}, \frac{\langle 0.3e^{j2\Pi(0.4)},\ 0.4e^{j2\Pi(0.6)},\ 0.9e^{j2\Pi(1)} \rangle}{u_2} \right\} \right\}, \\
&\left\{ (e_1, q, 0), \left\{ \frac{\langle 0.9e^{j2\Pi(0.7)},\ 0.8e^{j2\Pi(0.1)},\ 0.5e^{j2\Pi(0.2)} \rangle}{u_1}, \frac{\langle 0.5e^{j2\Pi(0.1)},\ 0.6e^{j2\Pi(0.5)},\ 0.9e^{j2\Pi(0.4)} \rangle}{u_2} \right\} \right\}, \\
&\left\{ (e_2, p, 0), \left\{ \frac{\langle 0.9e^{j2\Pi(0.1)},\ 0.8e^{j2\Pi(0.4)},\ 0.3e^{j2\Pi(0.9)} \rangle}{u_1}, \frac{\langle 0.4e^{j2\Pi(0.5)},\ 0.1e^{j2\Pi(0.3)},\ 0.4e^{j2\Pi(0.9)} \rangle}{u_2} \right\} \right\}, \\
&\left\{ (e_2, q, 0), \left\{ \frac{\langle 0.9e^{j2\Pi(0.2)},\ 0.8e^{j2\Pi(0.6)},\ 0.1e^{j2\Pi(0.2)} \rangle}{u_1}, \frac{\langle 0.2e^{j2\Pi(0.5)},\ 0.9e^{j2\Pi(0.9)},\ 0.2e^{j2\Pi(1)} \rangle}{u_2} \right\} \right\}, \\
&\left\{ (e_3, p, 0), \left\{ \frac{\langle 0.9e^{j2\Pi(0.6)},\ 0.8e^{j2\Pi(0.4)},\ 0.1e^{j2\Pi(0.8)} \rangle}{u_1}, \frac{\langle 0.3e^{j2\Pi(0.7)},\ 0.9e^{j2\Pi(0.5)},\ 0.9e^{j2\Pi(0.2)} \rangle}{u_2} \right\} \right\}, \\
&\left\{ (e_3, q, 0), \left\{ \frac{\langle 0.9e^{j2\Pi(0.7)},\ 0.8e^{j2\Pi(0.3)},\ 0.1e^{j2\Pi(0.1)} \rangle}{u_1}, \frac{\langle 0.5e^{j2\Pi(0.6)},\ 0.9e^{j2\Pi(0.4)},\ 0.9e^{j2\Pi(1)} \rangle}{u_2} \right\} \right\} \Bigg\}.
\end{aligned}
$$

medications by taking into account the degree of effectiveness and the time taken to overcome the disease which are represented by amplitude terms and phase terms, respectively. Let $U = \{u_1, u_2\}$ be a set of medication, $E = \{e_1, e_2, e_3\}$ a set of parameters that describes the degree of influence where $e_i (i = 1, 2, 3)$ denotes the decisions "high influence", "average influence" and "low influence" respectively and let $X = \{p, q\}$ be a set of experts.

Suppose that the company has distributed a questionnaire to the two experts to make decisions on these two new medication, then the CNSES (H, A) is defined as below:

In the CNSES (H, A), both the amplitude terms and phase terms lie between 0 and 1 such that an amplitude term with value close to 0 (1) implies that a medicine has a very little (strong) influence on a disease and a phase term with value close to 0 (1) implies that this medicine takes a very short (long) time to overcome the disease.

In the following, we introduce the concept of the subset of two CNSESs and the equality of two CNSESs.

Definition 4.3. For two CNSESs (H, A) and (G, B) over U, (H, A) is called a complex neutrosophic soft expert subset of (G, B) if

1. $A \subseteq B$,
2. $\forall \epsilon \in A$, $H(\epsilon)$ is complex neutrosophic subset of $G(\epsilon)$.

Definition 4.4. Two CNSESs (H, A) and (G, B) over U, are said to be equal if (H, A) is a complex neutrosophic soft expert subset of (G, B) and (G, B) is a complex neutrosophic soft expert subset of (H, A).

In the following, we propose the definition of the complement of a CNSES along with an illustrative example and give a proposition of the complement of a CNSES.

Let U be a universe of discourse and (H, A) be a CNSES on U, which is as defined below:

$$(H, A) = \left\{ \left\langle \alpha, T_{H(\alpha)}(u), I_{H(\alpha)}(u), F_{H(\alpha)}(u) \right\rangle : \alpha \in A, u \in U \right\}.$$

Definition 4.5. The complement of (H, A) is denoted by $(H, A)^c = (H^c, A)$, and is defined as:

$$(H, A)^c = \left\{ \left\langle \alpha, T_{H^c(\alpha)}(u), I_{H^c(\alpha)}(u), F_{H^C(\alpha)}(u) \right\rangle : \alpha \in A, u \in U \right\},$$

where $T_{H^c(\alpha)}(u) = p_{H^c(\alpha)}(u).e^{j\mu_{H^c(\alpha)}(u)} = r_{H(\alpha)}(u).e^{j(2\pi-\mu_{H(\alpha)}(u))}$, $I_{H^c(\alpha)}(u) = q_{H^c(\alpha)}(u).e^{j\nu_{H^c(\alpha)}(u)} = (1-q_{H(\alpha)}(u)).e^{j(2\pi-\nu_{H(\alpha)}(u))}$ and $F_{H^c(\alpha)}(u) = r_{H^c(\alpha)}(u).e^{j\omega_{H^c(\alpha)}(u)} = p_{H(\alpha)}(u).e^{j(2\pi-\omega_{H(\alpha)}(u))}$.

Example 4.6. Consider the approximation given in Example 4.2, where

$$H(e_1, p, 1) = \left\{ \frac{\langle 0.7e^{j2\Pi(0.3)},\ 0.2e^{j2\Pi(0.3)},\ 0.1e^{j2\Pi(0.2)} \rangle}{u_1}, \frac{\langle 0.9e^{j2\Pi(0.6)},\ 0.6e^{j2\Pi(0.4)},\ 0.3e^{j2\Pi(0)} \rangle}{u_2} \right\}.$$

By using the complex neutrosophic complement, we obtain the complement of the approximation given by

$$H(e_1, p, 1) = \left\{ \frac{\langle 0.1e^{j2\Pi(0.7)},\ 0.8e^{j2\Pi(0.7)},\ 0.7e^{j2\Pi(0.8)} \rangle}{u_1}, \frac{\langle 0.3e^{j2\Pi(0.4)},\ 0.4e^{j2\Pi(0.6)},\ 0.9e^{j2\Pi(1)} \rangle}{u_2} \right\}.$$

Proposition 4.7. *If (H, A) is a CNSES over U, then, $((H, A)^c)^c = (H, A)$.*

Proof. From Definition 4.5, we have $(H, A)^c = (H^c, A)$ where

$$\begin{aligned}
(H, A)^c &= \left\{ \left\langle \alpha, T_{H^c(\alpha)}(u), I_{H^c(\alpha)}(u), F_{H^C(\alpha)}(u) \right\rangle : \alpha \in A, u \in U \right\}, \\
&= \left\{ \left\langle \alpha, p_{H^c(\alpha)}(u).e^{j\mu_{H^c(\alpha)}(u)}, q_{H^c(\alpha)}(u).e^{j\nu_{H^c(\alpha)}(u)}, r_{H^c(\alpha)}(u).e^{j\omega_{H^c(\alpha)}(u)} \right\rangle : \alpha \in A, u \in U \right\}, \\
&= \left\{ \left\langle \alpha, r_{H(\alpha)}(u).e^{j(2\pi-\mu_{H(\alpha)}(u))}, (1-q_{H(\alpha)}(u)).e^{j(2\pi-\nu_{H(\alpha)}(u))}, p_{H(\alpha)}(u).e^{j(2\pi-\omega_{H(\alpha)}(u))} \right\rangle : \alpha \in A, u \in U \right\}.
\end{aligned}$$

Thus,

$$\begin{aligned}
((H, A)^c)^c &= \left\{ \left\langle \alpha, r_{H^c(\alpha)}(u).e^{j(2\pi-\mu_{H^c(\alpha)}(u))}, (1-q_{H^c(\alpha)}(u)).e^{j(2\pi-\nu_{H^c(\alpha)}(u))}, p_{H^c(\alpha)}(u).e^{j(2\pi-\omega_{H^c(\alpha)}(u))} \right\rangle : \alpha \in A,\ u \in U \right\}, \\
&= \left\{ \left\langle \alpha, p_{H(\alpha)}(u).e^{j\left(2\pi-(2\pi-\mu_{H(\alpha)}(u))\right)}, \left(1-(1-q_{H(\alpha)}(u))\right).e^{j\left(2\pi-(2\pi-\nu_{H(\alpha)}(u))\right)}, r_{H(\alpha)}(u).e^{j\left(2\pi-(2\pi-\omega_{H(\alpha)}(u))\right)} \right\rangle : \alpha \in A, u \in U \right\}, \\
&= \left\{ \left\langle \alpha, p_{H(\alpha)}(u).e^{j\mu_{H(\alpha)}(u)}, q_{H(\alpha)}(u).e^{j\nu_{H(\alpha)}(u)}, \right. \right.
\end{aligned}$$

$$r_{H(\alpha)}(u).e^{j\omega_{H(\alpha)}(u)}\Big\rangle : \alpha \in A, u \in U\Big\},$$

$$=\Big\{\Big\langle \alpha, T_{H(\alpha)}(u), I_{H(\alpha)}(u), F_{H(\alpha)}(u)\Big\rangle :$$

$$\alpha \in A, u \in U\Big\},$$

$$= (H, A).$$

This completes the proof.

Now, we put forward the definition of an agree- CNSES and the definition of a disagree- CNSES.

Definition 4.8. An agree- CNSES $(H, A)_1$ over U is a complex neutrosophic soft expert subset of (H, A) where the opinions of all experts are agree and is defined as follows:

$$(H, A)_1 = \Big\{H(e) : e \in Z \times X \times \{1\}\Big\}$$

Definition 4.9. A disagree- CNSES $(H, A)_0$ over U is a complex neutrosophic soft expert subset of (H, A) where the opinions of all experts are disagree and is defined as follows:

$$(H, A)_0 = \Big\{H(e) : e \in Z \times X \times \{0\}\Big\}$$

In the following, we introduce the definitions of the union and intersection of two CNSESs.

Definition 4.10. The union of two CNSESs (H, A) and (G, B) over a universe U is a CNSES (K, C), where $C = A \cup B$ and $\forall \epsilon \in C, \forall u \in U$,

$$T_{K(\epsilon)}(u) = \begin{cases} p_{H(\epsilon)}(u).e^{j\mu_{H(\epsilon)}(u)}, & \text{if } \epsilon \in A - B \\ p_{G(\epsilon)}(u).e^{j\mu_{G(\epsilon)}(u)}, & \text{if } \epsilon \in B - A \\ (p_{H(\epsilon)}(u) \vee p_{G(\epsilon)}(u)) & \\ .e^{j(\mu_{H(\epsilon)}(u) \vee \mu_{G(\epsilon)}(u))}, & \text{if } \epsilon \in A \cap B, \end{cases}$$

$$I_{K(\epsilon)}(u) = \begin{cases} q_{H(\epsilon)}(u).e^{j\nu_{H(\epsilon)}(u)}, & \text{if } \epsilon \in A - B \\ q_{G(\epsilon)}(u).e^{j\nu_{G(\epsilon)}(u)}, & \text{if } \epsilon \in B - A \\ (q_{H(\epsilon)}(u) \wedge q_{G(\epsilon)}(u)) & \\ .e^{j(\nu_{H(\epsilon)}(u) \wedge \nu_{G(\epsilon)}(u))}, & \text{if } \epsilon \in A \cap B, \end{cases}$$

$$F_{K(\epsilon)}(u) = \begin{cases} r_{H(\epsilon)}(u).e^{j\omega_{H(\epsilon)}(u)}, & \text{if } \epsilon \in A - B \\ r_{G(\epsilon)}(u).e^{j\omega_{G(\epsilon)}(u)}, & \text{if } \epsilon \in B - A \\ (r_{H(\epsilon)}(u) \wedge r_{G(\epsilon)}(u)) & \\ .e^{j(\omega_{H(\epsilon)}(u) \wedge \omega_{G(\epsilon)}(u))}, & \text{if } \epsilon \in A \cap B, \end{cases}$$

where $\vee = \max$, and $\wedge = \min$.

The union $(H, A)\,\tilde{\cup}\,(G, B) = (K, C)$.

Definition 4.11. The intersection of two CNSESs (H, A) and (G, B) over a universe U is a CNSES (K, C), where $C = A \cup B$ and $\forall \epsilon \in C, \forall u \in U$,

$$T_{K(\epsilon)}(u) = \begin{cases} p_{H(\epsilon)}(u).e^{j\mu_{H(\epsilon)}(u)}, & \text{if } \epsilon \in A - B \\ p_{G(\epsilon)}(u).e^{j\mu_{G(\epsilon)}(u)}, & \text{if } \epsilon \in B - A \\ (p_{H(\epsilon)}(u) \wedge p_{G(\epsilon)}(u)) & \\ .e^{j(\mu_{H(\epsilon)}(u) \wedge \mu_{G(\epsilon)}(u))}, & \text{if } \epsilon \in A \cap B, \end{cases}$$

$$I_{K(\epsilon)}(u) = \begin{cases} q_{H(\epsilon)}(u).e^{j\nu_{H(\epsilon)}(u)}, & \text{if } \epsilon \in A - B \\ q_{G(\epsilon)}(u).e^{j\nu_{G(\epsilon)}(u)}, & \text{if } \epsilon \in B - A \\ (q_{H(\epsilon)}(u) \vee q_{G(\epsilon)}(u)) & \\ .e^{j(\nu_{H(\epsilon)}(u) \vee \nu_{G(\epsilon)}(u))}, & \text{if } \epsilon \in A \cap B, \end{cases}$$

$$F_{K(\epsilon)}(u) = \begin{cases} r_{H(\epsilon)}(u).e^{j\omega_{H(\epsilon)}(u)}, & \text{if } \epsilon \in A - B \\ r_{G(\epsilon)}(u).e^{j\omega_{G(\epsilon)}(u)}, & \text{if } \epsilon \in B - A \\ (r_{H(\epsilon)}(u) \vee r_{G(\epsilon)}(u)) & \\ .e^{j(\omega_{H(\epsilon)}(u) \vee \omega_{G(\epsilon)}(u))}, & \text{if } \epsilon \in A \cap B, \end{cases}$$

where $\vee = \max$, and $\wedge = \min$.

The intersection $(H, A)\,\tilde{\cap}\,(G, B) = (K, C)$.

We show that De Morgan's law holds for the CNSES as follows.

Proposition 4.12. *If (H, A) and (G, B) are two CNSESs over U, then we have the following properties:*

1. $((H, A)\widetilde{\cup}(G, B))^c = (H, A)^c\widetilde{\cap}(G, B)^c$,
2. $((H, A)\widetilde{\cap}(G, B))^c = (H, A)^c\widetilde{\cup}(G, B)^c$.

Proof. (1) Assume that $(H, A)\widetilde{\cup}(G, B) = (K, C)$, where $C = A \cup B$ and $\forall\epsilon \in C$,

$$T_{K(\epsilon)}(u) = \begin{cases} p_{H(\epsilon)}(u).e^{j\mu_{H(\epsilon)}(u)}, & \text{if } \epsilon \in A - B \\ p_{G(\epsilon)}(u).e^{j\mu_{G(\epsilon)}(u)}, & \text{if } \epsilon \in B - A \\ (p_{H(\epsilon)}(u) \vee p_{G(\epsilon)}(u) & \\ .e^{j(\mu_{H(\epsilon)}(u) \vee \mu_{G(\epsilon)}(u))}, & \text{if } \epsilon \in A \cap B. \end{cases}$$

Since $(H, A)\widetilde{\cup}(G, B) = (K, C)$, then we have $((H, A)\widetilde{\cup}(G, B))^c = (K, C)^c = (K^c, C)$. Hence $\forall\epsilon \in C$,

$$T_{K^c(\epsilon)}(u)$$

$$= \begin{cases} r_{H(\epsilon)}(u).e^{j(2\pi - \mu_{H(\epsilon)}(u))}, & \text{if } \epsilon \in A - B \\ r_{G(\epsilon)}(u).e^{j(2\pi - \mu_{H(\epsilon)}(u))}, & \text{if } \epsilon \in B - A \\ (r_{H(\epsilon)}(u) \wedge r_{G(\epsilon)}(u)) & \\ .e^{j((2\pi - \mu_{H(\epsilon)}(u)) \wedge (2\pi - \mu_{G(\epsilon)}(u)))}, & \text{if } \epsilon \in A \cap B. \end{cases}$$

Since $(H, A)^c = (H^c, A)$ and $(G, B)^c = (G^c, B)$, then we have $(H, A)^c\widetilde{\cap}(G, B)^c = (H^c, A)\widetilde{\cap}(G^c, B)$. Suppose that $(H^c, A)\widetilde{\cap}(G^c, B) = (I, D)$, where $D = A \cup B$. Hence $\forall\epsilon \in D$,

$$T_{I(\epsilon)}(u) = \begin{cases} p_{H^c(\epsilon)}(u).e^{j\mu_{H^c(\epsilon)}(u)}, & \text{if } \epsilon \in A-B \\ p_{G^c(\epsilon)}(u).e^{j\mu_{G^c(\epsilon)}(u)}, & \text{if } \epsilon \in B-A \\ (p_{H^c(\epsilon)}(u) \wedge p_{G^c(\epsilon)}(u)) & \\ .e^{j(\mu_{H^c(\epsilon)}(u) \wedge \mu_{G^c(\epsilon)}(u))}, & \text{if } \epsilon \in A \cap B, \end{cases}$$

$$= \begin{cases} r_{H(\epsilon)}(u).e^{j(2\pi-\mu_{H(\epsilon)}(u))}, & \text{if } \epsilon \in A-B \\ r_{G(\epsilon)}(u).e^{j(2\pi-\mu_{H(\epsilon)}(u))}, & \text{if } \epsilon \in B-A \\ (r_{H(\epsilon)}(u) \wedge r_{G(\epsilon)}(u)) & \\ .e^{j((2\pi-\mu_{H(\epsilon)}(u)) \wedge (2\pi-\mu_{G(\epsilon)}(u)))}, & \text{if } \epsilon \in A \cap B. \end{cases}$$

Therefore, K^c and I are the same operators and $D = C$, which implies, $T_{(H(\epsilon)\cup G(\epsilon))^c}(u) = T_{H^c(\epsilon)\cap G^c(\epsilon)}(u), \forall u \in U$.

Similarly, on the same lines, we can show it also holds for the identity and falsity terms. Thus it follows that $((H, A)\widetilde{\cup}(G, B))^c = (H, A)^c\widetilde{\cap}(G, B)^c$ and this completes the proof.

(2) The proof is similar to that of (1).

We will now give the definitions of AND and OR operations with a proposition on these two operations.

Definition 4.13. Let (H, A) and (G, B) be any two CNSESs over a soft universe (U, Z). Then the operation (H, A) AND (G, B) denoted by $(H, A)\widetilde{\wedge}(G, B)$ is defined by $(H, A)\widetilde{\wedge}(G, B) = (K, A \times B)$, where $(K, A \times B) = K(\alpha, \beta)$, such that $K(\alpha, \beta) = H(\alpha) \cap G(\beta)$, for all $(\alpha, \beta) \in A \times B$, and $\cap$ represents the complex neutrosophic intersection.

Definition 4.14. Let (H, A) and (G, B) be any two CNSESs over a soft universe (U, Z). Then the operation (H, A) OR (G, B) denoted by $(H, A)\widetilde{\vee}(G, B)$ is defined by $(H, A)\widetilde{\vee}(G, B) = (K, A \times B)$, where $(K, A \times B) = K(\alpha, \beta)$, such that $K(\alpha, \beta) = H(\alpha) \cup G(\beta)$, for all $(\alpha, \beta) \in A \times B$, and $\cup$ represents the complex neutrosophic union.

Proposition 4.15. *If (H, A) and (G, B) are two CNSESs over U, then we have the following properties:*

1. $((H, A)\widetilde{\vee}(G, B))^c = (H, A)^c\widetilde{\wedge}(G, B)^c$,
2. $((H, A)\widetilde{\wedge}(G, B))^c = (H, A)^c\widetilde{\vee}(G, B)^c$.

Proof. The proof of (1) and (2) is similar to the proof of Propositions 4.12.

5. Decision-making under the complex neutrosophic soft expert environment

In this section, we present an application of CNSES in a decision-making problem by considering the following problem.

Example 5.1. Suppose we are interested in understanding the most important economic factors (indicators) affecting Malaysian economy in 2016. Suppose we take four factors which are represented in the universal set $U = \{u_1, u_2, u_3, u_4\}$ where u_1 = the plunge in commodity and oil prices, u_2 = China's economic slowdown, u_3 = goods and services tax (GST) implemented in this year and u_4 = the exchange rate variability. Our problem is to arrange these four factors in descending order from most important to least important. Let $E = \{e_1, e_2, e_3\}$ be the parameters set that represents the major sectors of the Malaysian economy, where e_1 = industry sector, e_2 = tourism sector, e_3 = external trade sector. Suppose $X = \{p, q\}$ be a set of economic experts who are assigned to analyze these four factors by determining the degree and the total time of the influence of these factors on the mentioned sectors of the Malaysian economy as in the following CNSES:

$$(H, A) = \Bigg\{\Bigg\{(e_1, p, 1), \frac{\langle 0.9e^{j2\Pi(11/12)}, 0.2e^{j2\Pi(1/12)}, 0.1e^{j2\Pi(0)}\rangle}{u_1}, \frac{\langle 0.5e^{j2\Pi(5/12)}, 0.6e^{j2\Pi(4/12)}, 0.7e^{j2\Pi(4/12)}\rangle}{u_2},$$
$$\Bigg\{\frac{\langle 0.4e^{j2\Pi(3/12)}, 0.2e^{j2\Pi(3/12)}, 0.6e^{j2\Pi(11/12)}\rangle}{u_3}, \frac{\langle 0.9e^{j2\Pi(8/12)}, 0.5e^{j2\Pi(5/12)}, 0.3e^{j2\Pi(0)}\rangle}{u_4}\Bigg\}\Bigg\},$$
$$\Bigg\{(e_1, q, 1), \frac{\langle 0.9e^{j2\Pi(8/12)}, 0.1e^{j2\Pi(2/12)}, 0.3e^{j2\Pi(1/12)}\rangle}{u_1}, \frac{\langle 0.7e^{j2\Pi(6/12)}, 0.4e^{j2\Pi(5/12)}, 0.9e^{j2\Pi(8/12)}\rangle}{u_2},$$
$$\Bigg\{\frac{\langle 0.3e^{j2\Pi(1/12)}, 0.9e^{j2\Pi(6/12)}, 0.9e^{j2\Pi(10/12)}\rangle}{u_3}, \frac{\langle 0.8e^{j2\Pi(8/12)}, 0.4e^{j2\Pi(6/12)}, 0.3e^{j2\Pi(2/12)}\rangle}{u_4}\Bigg\}\Bigg\},$$
$$\Bigg\{(e_2, p, 1), \frac{\langle 0.8e^{j2\Pi(8/12)}, 0.2e^{j2\Pi(4/12)}, 0.3e^{j2\Pi(1/12)}\rangle}{u_1}, \frac{\langle 0.4e^{j2\Pi(6/12)}, 0.5e^{j2\Pi(1/12)}, 0.4e^{j2\Pi(1/12)}\rangle}{u_2},$$

$$\left\{\frac{\langle 0.3e^{j2\Pi(6/12)},\ 0.5e^{j2\Pi(7/12)},\ 0.9e^{j2\Pi(8/12)}\rangle}{u_3},\frac{\langle 0.6e^{j2\Pi(7/12)},\ 0.3e^{j2\Pi(5/12)},\ 0.2e^{j2\Pi(6/12)}\rangle}{u_4}\right\}\Bigg\},$$

$$\Bigg\{(e_2,q,1),\frac{\langle 0.5e^{j2\Pi(11/12)},\ 0.2e^{j2\Pi(1/12)},\ 0.1e^{j2\Pi(1/12)}\rangle}{u_1},\frac{\langle 0.7e^{j2\Pi(6/12)},\ 0.1e^{j2\Pi(1/12)},\ 0.9e^{j2\Pi(4/12)}\rangle}{u_2},$$

$$\left\{\frac{\langle 0.2e^{j2\Pi(6/12)},\ 0.6e^{j2\Pi(4/12)},\ 0.9e^{j2\Pi(6/12)}\rangle}{u_3},\frac{\langle 0.5e^{j2\Pi(7/12)},\ 0.5e^{j2\Pi(4/12)},\ 0.2e^{j2\Pi(4/12)}\rangle}{u_4}\right\}\Bigg\},$$

$$\Bigg\{(e_3,p,1),\frac{\langle 0.7e^{j2\Pi(8/12)},\ 0.1e^{j2\Pi(2/12)},\ 0.4e^{j2\Pi(2/12)}\rangle}{u_1},\frac{\langle 0.9e^{j2\Pi(3/12)},\ 0.1e^{j2\Pi(6/12)},\ 0.5e^{j2\Pi(9/12)}\rangle}{u_2},$$

$$\left\{\frac{\langle 0.4e^{j2\Pi(1/12)},\ 0.2e^{j2\Pi(7/12)},\ 0.9e^{j2\Pi(2/12)}\rangle}{u_3},\frac{\langle 0.8e^{j2\Pi(9/12)},\ 0.3e^{j2\Pi(1/12)},\ 0.2e^{j2\Pi(1/12)}\rangle}{u_4}\right\}\Bigg\},$$

$$\Bigg\{(e_3,q,1),\frac{\langle 0.7e^{j2\Pi(6/12)},\ 0.5e^{j2\Pi(4/12)},\ 0.2e^{j2\Pi(2/12)}\rangle}{u_1},\frac{\langle 0.4e^{j2\Pi(7/12)},\ 0.1e^{j2\Pi(8/12)},\ 0.4e^{j2\Pi(6/12)}\rangle}{u_2},$$

$$\left\{\frac{\langle 0.3e^{j2\Pi(3/12)},\ 0.5e^{j2\Pi(8/12)},\ 0.9e^{j2\Pi(6/12)}\rangle}{u_3},\frac{\langle 0.4e^{j2\Pi(7/12)},\ 0.7e^{j2\Pi(5/12)},\ 0.7e^{j2\Pi(6/12)}\rangle}{u_4}\right\}\Bigg\},$$

$$\Bigg\{(e_1,p,0),\frac{\langle 0.1e^{j2\Pi(1/12)},\ 0.8e^{j2\Pi(11/12)},\ 0.9e^{j2\Pi(12/12)}\rangle}{u_1},\frac{\langle 0.7e^{j2\Pi(7/12)},\ 0.4e^{j2\Pi(8/12)},\ 0.5e^{j2\Pi(8/12)}\rangle}{u_2},$$

$$\left\{\frac{\langle 0.6e^{j2\Pi(9/12)},\ 0.8e^{j2\Pi(9/12)},\ 0.4e^{j2\Pi(1/12)}\rangle}{u_3},\frac{\langle 0.3e^{j2\Pi(4/12)},\ 0.5e^{j2\Pi(7/12)},\ 0.9e^{j2\Pi(12/12)}\rangle}{u_4}\right\}\Bigg\},$$

$$\Bigg\{(e_1,q,0),\frac{\langle 0.3e^{j2\Pi(4/12)},\ 0.9e^{j2\Pi(10/12)},\ 0.1e^{j2\Pi(11/12)}\rangle}{u_1},\frac{\langle 0.9e^{j2\Pi(6/12)},\ 0.6e^{j2\Pi(7/12)},\ 0.7e^{j2\Pi(4/12)}\rangle}{u_2},$$

$$\left\{\frac{\langle 0.9e^{j2\Pi(11/12)},\ 0.1e^{j2\Pi(6/12)},\ 0.7e^{j2\Pi(2/12)}\rangle}{u_3},\frac{\langle 0.3e^{j2\Pi(4/12)},\ 0.6e^{j2\Pi(6/12)},\ 0.8e^{j2\Pi(10/12)}\rangle}{u_4}\right\}\Bigg\},$$

$$\Bigg\{(e_2,p,0),\frac{\langle 0.3e^{j2\Pi(4/12)},\ 0.8e^{j2\Pi(8/12)},\ 0.8e^{j2\Pi(11/12)}\rangle}{u_1},\frac{\langle 0.4e^{j2\Pi(6/12)},\ 0.5e^{j2\Pi(11/12)},\ 0.4e^{j2\Pi(11/12)}\rangle}{u_2},$$

$$\left\{\frac{\langle 0.9e^{j2\Pi(6/12)},\ 0.5e^{j2\Pi(5/12)},\ 0.7e^{j2\Pi(4/12)}\rangle}{u_3},\frac{\langle 0.2e^{j2\Pi(5/12)},\ 0.7e^{j2\Pi(7/12)},\ 0.6e^{j2\Pi(6/12)}\rangle}{u_4}\right\}\Bigg\},$$

$$\Bigg\{(e_2,q,0),\frac{\langle 0.1e^{j2\Pi(1/12)},\ 0.8e^{j2\Pi(11/12)},\ 0.5e^{j2\Pi(11/12)}\rangle}{u_1},\frac{\langle 0.9e^{j2\Pi(6/12)},\ 0.9e^{j2\Pi(11/12)},\ 0.7e^{j2\Pi(8/12)}\rangle}{u_2},$$

$$\left\{\frac{\langle 0.9e^{j2\Pi(6/12)},\ 0.4e^{j2\Pi(8/12)},\ 0.2e^{j2\Pi(6/12)}\rangle}{u_3},\frac{\langle 0.2e^{j2\Pi(5/12)},\ 0.5e^{j2\Pi(8/12)},\ 0.5e^{j2\Pi(8/12)}\rangle}{u_4}\right\}\Bigg\},$$

$$\Bigg\{(e_3,p,0),\frac{\langle 0.4e^{j2\Pi(4/12)},\ 0.9e^{j2\Pi(10/12)},\ 0.3e^{j2\Pi(10/12)}\rangle}{u_1},\frac{\langle 0.5e^{j2\Pi(9/12)},\ 0.9e^{j2\Pi(6/12)},\ 0.9e^{j2\Pi(3/12)}\rangle}{u_2},$$

$$\left\{\frac{\langle 0.9e^{j2\Pi(11/12)},\ 0.8e^{j2\Pi(5/12)},\ 0.6e^{j2\Pi(10/12)}\rangle}{u_3},\frac{\langle 0.2e^{j2\Pi(3/12)},\ 0.7e^{j2\Pi(11/12)},\ 0.8e^{j2\Pi(11/12)}\rangle}{u_4}\right\}\Bigg\},$$

$$\Bigg\{(e_3,q,0),\frac{\langle 0.2e^{j2\Pi(6/12)},\ 0.5e^{j2\Pi(8/12)},\ 0.3e^{j2\Pi(10/12)}\rangle}{u_1},\frac{\langle 0.4e^{j2\Pi(5/12)},\ 0.9e^{j2\Pi(4/12)},\ 0.4e^{j2\Pi(6/12)}\rangle}{u_2},$$

$$\left\{\frac{\langle 0.9e^{j2\Pi(9/12)},\ 0.5e^{j2\Pi(4/12)},\ 0.7e^{j2\Pi(6/12)}\rangle}{u_3},\frac{\langle 0.7e^{j2\Pi(5/12)},\ 0.3e^{j2\Pi(7/12)},\ 0.4e^{j2\Pi(6/12)}\rangle}{u_4}\right\}\Bigg\}\Bigg\}.$$

In the context of this example, the amplitude terms measure the influence degree of the mentioned factors on the Malaysian economy, while the phase term represents the phase of this influence or the period of this influence.

Following in this direction, we provide an example of scenarios that could possibly occur in this context. For example, in the approximation

$$H(e_1, p, 1) = \left(\left\{ \frac{\langle 0.9e^{j2\Pi(11/12)},\ 0.2e^{j2\Pi(1/12)},\ 0.1e^{j2\Pi(0)} \rangle}{u_1}, \frac{\langle 0.5e^{j2\Pi(5/12)},\ 0.6e^{j2\Pi(4/12)},\ 0.7e^{j2\Pi(4/12)} \rangle}{u_2}, \dots \right\} \right),$$

the complex neutrosophic soft expert value (CNSEV)

$$\frac{\langle 0.9e^{j2\Pi(11/12)},\ 0.2e^{j2\Pi(1/12)},\ 0.1e^{j2\Pi(0)} \rangle}{u_1}$$

indicates that the plunge in commodity and oil prices has a big influence on the Malaysian economy. The complex-valued truth membership function $0.9e^{j2\Pi(11/12)}$ indicates that the expert p agrees that there is a strong influence of the plunge in commodity and oil prices on the industrial sector, since the amplitude term 0.9 is very close to one and this influence span of 11 months is considered a very long time of influence (phase term with value very close to one), the complex-valued indeterminacy membership function $0.2e^{j2\Pi(1/12)}$ can be interpreted as the expert p is unable to determine if there is influence or not with a degree of 0.2 and this influence is not evident for a month. For the complex-valued falsity membership function $0.1e^{j2\Pi(0)}$, expert p presumes with a degree of 0.1 that there is no influence and the time with no influence is 0.

Next the CNSES (H, A) is used together with a generalized algorithm to solve the decision-making problem stated at the beginning of this section. This algorithm is employed to rank the factors that affect the Malaysian economy corresponding to their influence strength. In this decision process the time of influence plays a key role where the factor which has a large degree of influence and a long time of influence will be more important than others. The algorithm given below converts the complex neutrosophic soft expert values (CNSEVs) to normalized single-valued neutrosophic soft expert values (SVNSEVs) and proceeds to the final decision using the single-valued neutrosophic soft expert method (SVNSEM) [50]. The algorithm steps are given as follows.

Algorithm:

1. Input the CNSES (H, A)

2. Convert the CNSES (H, A) to the SVNSES $(\widehat{H}, A)$ by obtaining the weighted aggregation values of $T_{\widehat{H}(\alpha)}(u)$, $I_{\widehat{H}(\alpha)}(u)$ and $F_{\widehat{H}(\alpha)}(u)$, $\forall \alpha \in A$ and $\forall u \in U$ as the following formulas:

$$T_{\widehat{H}(\alpha)}(u) = w_1 p_{H(\alpha)}(u) + w_2(1/2\pi)\mu_{H(\alpha)}(u),$$
$$I_{\widehat{H}(\alpha)}(u) = w_1 q_{H(\alpha)}(u) + w_2(1/2\pi)\nu_{H(\alpha)}(u),$$
$$F_{\widehat{H}(\alpha)}(u) = w_1 r_{H(\alpha)}(u) + w_2(1/2\pi)\omega_{H(\alpha)}(u),$$

where $p_{H(\alpha)}(u), q_{H(\alpha)}(u)$, $r_{H(\alpha)}(u)$ and $\mu_{H(\alpha)}(u), \nu_{H(\alpha)}(u), \omega_{H(\alpha)}(u)$ are the amplitude and phase terms in the CNSES (H, A), respectively. $T_{\widehat{H}(\alpha)}(u), I_{\widehat{H}(\alpha)}(u)$ and $F_{\widehat{H}(\alpha)}(u)$ are the truth membership function, indeterminacy membership function and falsity membership function in the SVNSES $(\widehat{H}, A)$, respectively and w_1, w_2 are the weights for the amplitude terms (degrees of influence) and the phase terms (times of influence), respectively, where w_1 and $w_2 \in [0, 1]$ and $w_1 + w_2 = 1$.

3. Find the values of $Z_{\widehat{H}(\alpha)}(u) = \frac{T_{\widehat{H}(\alpha)}(u)+(1-I_{\widehat{H}(\alpha)}(u))+(1-F_{\widehat{H}(\alpha)}(u))}{3}$, $\forall u \in U$ and $\forall \alpha \in A$.

Note that $Z_{\widehat{H}(\alpha)}(u)$ is the normalized values of $S_{\widehat{H}(\alpha)}(u) = T_{\widehat{H}(\alpha)}(u) - I_{\widehat{H}(\alpha)}(u) - F_{\widehat{H}(\alpha)}(u)$, $\forall u \in U$ and $\forall \alpha \in A$. We normalize the elements of $S = \{S_{\widehat{H}(\alpha)}(u), \forall u \in U \text{ and } \forall \alpha \in A\}$, since it represents the degree of the influence, where S takes its minimum value at -2 when $(T_{\widehat{H}(\alpha)}(u), I_{\widehat{H}(\alpha)}(u), F_{\widehat{H}(\alpha)}(u)) = (0, 1, 1)$, while its maximum takes the value 1 at $(T_{\widehat{H}(\alpha)}(u), I_{\widehat{H}(\alpha)}(u), F_{\widehat{H}(\alpha)}(u)) = (1, 0, 0)$.

4. Find the highest numerical grade for the agree-SVNSES and disagree-SVNSES.

5. Compute the score of each element $u_i \in U$ by taking the sum of the numerical grade of each element for the agree-SVNSES and disagree-SVNSES, denoted by K_i and S_i, respectively.

6. Find the values of the score $r_i = K_i - S_i$ for each element $u_i \in U$.

7. Determine the value of the highest score $m = max_{u_i \in U}\{r_i\}$. Then the decision is to choose element u_i as the optimal solution to the problem. If there are more than one element with the highest r_i score, then any one of those elements can be chosen as the optimal solution.

It is to be noted that this method is used to deal with decision-making problems with known weight information (complete weight information). To execute the above steps, we assume that the weight vector for the amplitude terms is $w_1 = 0.7$ and the weight vector for the phase terms is $w_2 = 0.3$.

Now, to convert the CNSES (H, A) to the SVNSES $(\widehat{H}, A)$, obtain the weighted aggregation values of $T_{\widehat{H}(\alpha)}(u)$, $I_{\widehat{H}(\alpha)}(u)$ and $F_{\widehat{H}(\alpha)}(u), \forall\alpha \in A$ and $\forall u \in U$. To illustrate this step, we calculate $T_{\widehat{H}(\alpha)}(u)$, $I_{\widehat{H}(\alpha)}(u)$ and $F_{\widehat{H}(\alpha)}(u)$, when $\alpha = (e_1, p, 1)$ and $u = u_1$ as shown below:

$$T_{\widehat{H}(e_1,p,1)}(u_1) = w_1 p_{H(e_1,p,1)}(u_1) + w_2(1/2\pi)\mu_{H(e_1,p,1)}(u_1) = 0.7(0.9) + 0.3(1/2\pi)(2\pi)(11/12) = 0.9$$

$$I_{\widehat{H}(e_1,p,1)}(u_1) = w_1 q_{H(e_1,p,1)}(u_1) + w_2(1/2\pi)\nu_{H(e_1,p,1)}(u_1) = 0.7(0.2) + 0.3(1/2\pi)(2\pi)(1/12) = 0.165$$

$$F_{\widehat{H}(e_1,p,1)}(u_1) = w_1 r_{H(e_1,p,1)}(u_1) + w_2(1/2\pi)\omega_{H(e_1,p,1)}(u_1) = 0.7(0.1) + 0.3(1/2\pi)(2\pi)(0) = 0.07.$$

Then, for $\alpha = (e_1, p, 1)$ and $u = u_1$, the SVNSEV $(T_{\widehat{H}(\alpha)}(u), I_{\widehat{H}(\alpha)}(u), F_{\widehat{H}(\alpha)}(u)) = (0.9, 0.165, 0.07)$.

In the same manner, we calculate the other SVNSEVs, $\forall\alpha \in A$ and $\forall u \in U$ as in the Table 1, which gives the values of $Z_{\widehat{H}(\alpha)}(u), \forall\alpha \in A$ and $\forall u \in U$.

It is to be noted that the upper and lower terms for each element in Table 1 represent the SVNSEVs,

Table 1
Values of $(\widehat{H}, A)$ and $Z_{\widehat{H}(\alpha)}(u)$

U	u_1	u_2	u_3	u_4
$(e_1, p, 1)$	⟨0.905, 0.165, 0.07⟩ 0.89	⟨0.475, 0.52, 0.59⟩ 0.455	⟨0.355, 0.215, 0.695⟩ 0.482	⟨0.83, 0.475, 0.21⟩ 0.715
$(e_1, q, 1)$	⟨0.83, 0.12, 0.235⟩ 0.825	⟨0.64, 0.405, 0.83⟩ 0.468	⟨0.235, 0.78, 0.88⟩ 0.192	⟨0.76, 0.43, 0.26⟩ 0.69
$(e_2, p, 1)$	⟨0.76, 0.24, 0.235⟩ 0.762	⟨0.43, 0.375, 0.305⟩ 0.583	⟨0.36, 0.525, 0.83⟩ 0.335	⟨0.595, 0.335, 0.29⟩ 0.657
$(e_2, q, 1)$	⟨0.625, 0.165, 0.095⟩ 0.788	⟨0.64, 0.095, 0.73⟩ 0.605	⟨0.29, 0.52, 0.78⟩ 0.33	⟨0.525, 0.45, 0.24⟩ 0.612
$(e_3, p, 1)$	⟨0.83, 0.12, 0.33⟩ 0.793	⟨0.705, 0.22, 0.757⟩ 0.576	⟨0.305, 0.315, 0.68⟩ 0.437	⟨0.785, 0.235, 0.165⟩ 0.795
$(e_3, q, 1)$	⟨0.64, 0.45, 0.19⟩ 0.667	⟨0.455, 0.27, 0.43⟩ 0.585	⟨0.285, 0.55, 0.78⟩ 0.318	⟨0.455, 0.615, 0.64⟩ 0.4
$(e_1, p, 0)$	⟨0.095, 0.835, 0.93⟩ 0.11	⟨0.665, 0.48, 0.55⟩ 0.545	⟨0.645, 0.785, 0.305⟩ 0.545	⟨0.31, 0.525, 0.93⟩ 0.285
$(e_1, q, 0)$	⟨0.31, 0.88, 0.345⟩ 0.362	⟨0.78, 0.715, 0.59⟩ 0.492	⟨0.905, 0.22, 0.54⟩ 0.715	⟨0.31, 0.57, 0.81⟩ 0.31
$(e_2, p, 0)$	⟨0.31, 0.76, 0.835⟩ 0.238	⟨0.43, 0.625, 0.555⟩ 0.417	⟨0.78, 0.475, 0.59⟩ 0.572	⟨0.265, 0.665, 0.57⟩ 0.343
$(e_2, q, 0)$	⟨0.095, 0.835, 0.625⟩ 0.212	⟨0.78, 0.905, 0.69⟩ 0.395	⟨0.78, 0.48, 0.29⟩ 0.67	⟨0.265, 0.55, 0.55⟩ 0.389
$(e_3, p, 0)$	⟨0.38, 0.88, 0.46⟩ 0.347	⟨0.575, 0.78, 0.705⟩ 0.363	⟨0.905, 0.685, 0.67⟩ 0.517	⟨0.215, 0.765, 0.835⟩ 0.205
$(e_3, q, 0)$	⟨0.29, 0.55, 0.46⟩ 0.427	⟨0.405, 0.73, 0.43⟩ 0.415	⟨0.855, 0.45, 0.64⟩ 0.588	⟨0.615, 0.385, 0.43⟩ 0.6

Table 2
Numerical grade for agree-SVNSES

U	u_i	Highest numerical grade
$(e_1, p, 1)$	u_1	0.89
$(e_1, q, 1)$	u_1	0.825
$(e_2, p, 1)$	u_1	0.762
$(e_2, q, 1)$	u_1	0.788
$(e_3, p, 1)$	u_4	0.795
$(e_3, q, 1)$	u_1	0.667

Table 3
Numerical grade for disagree-SVNSES

U	u_i	Highest numerical grade
$(e_1, p, 0)$	u_2, u_3	0.545
$(e_1, q, 0)$	u_3	0.715
$(e_2, p, 0)$	u_3	0.572
$(e_2, q, 0)$	u_3	0.67
$(e_3, p, 0)$	u_3	0.517
$(e_3, q, 0)$	u_4	0.6

Table 4
The score $r_i = K_i - S_i$

K_i	S_i	r_i
Score $(u_1) = 3.932$	Score $(u_1) = 0$	3.932
Score $(u_2) = 0$	Score $(u_2) = 0.545$	−0.545
Score $(u_3) = 0$	Score $(u_3) = 3.019$	−3.019
Score $(u_4) = 0.795$	Score $(u_4) = 0.6$	0.195

$\forall \alpha \in A$ and $\forall u \in U$ and the values of $Z_{\widehat{H}(\alpha)}(u)$, $\forall \alpha \in A$ and $\forall u \in U$, respectively.

Tables 2 and 3 give the highest numerical grade for the elements in the agree-SVNSES and disagree-SVNSES, respectively.

Let K_i and S_i, represent the score of each numerical grade for the agree-SVNSES and disagree-SVNSES, respectively. These values are given in Table 4.

Thus, from Table 4 $max_{u_i \in U}\{r_i\} = r_1$, followed by r_4 and r_2, where $min_{u_i \in U}\{r_i\} = r_3$. Therefore, the plunge in commodity and oil prices is the most important factor that affects the Malaysian economy, followed by the exchange rate variability and China's economic slowdown, where the goods and services tax lags behind these factors.

6. Comparison and discussion

In this section, we will compare our proposed complex neutrosophic soft expert method (CNSEM) to the SVNSEM [50] which is a generalization of intuitionistic fuzzy soft expert method (IFSEM) [48], fuzzy soft expert method [47] and soft expert method [33].

Compared with SVNSEM which uses the SVNSES to depict the decision-making information, the proposed CNSEM introduces a new descriptor, that is, CNSES to present actual decision-making information. From Example 5.1, it can be seen that the SVNSES cannot represent the degree of the influence and the time of the influence simultaneously, since it is unable to represent variables in two dimensions. However, the structure of CNSES provides the ability to describe these two variables simultaneously by virtue of the phase terms and amplitude terms. Thus the SVNSEM cannot directly solve such a decision-making problem with complex neutrosophic soft expert information.

In contrast, the CNSEM can directly address the single-valued neutrosophic soft expert problem, since the SVNSES is a special case of CNSES and can be easily represented in the form of CNSES. In other words, the SVNSES is a CNSES with phase terms equal zeros. For example the SVNSEV (0.3, 0.5, 0.7) can be represented as $(0.3e^{j2\Pi(0)}, 0.5e^{j2\Pi(0)}, 0.7e^{j2\Pi(0)})$ by means of CNSES.

Furthermore, our method is applicable for intuitionistic fuzzy soft expert problem, since IFSES is a special case of SVNSES and consequently of CNSES. For example the intuitionistic fuzzy soft expert value (0.3, 0.5) can be (0.3, 0.2, 0.5) by means of SVNSES and hence can be $(0.3e^{j2\Pi(0)}, 0.2e^{j2\Pi(0)}, 0.5e^{j2\Pi(0)})$ by means of CNSES, since the sum of the degrees of membership, nonmembership and indeterminacy of an intuitionistic fuzzy value equal to 1. Note that the indeterminacy degree in intuitionistic fuzzy set is provided by default and cannot be defined alone unlike the neutrosophic set where the indeterminacy is defined independently and quantified explicitly.

Thus, the proposed method has certain advantages. Firstly, this method uses the CNSES to represent the decision information and as an extension of SVNSES and IFSES, CNSES includes evaluation information missing in the first two models, such as the time frame which is presented by the phase terms. Our method highlights the impact that the time frame has on the final decision. Secondly, a practical formula is employed to convert the CNSEVs to the SVNSEVs, which maintains the entirety of the original data without reducing or distorting them. Thirdly, our method gives a decision-making with a simple computational process without the need to carry out directed operations on complex numbers. Finally, the CNSES that is used in our method has the ability to handle the imprecise, indeterminate, inconsistent, and incomplete information that is captured by the amplitude terms and phase terms simultaneously. As a result, the proposed method is capable of dealing with deeper uncertain data.

7. Conclusion

We established the concept of CNSES by extending the theories of SVNS and soft expert set to complex numbers. The basic operations on CNSES, namely complement, subset, union, intersection, AND, and OR operations, were defined. Subsequently, the basic properties of these operations such as De Morgan's laws and other relevant laws pertaining to the concept of CNSES were proven. Finally, a generalized algorithm is introduced and applied to the CNSES model to solve a hypothetical decision-making problem and its superiority and feasibility are further verified by comparison with other existing methods. This new extension will provide a significant addition to existing theories for handling indeterminacy, where time plays a vital rule in the decision process, and spurs more developments of further research and pertinent applications. For further research, we intend to take into account unknown weight information to develop some real applications of CNSES in other areas, where the phase term may represent other variables such as distance, speed and temperature.

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