User's manual galaxia

In AMUSE there are a set of analytic potentials that serve to model our galaxy, the Milky Way. To acquire these potentials from the AMUSE framework, it's necessary to write in a Python script the following line of code:

```
from amuse.community.galaxia.interface import BarAndSpirals3D
```

The module BarAndSpirals3D comprises the following potentials:

- Axisymmetric potential. This includes a:
  - bulge
  - disk
  - dark matter halo
- Potential of the central bar
- Potential of the spiral arms

Once we have imported the module BarAndSpirals3D, we need to create an instance of it. This can be done by writing the following line:

```
instance= BarAndSpirals3D()
```

When calling BarAndSpirals3D, the Galactic potential by default is the axisymmetric. A central bar and/or spiral arms can be added. Therefore, the types of Galactic models that can be constructed are:

- A pure axisymmetric Galaxy model (default model in BarAndSpirals3D)
- Axisymmetric + bar model
- Axisymmetric + spiral arms model
- Axisymmetric + bar + spiral arms model

In the following sections we will explain each potential in more detail.

1 Axisymmetric potential

This is the Galactic potential called by default in the module BarAndSpirals3D. The Axisymmetric potential is composed by a bulge, disk and a dark matter halo. Each of these components are explained in more detail below.

1.1 bulge

In the module BarAndSpirals3D, the bulge is modelled with a Plummer potential (Plummer 1911, *MNRAS*, 71, 460) which has the following form:

$$\Phi_{\rm bulge}(r) = -\frac{GM_{\rm b}}{\sqrt{r^2+b_1^2}} \,. \tag{1}$$

Here G corresponds to the gravitational constant, $M_{\rm b}$ is the mass of the bulge and b_1 is its corresponding scale length. The quantity $r = \sqrt{x^2+y^2+z^2}$ is the distance of a star to the Galactic center. The units of the model are such that $G = 1$.

The bulge parameters are set in BarAndSpirals3D through the following lines of code:

```
instance.parameters.mass_bulge
instance.parameters.b_bulge
```

The default values of the mass and scale length of the bulge are 1.4×10^{10} | units.MSun and 0.38 | units.kpc respectively (Allen & Santillan, 1991, Rev. Mex. Astron. Astrofis., 22, 255).

1.2 Disk

In BarAndSpirals3D the disk is modelled by using a Miyamoto-Nagai potential (Miyamoto & Nagai, 1975, PASJ, 27, 533), which is described by:

$$\Phi_{\rm disk}(R,z) = -\frac{GM_{\rm d}}{\sqrt{R^2+\left(a_2+\sqrt{z^2+b_2^2}\right)^2}} \,. \tag{2}$$

Here $M_{\rm d}$ corresponds to the mass of the disk. The parameters a_2 and b_2 are constants that modulate its shape. In particular, when $a_2 = 0$, Eq. **??** represents a spherical distribution of mass. In the case where $b_2 = 0$, Eq. **??** corresponds to the potential of a completely flattened disk. R and z represent the position of a star in a polar coordinate system.

The parameters of the disk are enabled by typing:

```
instance.parameters.mass_disk
instance.parameters.a_disk
instance.parameters.b_disk
```

The default values are: $M_{\rm d} = 8.5 \times 10^{10}$ | units.MSun; $a_2 = 5.31$ | units.kpc; $b_2 = 0.25$ | units.kpc (Allen & Santillan, 1991, Rev. Mex. Astron. Astrofis., 22, 255).

1.3 Dark matter halo

The dark matter halo is modelled with a logarithmic potential of the form:

$$\begin{aligned}\Phi_{\rm halo}(r) = &-\frac{GM(r)}{r} \\ &-\frac{GM_{\rm h}}{1.02a_3}\left[-\frac{1.02}{1+\Re^{1.02}} + \ln\left(1+\Re^{1.02}\right)\right]_r^{100} \,,\end{aligned} \tag{3}$$

where

$$M(r) = \frac{M_h \mathfrak{R}^{2.02}}{1+\mathfrak{R}^{1.02}} \quad \text{and}$$
$$\mathfrak{R} = \frac{r}{a_3}\,.$$

In the expression of above, M_h represents the mass of the halo and a_3 its scale length.

The parameters of the halo are set in the following way:

```
instance.parameters.mass_halo
instance.parameters.a_halo
```

The default values are: $M_h = 1.07 \times 10^{11}$ | units.MSun and $a_3 = 12$ | units.kpc (Allen & Santillan, 1991, Rev. Mex. Astron. Astrofis., 22, 255).

2 Potential of the central bar

BarAndSpirals3D uses the Ferrers potential (Ferrers, 1877, Pure Appl. Math.,14,1) to model the bar of the Milky Way. This potential is represented by the following density distribution:

$$\rho_{\text{bar}} = \begin{cases} \rho_0 \left(1-n^2\right)^k & n < 1 \\ 0 & n \geq 1 \end{cases} . \tag{4}$$

Here n^2 determines the shape of the bar potential, which comes from an ellipsoidal distribution of mass. ρ_0 represents the central density of the bar and k its concentration. Following the literature (Romero-Gomez et al. 2007, A&A, 472, 63; Romero-Gomez et al. 2011, MNRAS, 418, 1176 and references therein), $k = 1$.

The command to add the bar potential into the axisymmetric Galactic model is:

```
instance.parameters.bar_contribution = True (default: False)
```

The parameters to customize the bar potential are the following:

```
instance.parameters.mass_bar    # mass
instance.parameters.bar_phase    # initial orientation
instance.parameters.omega_bar    # pattern speed
instance.parameters.aaxis_bar    # semi-major axis
instance.parameters.axis_ratio_bar    # ratio between semi-minor and semi-major axis (b/a)
instance.parameters.caxis_bar    # vertical axis. Default: caxis_bar=0 which corresponds to a two-dimensional bar
```

The bar is build by subtracting mass from the bulge. For instance, if mass_bulge= 1.4×10^{10} | units.MSun and mass_bar= 1.4×10^{10} |units.MSun, the final mass of the bulge will be zero (after committing parameters). Therefore the mass of the bar can not be higher than the mass of the bulge.

In some simulations, it is desirable to have a bar growing adiabatically in time. To set this option in the module BarAndSpirals3D, the following line is necessary:

```
instance.parameters.nbt
```

the dimensionless parameter nbt is the number of revolutions needed to obtain a fully-grown bar. The default value is 0, which means that the bar is created instantaneously in the Galaxy model when the bar potential is enabled.

To obtain the growing time of the bar in Myr given the number of revolutions, just type:

```
print instance.parameters.tgrowth_bar.value_in(units.Myr)
```

If nbt = 0, tgrowth_bar = 0 .Myr.

To set the parameters of the bar that best fit the Milky Way, we refer the reader to:
Martínez-Barbosa et al., 2015, MNRAS, 446, 823 and
Martínez-Barbosa et al., 2016, MNRAS, 457, 1062.

3 Spiral arms

To add the potential of the spiral arms into the axisymmetric Galactic model, you need to type the following command:

```
instance.parameters.spiral_contribution= True (default:False)
```

In BarAndSpirals3D, there are three different models for the spirals arms:

- Tight winding approximation (TWA): This model treats the spiral arms as periodic perturbations of the Galactic disk. This model is two-dimensional.
- Cox & Gomez potential (C&G): This model also treats the spiral arms as periodic perturbations of the Galactic disk, but in the three-dimensional space.
- Composite model of Lepine: This model assumes the existence of multiple spiral arms with different amplitudes and pattern speeds. In BarAndSpirals3D, this model is two-dimensional.

Each of these potentials can be enabled in BarAndSpirals3D by typing:

```
instance.parameters.spiral_model= 0 # enables TWA potential
instance.parameters.spiral_model= 1 # enables C&G potential
instance.parameters.spiral_model= 2 # enables composite model potential
```

The parameters of each of these potentials are explained below.

3.1 TWA potential

The TWA potential models the spiral arms as periodic perturbations of a disk that rotates differentially. The potential of such perturbations in the plane is given by (Contopoulos & Grosbol, 1986, A&A, 155, 11):

$$\phi_{\rm sp}(R,\phi) = -A_{\rm sp} R e^{-R/R_\Sigma} \cos\left(m(\phi) - g(R)\right). \tag{5}$$

Here R and ϕ are the cylindrical coordinates of a star. $A_{\rm sp}$ is the amplitude of the spiral arms, R_Σ their scale length and m the number of arms. The function $g(R)$ defines the locus shape of the spiral arms, which is defined by the following expression (Antoja et al., 2011, MNRAS, 418, 1423):

$$g(R) = \left(\frac{m}{N\tan i}\right)\ln\left(1+\left(\frac{R}{R_{\rm sp}}\right)^N\right). \tag{6}$$

N is a parameter which measures how sharply the change from a bar to a spiral structure occurs in the inner regions of the Milky Way. $R_{\rm sp}$ is the separation distance at the beginning of the spiral shape locus and $\tan i$ is the tangent of the pitch angle.

The parameters to customize the TWA potential are the following:

```
instance.parameters.amplitude  # amplitude
instance.parameters.spiral_phase  # initial orientation
instance.parameters.omega_spiral  # pattern speed
instance.parameters.rsp  # separation distance at the beginning of the spiral locus
instance.parameters.m  # number of arms
instance.parameters.tan_pitch_angle  # tangent of the pitch angle
instance.parameters.N  # change from a bar to a spiral structure in the Galactic center
instance.parameters.r_sigma  # scale length
```

The values of these parameters that best fit the Milky Way can be found in Martínez-Barbosa et al., 2015, MNRAS, 446, 823.

3.2 C&G potential

In this model, the potential of the spiral arms is described by the following expression (Cox & Gómez, 2002, ApJS, 142, 261):

$$\Phi_{\rm sp}(r) = -4\pi G H A_{\rm sp}\exp\left(-\frac{r}{R_\Sigma}\right)\sum_n\left(\frac{C_n}{K_n D_n}\right)\cos(n\gamma)\left[\operatorname{sech}\left(\frac{K_n z}{\beta_n}\right)\right]^{\beta_n}. \tag{7}$$

Here H is the scale height, $A_{\rm sp}$ is the amplitude of the spiral arms and R_Σ is the scale length. In BarAndSpirals3D, $n = 1$ and $C_1 = 8/3\pi$. As a consequence, the parameters K_1, D_1 and β_1 are given by:

$$\begin{aligned} K_1 &= \frac{m}{r\sin i}, \\ \beta_1 &= K_1 H(1 + 0.4K_1 H), \\ D_1 &= \frac{1 + K_1 H + 0.3(K_1 H)^2}{1 + 0.3K_1 H}. \end{aligned}$$

Here m is the number of arms of the spiral structure and i correspond to the pitch angle.

The term γ in Eq. **??** represents the shape of the spiral structure, which is described by the expression:

$$\gamma = m\left[\varphi - \frac{\ln(r/r_0)}{\tan i}\right],$$

where r_0 is a fiducial radius where the density amplitude and the spiral phase are measured. According to Cox & Gómez (2002), $r_0 = 8$ kpc.

The parameters that customize the C&G potential are the following:

```
instance.parameters.spiral_density_amplitude   # amplitude
instance.parameters.spiral_phase   # initial orientation
instance.parameters.omega_spiral   # pattern speed
instance.parameters.m   # number of arms
instance.parameters.tan_pitch_angle   # tangent of the pitch angle
instance.parameters.r_sigma   # scale length
instance.parameters.scale_height   # scale height
instance.parameters.fiducial_radius   # fiducial radius
```

The values of these parameters that best fit the Milky Way are explained in
Jílková et al., 2012, A&A, 541, A64 and
Martínez-Barbosa et al., 2016b in prep.

3.3 Composite model of Lépine

This model assumes that the spiral structure of the Galaxy has multiple spiral arms with different amplitudes and pattern speeds. The Lépine model is often called the $(2 + 2)$ composite model because the main and secondary structures contain two spiral arms respectively.

In BarAndSpirals3D, the Lépine model uses the TWA potential to model the spiral arms; therefore, the composite model is defined in the two-dimensional space.

The parameters to customize the main spiral structure are:

```
instance.parameters.amplitude   # amplitude
instance.parameters.spiral_phase   # initial orientation
instance.parameters.omega_spiral   # pattern speed
instance.parameters.m   # number of arms
instance.parameters.tan_pitch_angle   # tangent of the pitch angle
```

The parameters to customize the secondary spiral structure are:

```
instance.parameters.phi21_spiral   # initial orientation with respect to the main spiral structure
instance.parameters.amplitude2   # amplitude
instance.parameters.omega_spiral2   # pattern speed
instance.parameters.m2   # number of arms
instance.parameters.tan_pitch_angle2   # tangent of the pitch angle
```

In the composite model the following parameters are the same for both the main and secondary structures:

```
instance.parameters.rsp   # separation distance at the beginning of the spiral locus
instance.parameters.N   # change from a bar to a spiral structure in the Galactic center
instance.parameters.r_sigma   # scale length of the spiral arms
```

4 Committing the parameters

Once you have enabled the desired Galactic model and parameters you must commit these. The parameters are commited by typing the following command:

```
instance.commit_parameters()
```

5 Other important parameters

In an inertial coordinate system, the bar and spiral arms rotate with different pattern speeds; therefore, the force exerted by each of these components on the stars will be time-dependent. It is however, convenient to integrate the stellar motion in a frame corotating with the bar or with the spiral arms so that the forces will not depend on time. According to the Galactic model selected (see options in the first page), BarAndSpirals3D decides the rotating frame that is more suitable. In the case where the Galactic model contains bar and spiral arms, BarAndSpirals3D chooses a frame that co-rotates with the bar.

We however emphasize that the equations of motion are NOT computed using BarAndSpirals3D. This module only provides the Galactic potential and the parameters of the rotating frame, such as its initial phase and angular velocity. The integrator that is used to compute the stellar motion is the ROTATING BRIDGE and it takes the output parameters of BarAndSpirals3D.

To obtain the parameters of the rotating frame where the stellar motion is computed, you first need to commit the Galaxy model parameters, as was explained in Sect. 4. and type the following commands:

```
pattern_speed= instance.parameters.omega_system     # angular velocity of the rotating frame
initial_phase = instance.parameters.initial_phase     # initial phase of the rotating frame
```

In the folder *examples_galaxia* you will find several scripts where BarAndSpirals3D is used, including the computation of the orbit of the Sun; how to make the evolution of a star cluster in the Galaxy; how to compute the resonances produced by the bar or the spiral arms and how to calculate the circular velocity of the Milky Way.