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Complex neutrosophic concept lattice and its applications to air quality analysis

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ABSTRACT

In the current year, the precise measurement of uncertainty and fluctuation exists in a complex fuzzy attributes is addressed as computationally and mathematically expensive tasks with regard to its graphical analytics. To deal with this problem the calculus of complex neutrosophic sets are recently introduced to characterize the uncertainty and its changes based on its truth, indeterminacy, and falsity membership–value, independently. This given a way to represent the given data sets in form of complex neutrosophic matrix for further analysis towards knowledge processing tasks. In this process, a major problem arises when an expert wants to find some of the interesting patterns in the given complex neutrosophic data sets to solve the particular problem. To resolve this issue, the current paper proposes a method for step by step demonstration to investigate the complex neutrosophic concepts and their graphical structure visualization based on their Lower Neighbors. One of the suitable examples of the proposed method is also given for precise measurement of uncertainty exists in Air Quality Index (AQI) and its pattern at given phase of time.

1. Introduction

Recently, the calculus of complex vague set concept lattice [25] and its properties [26–28,34,35] is introduced for measuring the changes in uncertainty using amplitude and phase term of a complex set. It has given a new mathematical way to characterize the uncertainty and vagueness in attributes more understandable manner when compared to approaches available in unipolar fuzzy space [11,16,23,24]. The reason is that the calculus of complex set [12,13] and concept lattice theory [15,24] provides a well-established mathematical framework to measure the human cognitive thought [14]. To measure the fluctuation in uncertainty exists in fuzzy attributes the calculus of complex fuzzy sets [12,13] become more helpful in its precise representation using amplitude and phase term in bipolar [22–25] or three–way space [26,27] for multi–decision process [28]. In this process, an important problem was addressed while handling the three–way fuzzy attributes [26–28] that how to measure their changes at given phase of time. To achieve this goal, properties of complex neutrosophic sets [3–5] and its graphs [10] are introduced for handling multi–decision attributes [9,25,28,33]. This extensive version of complex fuzzy set [1,2,29,30] and its properties in the neutrosophic or three–way polar space [31–33] given a new orientation to analyze the data sets based on applied abstract algebra [1,2,15,21,37–39]. Towards this extension recently, Singh [26–28] introduces properties of neutrosophic [26,27] and complex vague set [25] based concept lattice for precise approximation of computational linguistics exists in three–way decision space [47,48]. In this process a problem is addressed while handling the changes in three–way fuzzy attributes based on its truth, indeterminacy and falsity membership–values [3–5,10]. One of the suitable example is 22 °C temperature used to consider as a cool in summer seasons, warm in winter season whereas fair (or uncertain) in spring season. This interpretation of human cognitive thought used to exists in several real life examples from morning to evening while taking veg, non-veg or indeterminant spices. The precise representation of these types of attributes using a mathematical model is rigorous tasks for the research communities. This problem is dovetail which affect the human life directly in form of Air Quality Index (AQI)[1] or Bushfire.[2] All of these cases characterizing the uncertainty and fluctuations based on its acceptation, rejection and uncertain part is major concern. Second problem arises with their mathematical representation and graphical analytics for further analysis. Hence the current paper focuses on solving these issues of complex fuzzy attributes. The motivation is to provide a mathematical model for easier of understanding the information contained in complex neutrosophic data sets

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[1] https://en.wikipedia.org/wiki/Air_quality_index.
[2] https://en.wikipedia.org/wiki/Bushfires_in_Australia.

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0960-0779/

Table 1
Some necessary conditions for the uses of complex neutrosophic set.

	Complex fuzzy set	Complex vague set	Complex neutrosophic set
Domain	Universe of Discourse	Universe of Discourse	Universe of Discourse
Co–domain	Unipolar–value in unit circle [0, 1]	Bipolar–valued in unit in circle [0, 1]	Three–valued in unit circle $[0, 1]^3$
Truth membership	Yes in [0, 1]	Yes in $[0, 1]^2$	Yes in $[0, 1]^3$
False membership	No	Yes in [0, 1]	Yes in $[0, 1]^3$
Indeterminacy membership	No	1–True –false	Yes in $[0, 1]^3$
Amplitude term	Yes in [0, 1]	Yes in $[0, 1]^2$	Yes in $[0, 1]^3$
Phase term measurement	Yes $[0, 2\pi]$	Yes $[0, 2\pi]$	Yes in $[0, 2\pi]$
Uncertainty measurement	Yes in [0, 1]	Yes in $[0, 1]^2$	Yes in $[0, 1]^3$
Fluctuation measurement	Yes	Yes	Yes
Graph	Yes	Yes	Yes

based on its maximal acceptation, minimal rejection or uncertain regions, independently. To achieve this goal, the current paper focuses on depth analysis of complex neutrosophic context and its graphical structure visualization based on applied abstract algebra.

Recently, some of the researchers started analyzing the indeterminacy based on their partial ordering [17–20] graphical visualization [26–28] to approximate it more prominently via three–way decision space [40–42]. All of these approaches fail in precise measurement of periodic changes in three–way or neutrosophic fuzzy attributes. One of the suitable example is Air Quality Index (AQI) of any country changes at each interval of time. In this case, measuring the AQI based on its acceptation, rejection or uncertain regions is a computationally expensive tasks for the researchers. The reason is that the values of AQI used to change several times in a day due to change in level of $PM_{2.5}$, PM_{10}, NO_2 and other parameters. In this case precise representation of uncertainty and its changes based on its acceptation, rejection and uncertain regions at given phase of time is mathematically expensive tasks. To conquer this problem recently, some of the researchers tried to represent them using the calculus of complex neutrosophic sets [3–5] and its graph theory [10] for multi–decision process [9,18,28,36] at δ–granulation [43–48]. However, none of the available approaches described any ways to find some of the useful pattern exists in the complex neutrosophic contexts for knowledge processing tasks. Due to which, the current paper focuses on introducing a method for finding some of the interesting pattern in complex neutrosophic contexts based on applied lattice theory. The reason is it provides a more descriptive measurement of uncertainty and its changes in the complex fuzzy attributes based on its truth, indeterminacy and uncertain regions, independently when compared to other extensions of neutrosophic sets as shown in Table 1. To acquire this advantages the calculus of applied lattice theory [11,15,16,38,39] and its extensive properties [22–28] is utilized in this paper for generating the complex neutrosophic concepts and its hierarchical order visualization in the concept lattice using their Lower Neighbors. The reason to utilize the Lower Neighbor method is that it provides an easier way to investigate the concepts within limited time complexity when compared to other approaches [6–8]. In this way, the proposed method provides a basis of an algorithm for compressed graphical visualization of complex neutrosophic context in the concept lattice. The motivation is to provide a mathematical model to analyze the complex neutrosophic data sets more precisely when compared to its numerical representation. The objective is to extract some of the useful pattern in the given complex neutrosophic context for multi–decision process as shown in Fig. 1. It can be considered as one of the significant outputs of the proposed method in the field of complex data set analysis.

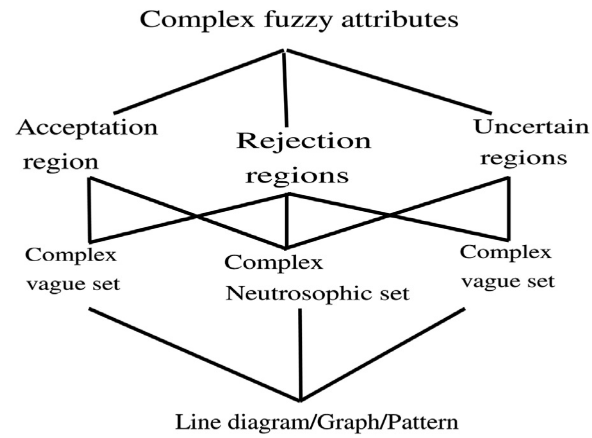

Fig. 1. The motivation for introducing the complex neutrosophic concept lattice.

Remaining part of the paper is organized as follows: Section 2 provides some basic preliminaries about complex neutrosophic sets. Section 3 provides a method for generating the complex neutrosophic concepts using their Lower Neighbor. Section 4 provides illustration of the proposed method with an example. Section 5 contains discussions followed by conclusions, and references.

2. Complex neutrosophic context and its graphical visualization

Recently, it seems that. handling complex neutrosophic data set like measuring the quality of AQI is mathematically rigorous tasks. To deal with these types of complex or seasonal data sets one solution is to represent them matrix format and try to visualize them in the graph. The current section contains some useful definitions to achieve this goal as given below:

Definition 1 (Complex fuzzy set [21,29,30])**.** A complex fuzzy set Z can be defined over a universe of discourse U having a single fuzzy membership–value at given phase of time. The complex–valued grade of membership of an element $z \in U$ can be characterized by $\mu_Z(z)$. The membership–values that $\mu_Z(z)$ may receive all values within the unit circle of a defined complex plane in the form $\mu_Z(z) = r_z(x)e^{iw_z(x)}$, where i=$\sqrt{-1}$, both $r_Z(z)$ and $w_Z(z)$ are real–valued and $r_Z(z) \in [0, 1]$. The complex fuzzy set Z may be represented as the set of ordered pairs:

$$Z = \{(z, \mu_Z(z)) : z \in U\} = \left\{(z, r_Z(z)e^{iw_Z(z)}) : z \in U\right\}$$

The union, intersection and other operator among complex fuzzy set can be studied in [1–3] with an illustrative example for better understanding.

Example 1. Let us suppose, an expert wants to measure the level of AQI index of the given geographical regions (i.e. object–x_1) based on its saturation value of PM_{10} (i.e. attribute y_1). The user collected the data and saw that the saturation value of PM_{10} changes 50 percent in six to seven months. This complex fuzzy attributes can be written using the properties of complex fuzzy set as follows: $0.5e^{i1.2\pi}$. In case the user want to represent the indeterminacy and falsity regions then properties of neutrosophic set can be useful.

Definition 2 (Neutrosophic set [32])**.** It provides a way to characterize the uncertainty and vagueness in attributes $y \in Y$ based on truth–membership function $T_N(y)$, a indeterminacy–membership function $I_N(y)$ and a falsity–membership function $F_N(y)$. The $T_N(y)$, $I_N(y)$ and $F_N(y)$ are real standard or non-standard subsets of $]0^-, 1^+[$as given below:

$T_N : Y \to]0^-, 1^+[$,
$I_N : Y \to]0^-, 1^+[$,
$F_N : Y \to]0^-, 1^+[$.

The neutrosophic set can be represented as follows:

$$N = \{(x, T_N(y), I_N(x), F_N(y)) : y \in Y\} \quad \text{where} \quad 0^- \le T_N(y) + I_N(y) + F_N(y) \le 3^+.$$

It is noted that $0^- = 0 - \epsilon$ where 0 is its standard part and ϵ is its non-standard part. Similarly, $1^+ = 1 + \epsilon$ $(3^+ = 3 + \epsilon)$ where 1 (or 3) is standard part and ϵ is its non-standard part. The real standard (0, 1) or [0, 1] can be also used to represent the neutrosophic set. The union and intersection among neutrosophic sets N_1 and N_2 can be computed as follows:

$\bullet N_1 \bigcup N_2 = \{(x, T_{N_1}(x) \vee T_{N_2}(x), I_{N_1}(x) \wedge I_{N_2}(x), F_{N_1}(x) \wedge F_{N_2}(x)) : x \in X\}$

The intersection of N_1 and N_2 can be defined as follows:

$\bullet N_1 \bigcap N_2 = \{(x, T_{N_1}(x) \wedge T_{N_2}(x), I_{N_1}(x) \vee I_{N_2}(x), F_{N_1}(x) \vee F_{N_2}(x)) : x \in X\}$.

Example 2. Example 1 represents the acceptation part of PM_{10} in the given year. In case, the expert wants to measure the acceptation, rejection or indeterminacy part exists in AQI then the properties of neutrosophic set can be useful. To illustrate the problem, let us consider an expert founds that the level of PM_{10} in the given area is 60 percent accepted, 20 rejected and 10 percent uncertain for the health of citizens. This neutrosophic value can be written as (0.6, 0.2, 0.1) where 0.6 represents the truth–membership value, 0.2 indeterminacy–membership value, and 0.1 falsity–membership value. Now suppose the user want to measure the changes on the acceptation, rejection and uncertain regions of neutrosophic value at the given year. In this case, the properties of complex neutrosophic set can be useful.

Definition 3 (Complex neutrosophic set [3–5])**.** A complex neutrosophic set Z can be defined over a universe of discourse U. The uncertainty in the attributes $z \in U$ can be characterized by true $^-0 < r_{T_Z} < 1^+$, indeterminacy $^-0 \le r_{I_Z} < 1^+$ and falsity membership-value $^-0 \le r_{F_Z} < 1^+$, independently with a given phase of time $(0, 2\pi)$. It can be observed that, the “amplitude” term in complex neutrosophic set satisfies the property $^-0 \le r_{T_Z} + r_{I_Z} + r_{F_Z} \le 3^+$ whereas the “phase” term can be characterized by $w^r_{T_Z}$, $w^r_{I_Z}$ and $w^r_{F_Z}$ in real–valued interval $[0, 2\pi]$. It can be represented as $Z = \left\{(z, (r_{T_Z}e^{w^r_{T_Z}}, r_{I_Z}e^{w^r_{I_Z}}, r_{F_Z}e^{w^r_{F_Z}})) : z \in U\right\}$.

Example 3. Let us extend the Example 1, that the expert agreed that quality of PM_{10} (i.e. y_1) is accepted 60 percent at the end of six to seven months, 20 percent rejected at the end of four to five months whereas the user is uncertain 10 percent at the end of nine to tenth month of a year. This complex query can be written using the complex neutrosophic set as given below:

$x_1 = (0.6e^{1.2\pi}, 0.2e^{0.7\pi}, 0.1e^{1.6\pi})/y_1$ where 2π is considered as phase term to represent the year.

Definition 4 (Complex neutrosophic graph [10,25])**.** A complex neutrosophic fuzzy graph $G=(V, \mu_c, \rho_c)$ is a non–empty set in which the value of vertices μ_c: $V \to (r^T_c(v).e^{iarg^T_c(v)}, r^I_c(v).e^{iarg^I_c(v)}, r^F_c(v).e^{iarg^F_c(v)})$ and edges ρ_c: $V \times V \to (r^T_c(v \times v).e^{iarg^T_c(v \times v)}, r^I_c(v \times v).e^{iarg^I_c(v \times v)}, r^F_c(v \times v).e^{iarg^F_c(v \times v)})$. It means the membership–values can be characterized by the truth, indeterminate and falsity membership–values within the unit circle [0, 1] at given period of time. It can be represented through amplitude and phase term of defined complex neutrosophic set as follows:

Table 2
A representation of PM_{10} and its fluctuation at the given areas using complex neutrosophic set.

Vertex	y_1
x_1	$(0.5e^{i0.7\pi}, 0.3e^{i1.2\pi}, 0.2e^{i1.8\pi})$
x_2	$(0.7e^{i0.2\pi}, 0.6e^{i1.6\pi}, 0.1e^{i0.4\pi})$
x_3	$(0.4e^{i0.4\pi}, 0.5e^{i0.8\pi}, 0.6e^{i2\pi})$
x_4	$(0.8e^{i0.3\pi}, 0.7e^{i1.7\pi}, 0.3e^{i0.7\pi})$

Table 3
A complex neutrosophic relation among the given areas using their PM_{10}.

Edges	y_1
(x_1, x_2)	$(0.5e^{i0.2\pi}, 0.3e^{i1.2\pi}, 0.1e^{i0.4\pi})$
(x_1, x_3)	$(0.4e^{i0.4\pi}, 0.3e^{i0.8\pi}, 0.2e^{i1.8\pi})$
(x_2, x_4)	$(0.7e^{i0.2\pi}, 0.6e^{i1.6\pi}, 0.1e^{i0.4\pi})$
(x_3, x_4)	$(0.4e^{i0.3\pi}, 0.5e^{i0.8\pi}, 0.3e^{i0.7\pi})$

$r^T_c(v_i \times v_j).e^{iarg^T_c(v_i \times v_j)} \le min\Big(r^T_c(v_i), r^T_c(v_j)\Big).e^{imin\left(arg^T_c(v_i), arg^T_c(v_j)\right)}$.

$r^I_c(v_i \times v_j).e^{iarg^I_c(v_i \times v_j)} \ge max\Big(r^I_c(v_i), r^I_c(v_j)\Big).e^{imax\left(arg^I_c(v_i), arg^I_c(v_j)\right)}$.

$r^F_c(v_i \times v_j).e^{iarg^F_c(v_i \times v_j)} \ge max\Big(r^F_c(v_i), r^F_c(v_j)\Big).e^{imax\left(arg^F_c(v_i), arg^F_c(v_j)\right)}$.

The given complex fuzzy graph is complete iff:

$r_c(v_i \times v_j).e^{iarg_c(v_i \times v_j)} = min(r_c(v_i), r_c(v_j)).e^{imin\left(arg_c(v_i), arg_c(v_j)\right)}$

for the truth, indeterminacy and falsity membership functions, independently.

Example 4. Let us suppose, the expert wants to analyze the four given areas x_1, x_2, x_3, x_4 based on the level of PM_{10} and its changes as shown in Table 2. The corresponding relationship among them is shown in Table 3. The obtained complex neutrosophic contexts shown in Tables 3 and 4 can be visualized in using the vertices V and edges E of a defined complex neutrosophic graph as shown in Fig. 2.

Definition 5 (Lattice structure of neutrosophic set [31,32])**.** Let N_1 and N_2 be neutrosophic sets in the universe of discourse X. Then $N_1 \subseteq N_2$ iff $T_{N_1}(x) \le T_{N_2}(x)$, $I_{N_1}(x) \ge I_{N_2}(x)$, $F_{N_1}(x) \ge F_{N_2}(x)$ for any $x \in X$. $(N, \wedge, \vee)$ is bounded lattice. Also the structure $(N, \wedge, \vee, (1, 0, 0), (0, 1, 1), \neg)$ follow the D–Morgan algebra. Similarly, this lattice structure can be used to represent the three–way fuzzy concept lattice and their concept using Gödel logic.

Definition 6 (Neutrosophic fuzzy concepts [26,27])**.** Let us suppose, a set of attribute i.e. $(B) = \{y_j, (T_B(y_j), I_B(y_j), F_N(y_j)) \in [0, 1]^3$: $\forall y_j \in Y\}$ where $j \le m$. For the selected three–polar attribute set find their covering objects set in the given fuzzy context i.e.

$$(A) = \{x_i, (T_A(x_i), I_A(x_i), F_A(x_i)) \in [0, 1]^3: \forall x_i \in X\} \text{ where } i \le n.$$

The obtain pair (A, B) is called as a neutrosophic fuzzy concept iff: $A^{\uparrow} = B$ and $B^{\downarrow} = A$. It can be interpreted as neutrosophic set of objects having maximal truth membership value, minimum indeterminacy and minimum falsity membership value with respect to integrating the information from the common set of neutrosophic attributes in the defined three–way fuzzy space $[0, 1]^3$ using the component–wise Gödel residuated lattice. After that, none of the neutrosophic set of objects (or attributes) can be found which can make the membership value of the obtained neutrosophic set of attributes (or objects) bigger. Then obtained pair of neutrosophic set (A, B) is called as a formal concepts, where A is called as extent, and B is called as intent. In this process, a problem arises when the truth, falsity and indeterminacy value of a neutrosophic attributes changes at each given phase of time. To overcome from this issue, a method is proposed in the next section for generating the complex neutrosophic concepts based on their Lower Neighbors [6] as it is considered as one of the easier and cost effective method.

Table 4
A proposed algorithm for building the complex neutrosophic concept lattice.

Input: A three–way complex fuzzy context $\mathbf{K}=(X, Y, \tilde{R})$
where $|X|=n$, $|Y|=m$.
Output: The set of three–way complex fuzzy concepts
1. Find the maximal covering attributes for the objects set (X) using ($\uparrow$):
(i) $\left(x_i, (r_{R_{x_i}} e^{w^r_{T_{x_i}}}, r_{I_{x_i}} e^{w^r_{I_{x_i}}}, r_{F_{x_i}} e^{w^r_{F_{x_i}}})\right)^{\uparrow} = \left(y_j, (r_{R_{y_j}} e^{w^r_{T_{y_j}}}, r_{I_{y_j}} e^{w^r_{I_{y_j}}}, r_{F_{y_j}} e^{w^r_{F_{y_j}}})\right)$.
(ii) Compute the neutrosophic membership–value for the obtained attributes:
min $(y_j, r_{T_{y_j}} e^{w^r_{T_{x_j}}})$ for true membership,
max $(y_j, r_{I_{y_j}} e^{w^r_{I_{y_j}}})$ for indeterminacy membership,
max $(y_j, r_{F_{y_j}} e^{w^r_{F_{y_j}}})$ for false membership,
(iii) Apply the operator ($\downarrow$) on the obtained attribute set:
$\left(y_j, (r_{R_{y_j}} e^{w^r_{T_{y_j}}}, r_{I_{y_j}} e^{w^r_{I_{y_j}}}, r_{F_{y_j}} e^{w^r_{F_{y_j}}})\right)^{\downarrow} = \left(x_i, (r_{R_{x_i}} e^{w^r_{T_{x_i}}}, r_{I_{x_i}} e^{w^r_{x_{x_i}}}, r_{F_{x_i}} e^{w^r_{F_{x_i}}})\right)$
(iv) This gives first complex neutrosophic concept (A, B).
2. Find its Lower Neighbor:
3. **for** ($k=0$ to m)
$Y_k = Y - y_j$ where $j, k \leq m$
(i). New attribute set: $y_k = \{y_j, y_k\}$
(ii). Set maximal acceptance for the complex neutrosophic attributes
i.e. Amplitude = (1.0, 0.0, 0.0) and Phase=(0, 2π)
(iii). Apply the operator ($\downarrow$) on the attributes
$\left(y_j, (r_{R_{y_j}} e^{w^r_{T_{y_j}}}, r_{I_{y_j}} e^{w^r_{I_{y_j}}}, r_{F_{y_j}} e^{w^r_{F_{y_j}}})\right)^{\downarrow} = \left(x_i, (r_{R_{x_i}} e^{w^r_{T_{x_i}}}, r_{I_{x_i}} e^{w^r_{x_{x_i}}}, r_{F_{x_i}} e^{w^r_{F_{x_i}}})\right)$
(iv). Compute the membership of the obtained objects using Step 1 (ii):
(v). Apply the operator ($\uparrow$) on the constituted set of objects:
$\left(x_i, (r_{R_{x_i}} e^{w^r_{T_{x_i}}}, r_{I_{x_i}} e^{w^r_{x_{x_i}}}, r_{F_{x_i}} e^{w^r_{F_{x_i}}})\right)^{\uparrow} = \left(y_j, (r_{R_{y_j}} e^{w^r_{T_{y_j}}}, r_{I_{y_j}} e^{w^r_{I_{y_j}}}, r_{F_{y_j}} e^{w^r_{F_{y_j}}})\right)$.
(vi). Compute the membership of obtained attributes as per Step 1 (ii).
End **for**.
4. Distinct Lower Neighbor is considered as Next Neighbor.
5. Similarly, generate all the Next Neighbor using uncovered attributes.
6. Build the complex neutrosophic concept lattice for knowledge extraction.

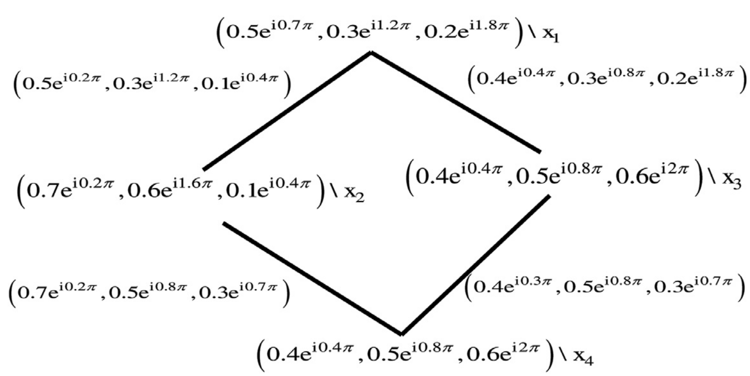

Fig. 2. A three–way complex neutrosophic graph visualization of Tables 2 and 3.

3. A proposed method for generating the complex neutrosophic concept

Generating the complex neutrosophic concepts is addressed as one of the major issues for precise analysis of complex data sets based on its acceptation, rejection, and uncertain regions. To deal with this problem recently subset based algorithms are introduced to handle the neutrosophic context [25–28]. This paper focuses on generating the complex neutrosophic concepts based on their Lower neighbor algorithm. One of the most suitable reason behind this method is that it provides an easier way to understand the concept generation when compared to other algorithms. The steps of the proposed method are as follows:

Step (1) The first complex neutrosophic concepts can be investigated by exploring all the objects set $\uparrow$ i.e.

$$\left(x_i, (r_{R_{x_i}} e^{w^r_{T_{x_i}}}, r_{I_{x_i}} e^{w^r_{I_{x_i}}}, r_{F_{x_i}} e^{w^r_{F_{x_i}}})\right)^{\uparrow} = \left(y_j, (r_{R_{y_j}} e^{w^r_{T_{y_j}}}, r_{I_{y_j}} e^{w^r_{I_{y_j}}}, r_{F_{y_j}} e^{w^r_{F_{y_j}}})\right).$$

The membership–value for the complex neutrosophic set of attributes can be computed as follows:

Amplitude:
min $(y_j, r_{T_{y_j}})$ for true membership,
max $(y_j, r_{I_{y_j}})$ for indeterminacy membership,
max $(y_j, r_{F_{y_j}})$ for false membership,

Phase term:

min $(y_j, e^{w^r_{T_{x_j}}})$ for true phase term,
max $(y_j, e^{w^r_{I_{y_j}}})$ for indeterminacy phase term.
max $(y_j, e^{w^r_{F_{y_j}}})$ for false phase term.

Step (2) The Lower Neighbor of the complex fuzzy concepts generated at Step (1) can be investigated using uncovered attributes i.e.: $y_k = Y - y_j$ where $j \leq m$ and $k \leq m|$.

Step (3) The obtained complex neutrosophic set of attributes set can be explored using the Galois connection ($\downarrow$) on Amplitude =

(1.0, 0.0, 0.0) and Phase=(0, 2π) term. The covering objects set can be found by (↓) as follows:

$$\left(y_j, (r_{R_{y_j}} e^{w^r_{T_{y_j}}}, r_{I_{y_j}} e^{w^r_{I_{y_j}}}, r_{F_{y_j}} e^{w^r_{F_{y_j}}})\right)^{\downarrow} = \left(x_i, (r_{R_{x_i}} e^{w^r_{T_{x_i}}}, r_{I_{x_i}} e^{w^r_{x_{x_i}}}, r_{F_{x_i}} e^{w^r_{F_{x_i}}})\right).$$

Compute the membership–values for the obtained objects:
Amplitude:

min $(x_i, r_{T_{x_i}})$ for true membership,
max$(x_i, r_{I_{x_i}})$ for Indeterminacy membership,
max$(x_i, r_{F_{x_i}})$ for false membership,

Phase term:

min $(x_i, e^{w^r_{T_{x_i}}})$ for true phase term,
max $(x_i, e^{w^r_{I_{x_i}}})$ for indeterminacy phase term.
max $(x_i, e^{w^r_{F_{x_i}}})$ for false phase term.

Step (4). Apply the up operator ↑ on the constituted objects set:

$$\left(x_i, (r_{R_{x_i}} e^{w^r_{T_{x_i}}}, r_{I_{x_i}} e^{w^r_{x_{x_i}}}, r_{F_{x_i}} e^{w^r_{F_{x_i}}})\right)^{\uparrow} = \left(y_j, (r_{R_{y_j}} e^{w^r_{T_{y_j}}}, r_{I_{y_j}} e^{w^r_{I_{y_j}}}, r_{F_{y_j}} e^{w^r_{F_{y_j}}})\right).$$

Compute the neutrosophic membership–value for the obtained attributes:
Amplitude:

min $(y_j, r_{T_{y_j}})$ for true membership,
max $(y_j, r_{I_{y_j}})$ for indeterminacy membership,
max $(y_j, r_{F_{y_j}})$ for false membership,

Phase term:

min $(y_j, e^{w^r_{T_{x_j}}})$ for true phase term,
max $(y_j, e^{w^r_{I_{y_j}}})$ for indeterminacy phase term,
max $(y_j, e^{w^r_{F_{y_j}}})$ for false phase term.

Step (5) The obtained pair of complex neutrosophic set of objects and attributes (A, B) represents the Lower Neighbor of given concept. The distinct Lower Neighbors having maximal acceptance of complex neutrosophic membership value while integrating the information among objects and attributes set can be considered as Next Neighbor.

Step (6) Similarly, all the complex neutrosophic concepts can be discovered using the uncovered attributes.

Step (7) The complex neutrosophic concepts lattice can be build using their Next Neighbor.

Step (8) Extract some of the meaningful information from the obtained lattice. The pseudo code for the proposed algorithm is shown in Table 4.

Complexity: Let us suppose, the number of objects and the number of attributes in the given three–way complex fuzzy context is n and m, respectively. To discover the Lower Neighbor of three–way complex fuzzy attributes takes O($n^3.m$) time complexity for the amplitude and phase term, respectively. The removal of similar Lower Neighbor takes at most O(n^3*m^3) time complexity for the amplitude and phase term, independently. This computation gives the proposed method takes O(|C|.$n^6.m^6$) where, C is Lower Neighbor. In this way the proposed method shown in Table 4 takes less computation when compared to any of the available approaches for processing the complex neutrosophic data sets.

Table 5
A truth complex membership value for the PM_{10}, $PM_{2.5}$ and NO_2.

	y_1	y_2	y_3
x_1	$0.5e^{i0.7\pi}$	$0.8e^{i1.7\pi}$	$0.4e^{i0.4\pi}$
x_2	$0.3e^{i0.5\pi}$	$0.4e^{i0.3\pi}$	$0.5e^{i0.4\pi}$
x_3	$0.4e^{i1.5\pi}$	$0.6e^{i1.6\pi}$	$0.3e^{i0.5\pi}$
x_4	$0.4e^{i0.2\pi}$	$0.2e^{i0.9\pi}$	$0.7e^{i1.2\pi}$

Table 6
An indeterminacy complex membership value for the PM_{10}, $PM_{2.5}$ and NO_2.

	y_1	y_2	y_3
x_1	$0.3e^{i1.6\pi}$	$0.7e^{i1.1\pi}$	$0.5e^{i0.2\pi}$
x_2	$0.5e^{i1.3\pi}$	$0.1e^{i0.8\pi}$	$0.4e^{i1.4\pi}$
x_3	$0.6e^{i1.9\pi}$	$0.6e^{i1.2\pi}$	$0.3e^{i1.4\pi}$
x_4	$0.7e^{i0.2\pi}$	$0.1e^{i0.5\pi}$	$0.2e^{i0.7\pi}$

4. Complex neutrosophic concept lattice in context of AQI measurement

Recently, Singh [25–28] has paid attention towards analysis of uncertainty in data beyond the unipolar [23] or bipolar fuzzy space [22]. In this process, a major problem was addressed when the uncertainty and vagueness in the attributes changes at each given phase of time. In this case, characterization of uncertainty based on its acceptation, rejection and uncertain regions is computationally expensive tasks. One of the most suitable example to understand this situation is Air Quality Index. It used to change each phase of time which effect the human life directly in India.[3] Hence it is a major problem for the researchers to measure the pattern of AQI to control or reduce its effect on human life via providing some guidelines. To deal with this type of data sets recently one of the researcher tried to measure the uncertainty and its fluctuations based on its acceptation, rejection and uncertain part [3–5]. This method gives a way to characterize the complex data set based on its truth, falsity and indeterminacy–membership–values, independently with their periodic phase of time in the graphs [10]. This paper put forward effort to extract some meaningful pattern in the complex neutrosophic data sets using the properties of applied lattice theory as shown previously [26,27]. To achieve this goal, the current paper introduces a method in Section 3 for discovery of complex neutrosophic concepts based on properties of Next Neighbor algorithm as shown in Table 4. To illustrate the proposed method one of the real–life examples for measuring the changes in AQI and its pattern is illustrated below:

Example 5. Let us suppose, an expert wants to analyze the Air Quality Index (AQI) of four geographical regions (x_1, x_2, x_3, x_4) based on periodic changes in several parameters like PM_{10}, $PM_{2.5}$, NO_2, Carbon monoxide (Co), Lead (Pb), Ozone (O_3), Sulphur dioxide(So_2), Ammonia (NH_3) etc.[4] To illustrate the proposed method first three parameters PM_{10} (y_1), $PM_{2.5}$ (y_2), NO_2 (y_3) is considered in this paper. The expert can write the changes in the level of these parameters at the given year based on acceptation, rejection and indeterminacy regions as shown in Tables 5–7, respectively. Table 8 represents the compact form of these contexts using the properties of complex neutrosophic sets. It can be called as three–way complex fuzzy context which is central notion of this paper. To understand the entries in Table 8 let us suppose: $\tilde{R}_{(x_1,y_1)}$ =($0.5e^{i0.7\pi}$, $0.3e^{i1.6\pi}$, $0.3e^{i1.4\pi}$). This entry shows that the saturation

[3] http://indianexpress.com/article/india/india-news-india/delhi-air-pollution-smog-health-effects-3739848/.
[4] https://www.dpcc.delhigovt.nic.in/indexdup.php.

Table 7
A falsity complex membership value for the PM_{10}, $PM_{2.5}$ and NO_2.

	y_1	y_2	y_3
x_1	$0.3e^{i1.4\pi}$	$0.2e^{i0.5\pi}$	$0.4e^{i0.7\pi}$
x_2	$0.4e^{i1.3\pi}$	$0.4e^{i1.7\pi}$	$0.3e^{i0.5\pi}$
x_3	$0.5e^{i0.2\pi}$	$0.8e^{i0.9\pi}$	$0.4e^{i1.5\pi}$
x_4	$0.2e^{i0.5\pi}$	$0.9e^{i1.9\pi}$	$0.4e^{i0.2\pi}$

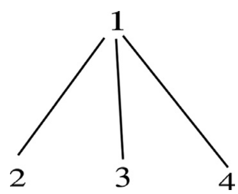

Fig. 3. The three–way complex neutrosophic line diagram build at Step 2 using the proposed algorithm.

values of PM_{10} is 50 percent acceptable in third to fourth months, 30 percent unacceptable in ninth to tenth months whereas it is 30 percent unpredictable in sixth to the seventh month of the given year. Similarly, other entries of three–way complex fuzzy matrix can be interpreted. Fig. 3 represents its graphical visualization using the proposed method.

Step 1. The proposed algorithm shown in Section 3 starts the investigation for three–way complex fuzzy concepts using those attributes which covers the objects set maximally. The attribute which covers objects set maximally i.e. $\{(1.0, 1.0)/x_1 + (1.0, 1.0)/x_2 + (1.0, 1.0)/x_3 + (1.0, 1.0)/x_4\}$ can be found using the operator ($\uparrow$) as shown below:

$$\{(1.0e^{i2\pi}, 0.0e^{i2\pi}, 0.0e^{i2\pi})/x_1 + (1.0e^{i2\pi}, 0.0e^{i2\pi}, 0.0e^{i2\pi})/x_2 + (1.0e^{i2\pi}, 0.0e^{i2\pi}, 0.0e^{i2\pi})/x_3 + (1.0e^{i2\pi}, 0.0e^{i2\pi}, 0.0e^{i2\pi})/x_4\}^{\uparrow} = \{(0.3e^{i0.4\pi}, 0.7e^{i1.9\pi}, 0.5e^{i1.3\pi})/y_1 + (0.2e^{i0.3\pi}, 0.7e^{i1.2\pi}, 0.7e^{i1.9\pi})/y_2 + (0.3e^{i0.4\pi}, 0.5e^{i1.4\pi}, 0.4e^{i1.5\pi})/y_3\}.$$

Now, apply the operator $\downarrow$ to find maximal vague set of objects while integrating the information from these constitutes attributes as given below:

$$\{(0.3e^{i0.4\pi}, 0.7e^{i1.9\pi}, 0.5e^{i1.3\pi})/y_1 + (0.2e^{i0.3\pi}, 0.7e^{i1.2\pi}, 0.7e^{i1.9\pi})/y_2 + (0.3e^{i0.4\pi}, 0.5e^{i1.4\pi}, 0.4e^{i1.5\pi})/y_3\}^{\downarrow} = \{(1.0e^{i2\pi}, 0.0e^{i2\pi}, 0.0e^{i2\pi})/x_1 + (1.0e^{i2\pi}, 0.0e^{i2\pi}, 0.0e^{i2\pi})/x_2 + (1.0e^{i2\pi}, 0.0e^{i2\pi}, 0.0e^{i2\pi})/x_3 + (1.0e^{i2\pi}, 0.0e^{i2\pi}, 0.0e^{i2\pi})/x_4\}.$$

It provides following three–way complex neutrosophic concepts: 1. Extent:

$$\{(1.0e^{i2\pi}, 0.0e^{i2\pi}, 0.0e^{i2\pi})/x_1 + (1.0e^{i2\pi}, 0.0e^{i2\pi}, 0.0e^{i2\pi})/x_2 + (1.0e^{i2\pi}, 0.0e^{i2\pi}, 0.0e^{i2\pi})/x_3 + (1.0e^{i2\pi}, 0.0e^{i2\pi}, 0.0e^{i2\pi})/x_4\}$$

Intent:

$$\{(0.3e^{i0.4\pi}, 0.7e^{i1.9\pi}, 0.5e^{i1.3\pi})/y_1 + (0.2e^{i0.3\pi}, 0.7e^{i1.2\pi}, 0.7e^{i1.9\pi})/y_2 + (0.3e^{i0.4\pi}, 0.5e^{i1.4\pi}, 0.4e^{i1.5\pi})/y_3\}$$

Step 2. The Lower Neighbors of concepts shown in Step 1 can be found as follows:

(i) $\{(0.3e^{i0.4\pi}, 0.7e^{i1.9\pi}, 0.5e^{i1.3\pi})/y_1 + (0.2e^{i0.3\pi}, 0.7e^{i1.2\pi}, 0.7e^{i1.9\pi})/y_2 + (0.3e^{i0.4\pi}, 0.5e^{i1.4\pi}, 0.4e^{i1.5\pi})/y_3\} \cup \left\{(1.0e^{i2\pi}, 0.0e^{i2\pi}, 0.0e^{i2\pi})/y_1\right\}$.

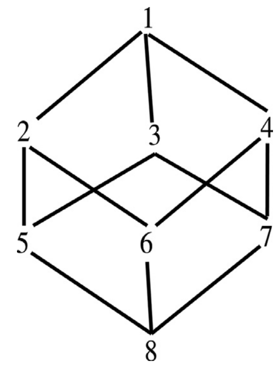

Fig. 4. A three–way complex neutrosophic concept lattice generated from Table 8.

It provides $\{(1.0e^{i2\pi}, 0.0e^{i2\pi}, 0.0e^{i2\pi})/y_1 + (0.2e^{i0.3\pi}, 0.7e^{i1.2\pi}, 0.7e^{i1.9\pi})/y_2 + (0.3e^{i0.4\pi}, 0.5e^{i1.4\pi}, 0.4e^{i1.5\pi})/y_3\}$.

(ii) $\{(0.3e^{i0.4\pi}, 0.7e^{i1.9\pi}, 0.5e^{i1.3\pi})/y_1 + (0.2e^{i0.3\pi}, 0.7e^{i1.2\pi}, 0.7e^{i1.9\pi})/y_2 + (0.3e^{i0.4\pi}, 0.5e^{i1.4\pi}, 0.4e^{i1.5\pi})/y_3\} \cup \left\{(1.0e^{i2\pi}, 0.0e^{i2\pi}, 0.0e^{i2\pi})/y_2\right\}$.

It provides: $\{(0.3e^{i0.4\pi}, 0.7e^{i1.9\pi}, 0.5e^{i1.3\pi})/y_1 + (1.0e^{i2\pi}, 0.0e^{i2\pi}, 0.0e^{i2\pi})/y_2 + (0.3e^{i0.4\pi}, 0.5e^{i1.4\pi}, 0.4e^{i1.5\pi})/y_3\}$.

(iii) $\{(0.3e^{i0.4\pi}, 0.7e^{i1.9\pi}, 0.5e^{i1.3\pi})/y_1 + (0.2e^{i0.3\pi}, 0.7e^{i1.2\pi}, 0.7e^{i1.9\pi})/y_2 + (0.3e^{i0.4\pi}, 0.5e^{i1.4\pi}, 0.4e^{i1.5\pi})/y_3\} \cup \left\{(1.0e^{i2\pi}, 0.0e^{i2\pi}, 0.0e^{i2\pi})/y_3\right\}$.

It provides: $\{(0.3e^{i0.4\pi}, 0.7e^{i1.9\pi}, 0.5e^{i1.3\pi})/y_1 + (0.2e^{i0.3\pi}, 0.7e^{i1.2\pi}, 0.7e^{i1.9\pi})/y_2 + (1.0e^{i2\pi}, 0.0e^{i2\pi}, 0.0e^{i2\pi})/y_3\}$

Now following Lower Neighbor can be generated from above obtained complex neutrosophic attribute using the Galois connection (as illustrated in Step 1):

2. Extent:
$\{(0.5e^{i0.7\pi}, 0.3e^{i1.6\pi}, 0.3e^{i1.4\pi})/x_1 + (0.3e^{i0.5\pi}, 0.5e^{i0.4\pi}, 0.4e^{i1.3\pi})/x_2 + (0.4e^{i1.5\pi}, 0.6e^{i1.9\pi}, 0.5e^{i0.2\pi})/x_3 + (0.4e^{i0.2\pi}, 0.7e^{i0.2\pi}, 0.2e^{i0.5\pi})/x_4\}$

Intent:

$\{(1.0e^{i2\pi}, 0.0e^{i2\pi}, 0.0e^{i2\pi})/y_1 + (0.2e^{i0.3\pi}, 0.7e^{i1.2\pi}, 0.9e^{i1.4\pi})/y_2 + (0.3e^{i0.4\pi}, 0.5e^{i1.4\pi}, 0.4e^{i1.5\pi})/y_3\}$

3. Extent: $\{(0.8e^{i1.7\pi}, 0.7e^{i1.1\pi}, 0.2e^{i0.5\pi})/x_1 + (0.4e^{i0.3\pi}, 0.1e^{i0.8\pi}, 0.4e^{i1.7\pi})/x_2 + (0.6e^{i1.6\pi}, 0.6e^{i1.2\pi}, 0.8e^{i0.9\pi})/x_3 + (0.2e^{i0.9\pi}, 0.1e^{i0.5\pi}, 0.9e^{i1.9\pi})/x_4\}$

Intent: $\{(0.3e^{i0.4\pi}, 0.7e^{i1.9\pi}, 0.5e^{i1.3\pi})/y_1 + (1.0e^{i2\pi}, 0.0e^{i2\pi}, 0.0e^{i1.9\pi})/y_2 + (0.3e^{i0.4\pi}, 0.5e^{i1.4\pi}, 0.4e^{i1.5\pi})/y_3\}$

4. Extent: $\{(0.4e^{i0.4\pi}, 0.5e^{i0.2\pi}, 0.4e^{i0.7\pi})/x_1 + (0.5e^{i0.4\pi}, 0.4e^{i1.4\pi}, 0.3e^{i0.5\pi})/x_2 + (0.3e^{i0.5\pi}, 0.3e^{i1.4\pi}, 0.4e^{i1.5\pi})/x_3 + (0.7e^{i1.2\pi}, 0.2e^{i0.7\pi}, 0.4e^{i0.2\pi})/x_4\}$

Intent: $\{(0.3e^{i0.4\pi}, 0.7e^{i1.9\pi}, 0.5e^{i1.3\pi})/y_1 + (0.2e^{i0.3\pi}, 0.7e^{i1.2\pi}, 0.7e^{i1.9\pi})/y_2 + (1.0e^{i2\pi}, 0.0e^{i2\pi}, 0.0e^{i2\pi})/y_3\}$

It can be observed that each of the obtained Lower Neighbors are distinct. In this case, each of them can be considered as Next Neighbor as shown in Fig. 4.

Step 3 Similarly, following concepts can be generated using the Next Neighbor of concept generated at Step 2:

5. Extent: $\{(0.5e^{i0.7\pi}, 0.7e^{i1.6\pi}, 0.3e^{i1.4\pi})/x_1 + (0.3e^{i0.3\pi}, 0.5e^{i0.8\pi}, 0.4e^{i1.7\pi})/x_2 + (0.4e^{i1.5\pi}, 0.6e^{i1.9\pi}, 0.8e^{i0.9\pi})/x_3 + (0.2e^{i0.2\pi}, 0.7e^{i0.5\pi}, 0.9e^{i1.4\pi})/x_4\}$

Intent: $\{(1.0e^{i2\pi}, 0.0e^{i2\pi}, 0.0e^{i2\pi})/y_1 + (1.0e^{i2\pi}, 0.0e^{i2\pi}, 0.0e^{i2\pi})/y_2 + (0.3e^{i0.4\pi}, 0.5e^{i1.4\pi}, 0.4e^{i1.5\pi})/y_3\}$

6. Extent: $\{(0.4e^{i0.4\pi}, 0.5e^{i1.6\pi}, 0.4e^{i1.4\pi})/x_1 + (0.3e^{i0.4\pi}, 0.5e^{i1.4\pi}, 0.4e^{i1.3\pi})/x_2 + (0.3e^{i0.5\pi}, 0.6e^{i1.9\pi}, 0.8e^{i1.5\pi})/x_3 + (0.4e^{i0.2\pi}, 0.7e^{i0.7\pi}, 0.9e^{i1.9\pi})/x_4\}$

Table 8
A complex neutrosophic context representation of Tables 5–7.

	y_1	y_2	y_3
x_1	$(0.5e^{i0.7\pi}, 0.3e^{i1.6\pi}, 0.3e^{i1.4\pi})$	$(0.8e^{i1.7\pi}, 0.7e^{i1.1\pi}, 0.2e^{i0.5\pi})$	$(0.4e^{i0.4\pi}, 0.5^{i0.2\pi}, 0.4e^{i0.7\pi})$
x_2	$(0.3e^{i0.5\pi}, 0.5e^{i0.4\pi}, 0.4e^{i1.3\pi})$	$(0.4e^{i0.3\pi}, 0.1e^{i0.8\pi}, 0.4e^{i1.7\pi})$	$(0.5e^{i0.4\pi}, 0.4^{i1.4\pi}, 0.3e^{i0.5\pi})$
x_3	$(0.4e^{i1.5\pi}, 0.6e^{i1.9\pi}, 0.5e^{i0.2\pi})$	$(0.6e^{i1.6\pi}, 0.6e^{i1.2\pi}, 0.8e^{i0.9\pi})$	$(0.3e^{i0.5\pi}, 0.3^{i1.4\pi}, 0.4e^{i1.5\pi})$
x_4	$(0.4e^{i0.2\pi}, 0.7e^{i0.2\pi}, 0.2e^{i0.5\pi})$	$(0.2e^{i0.9\pi}, 0.1e^{i0.5\pi}, 0.9e^{i1.9\pi})$	$(0.7e^{i1.2\pi}, 0.2^{i0.7\pi}, 0.4e^{i0.2\pi})$

Table 9
Significant distinction of the proposed method when compared to other approaches.

	Complex fuzzy set [29,30]	Complex vague set [25] [24–35]	Complex neutrosophic set [2,10]	Proposed method
Domain	Universe of Discourse	Universe of Discourse	Universe of Discourse	Universe of Discourse
Co-domain	Three–polar Single–valued	Three–polar Interval–valued	Unit circle	Three–polar circle
True–region	[0, 1]	[0, 1]	[0, 1]	[0, 1]
False–region	[0, 1]	[0, 1]	[−1, 0)	[0, 1]
Uncertain regions	[0, 1]	[0, 1]	1-true -false	[0, 1]
Amplitude term	Yes	Yes	Yes	**Yes**
Phase term	No	No	Yes	**Yes**
Pattern	Yes	Yes	Yes	**Yes**
Graph	Yes	Yes	No	**Yes**
Lattice	Yes	Yes	No	**Yes**
Methodology	Subset	Subset	δ–equal	**Lower Neighbor**
Time complexity	$O(2^m.n)$	$O(2^m.n)$	$O(2^m.n^2)$	$O(\|C\|.n^6.m^6)$

Intent: $\{(1.0e^{i2\pi}, 0.0e^{i2\pi}, 0.0e^{i2\pi})/y_1 + (0.2e^{i0.3\pi}, 0.7e^{i1.2\pi}, 0.7e^{i1.9\pi})/y_2 + (1.0e^{i2\pi}, 0.0e^{i2\pi}, 0.0e^{i2\pi})/y_3\}$

7. Extent: $\{(0.3e^{i0.4\pi}, 0.7e^{i1.1\pi}, 0.5e^{i0.7\pi})/x_1 + (0.4e^{i0.3\pi}, 0.5e^{i1.4\pi}, 0.4e^{i1.7\pi})/x_2 + (0.3e^{i0.5\pi}, 0.6e^{i1.2\pi}, 0.8e^{i1.5\pi})/x_3 + (0.2e^{i0.9\pi}, 0.2e^{i0.7\pi}, 0.9e^{i1.4\pi})/x_4\}$

Intent: $\{(0.3e^{i0.4\pi}, 0.7e^{i1.9\pi}, 0.5e^{i1.3\pi})/y_1 + (1.0e^{i2\pi}, 0.0e^{i2\pi}, 0.0e^{i2\pi})/y_2 + (1.0e^{i2\pi}, 0.0e^{i2\pi}, 0.0e^{i2\pi})/y_3\}$

8. Extent: $\{(0.4e^{i0.4\pi}, 0.7e^{i1.6\pi}, 0.4e^{i1.4\pi})/x_1 + (0.3e^{i0.3\pi}, 0.5e^{i1.4\pi}, 0.4e^{i1.7\pi})/x_2 + (0.3e^{i0.5\pi}, 0.6e^{i1.9\pi}, 0.8e^{i1.5\pi})/x_3 + (0.2e^{i0.2\pi}, 0.7e^{i0.7\pi}, 0.9e^{i1.9\pi})/x_4\}$

Intent: $\{(1.0e^{i2\pi}, 0.0e^{i2\pi}, 0.0e^{i2\pi})/y_1 + (1.0e^{i2\pi}, 0.0e^{i2\pi}, 0.0e^{i2\pi})/y_2 + (1.0e^{i2\pi}, 0.0e^{i2\pi}, 0.0e^{i2\pi})/y_3\}$

It can be observed that the above generated three–way complex fuzzy concepts and their compact visualization is shown in Fig. 4 using the properties of complex neutrosophic graph. This graph shows that concept 1 is most generalized concept whereas concept number 8 is most specialized concepts. The concept number 1 represents that each of the chosen regions has 20–30 percent acceptable saturation value for the PM_{10} in month of second to four months, 50–70 percent un–acceptable level in month of seventh to eighth whereas 40–70 percent uncertain from ninth to eleven months. In this case, the expert can refer to authorized government body for extra preparation in those months to reduce their health effects on the citizen. In a more precise way the expert may interpret the concept numbers 8. It represents that, the region x_1 has 40 percent acceptance level of each parameter in the month of second, 70 percent un–acceptable level in the month of ninth whereas 40 percent unpredictable in the month of seventh. The region x_2 has 30 percent acceptance level of each parameter in the month of first to second, 50 percent un–acceptable level in the month of eighth to ninth whereas 40 percent un–predictable in the month of ninth to tenth. The region x_3 has 30 percent acceptance level of each parameter in the month of second to third, 60 percent un–acceptable level in the month of tenth to eleven, whereas 80 percent un–predictable in the month of ninth to tenth. The region x_4 has 20 percent acceptance level of each parameter in the month of first, 70 percent un–acceptable level in the month of third to fourth whereas 90 percent un–predictable in the month of tenth to eleven. It can be observed that, these extracted patterns are more helpful in controlling or measuring the effect of AQI on the health of citizens in those areas. This will help in reducing the level of AQI and its fluctuation in those particular months to the certain levels using following methods:

1. Controlling emission from coal based power station,
2. Controlling hospitals waste,
3. Controlling diesel vehicles,
4. Controlling road or building construction dust,
5. Controlling the old and private vehicles etc.

It is one of the major and significant advantages of the proposed method towards measuring the pattern of AQI and its reduction which will help to the society.

Table 9 shows that, the proposed method has several advantages while dealing with complex neutrosophic context when compared to recently introduced methods. One of the most significant output of the proposed method provides a compressed line diagram and graphical analytics of the given complex neutrosophic context $O(|C|.n^6.m^6)$ time complexity. However, the proposed method unable to provide a mechanism to incorporate the opinion of all experts in one model to refine the pattern at user required information granules. To overcome from this problem the author will focus on introducing connection of granular computing [46–48] to refine the multi–valued neutrosophic [9,33] contexts for multi–decision process [19,20,28] at user required complex granulation.

5. Conclusions and future research

This paper aimed at measuring changes in complex fuzzy attributes and its pattern based on truth, false and indeterminacy membership–values at given phase of time using the properties of complex neutrosophic concept lattice. To achieve this goal, a method is proposed in this paper for graphical analytics of complex neutrosophic data set using the calculus of Lower Neighbors algorithm which takes $O(|C|.n^6.m^6)$ time complexity. One of the suitable applications of the proposed method is also demonstrated for precise analysis of AQI and its interested pattern in the given year. However, the proposed method unable to provide an adequate analysis based on user required complex granules exists beyond the bipolar space. To deal with this problem author will focus on refined neutrosophic sets [9,33] or complex multi–fuzzy sets [28,47] at different granulation for further applications.

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