

## WAVENUMBER SPECTRA OF SHORT WIND WAVES: IMPLICATIONS FROM LABORATORY STUDIES

Jochen Klinke and Bernd Jähne

*Physical Oceanography Research Division, Scripps Institution of Oceanography, La Jolla, CA  
92093-0230, U.S.A*

**Abstract.** Directional wavenumber spectra of short wind waves as measured in three different wind-wave tanks at fetches from 5 m to infinity and wind speeds from 2 to 15 ms<sup>-1</sup> are analyzed. The data were collected using a refraction-based optical technique that allows the measurement of either the along-wind or cross-wind slope component of the water surface in areas of up to 30 × 40 cm<sup>2</sup>. The comparison of the directional wavenumber spectra from the different facilities shows that the angular spreading of the waves is most sensitive to the geometry of the facility, especially the width of the water channel. However, all unidirectional wavenumber spectra show two regimes that are also expected to be found in oceanic conditions. The first regime is characterized by a  $k^{-3.5}$  dependence of the wave height spectrum. It ranges from short gravity waves well into the capillary wave range. At a wavenumber of  $k \approx 1200 \text{ m}^{-1}$  (wavelength  $\lambda \approx 0.5 \text{ cm}$ ) a sharp and almost wind speed independent cutoff of the spectral densities occurs. This implies that viscous damping is not the dominant dissipation mechanism for capillary waves. The increase of the spectral densities with friction velocity is found to be dependent on wave number as well as on fetch.

### 1 Introduction

In the past decade, short ocean waves have received increasing scientific attention for a variety of reasons. Especially important is the knowledge of the wind speed dependence of short ocean wave spectra when modelling the radar backscatter from the ocean surface in remote sensing applications (van Halsema *et al.*, 1991). Although many measurements of frequency spectra have been performed in the ocean, no field data of directional wavenumber spectra in the gravity/capillary and capillary regimes exist. However, frequency spectra cannot readily be transformed to wavenumber spectra due to the modulation of the phase speed of small waves by larger ones. Therefore, any estimate of short wave spectra still relies on laboratory data. Since no high wave ages can be achieved in the laboratory, it is not clear whether the characteristics of the wavenumber spectra obtained in laboratory experiments can be extrapolated to field conditions as found in the open ocean.

A first laboratory study on wavenumber spectra at 100 m fetch in the Delft wind wave flume was published previously (Jähne and Riemer, 1990). In this paper this research is continued with a systematic study of wavenumber spectra from three considerably different wind wave facilities—Marseille, Delft, and Heidelberg—obtained under a wide range of experimental conditions. Data from different facilities not only provide more reliable estimates for the short wave spectra in the open ocean than data from only one facility obtained at a single fetch, but also allow for a detailed investigation of the influence of the channel geometry and the fetch on the wavenumber spectra.

## 2 Measurement Technique

The wave imaging technique makes use of the fact that different slopes of the water surface correspond to brightness variations in the image and is described in detail in Klinke (1991), Klinke and Jähne (1992), and Jähne and Schultz (1992). From the wave slope images, the wave height spectra can be obtained since the total slope wavenumber spectrum  $S$  and the corresponding height wavenumber spectrum  $F$  are related in the following way (Jähne and Riemer, 1990):

$$F(\mathbf{k}) = k^{-2}S(\mathbf{k}), \quad S(\mathbf{k}) = k^2F(\mathbf{k}). \quad (1)$$

In the case of slope measurements, however, it is necessary to measure both wave slope components and to add the along-wind and the cross-wind spectra to obtain the total slope spectrum. All wavenumber spectra presented in this paper are shown only as the dimensionless degree of saturation  $B(\mathbf{k})$  (Phillips, 1985). The degree of saturation is related to the height and the total slope spectra in the following way

$$B(\mathbf{k}) = F(\mathbf{k})k^4 = S(\mathbf{k})k^2. \quad (2)$$

## 3 Energy Balance of Short Wind Waves

Generally, the spectrum of a stationary wave field is determined by the energy input by the turbulent wind field, the energy transfer between waves of different wavenumbers by nonlinear wave-wave interaction, and the dissipation of energy by wave breaking, viscous and turbulent diffusion (Phillips, 1985; Kinsman, 1965).

Unfortunately, the energy balance cannot be determined uniquely from wave number spectra. Well known examples are the theories of Kitaigorodskii, (1983) and Phillips, (1985) which derive the same spectral form (3) with quite different assumptions about the energy balance

$$B(\mathbf{k}) \propto k^{0.5}, \quad F(\mathbf{k}) \propto k^{-3.5}. \quad (3)$$

On the one hand, Kitaigorodski assumed that in the case of a stationary and homogeneous wave field the energy input by the wind field primarily occurs at wavenumbers near the spectral peak, while the dissipation of energy by wave breaking and viscous damping takes places at large wavenumbers. On the other hand, Phillips argued that in the equilibrium range neither the energy input by the wind, nor the energy dissipation by wave breaking can be disregarded. Then, a local balance of all three spectral fluxes yields the same spectral form.

## 4 Results

Although the two-dimensional wavenumber spectra are useful for studying the angular dispersion of the waves qualitatively, angle-independent information contained in the wavenumber spectra cannot be readily extracted from such a representation. Unidirectional wavenumber spectra, i.e. a profile of the

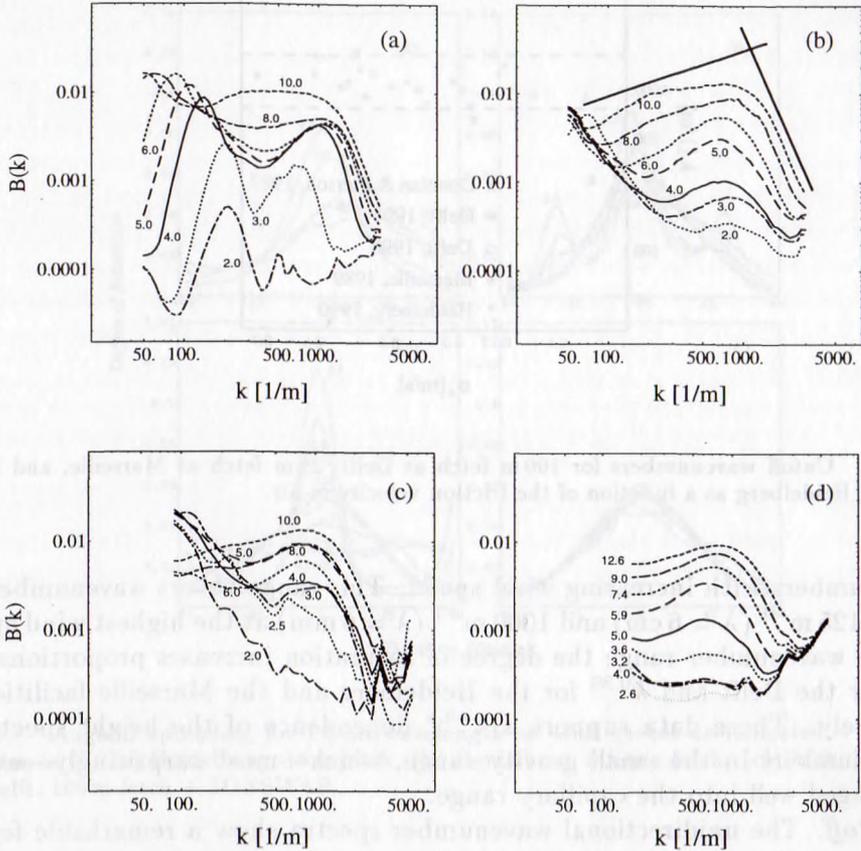


Fig. 1. Unidirectional wavenumber spectra at wind speeds as indicated. (a) Marseille, 7 m fetch. (b) Marseille, 29 m fetch. (c) Heidelberg, infinite fetch. (d) Delft, 100 m fetch.

wavenumber spectrum integrated over all angles, are much better suited for a further-reaching quantitative analysis of the data. Fig. 1 shows examples of unidirectional wavenumber spectra from all three wind-wave facilities.

*Spectral Shape.* In all facilities the dominant wave peak is included in the measured wavenumber range only for the lowest wind speeds and the shortest fetches. Except for low wind speeds at 7 m fetch in Marseille, no spectral gap around the wavenumber for minimum phase speed is found. At the highest wind speeds an equilibrium range has been established in all wind-wave facilities, with the spectral densities in the capillary range exceeding the spectral densities at lower wavenumbers. The equilibrium range extends towards smaller

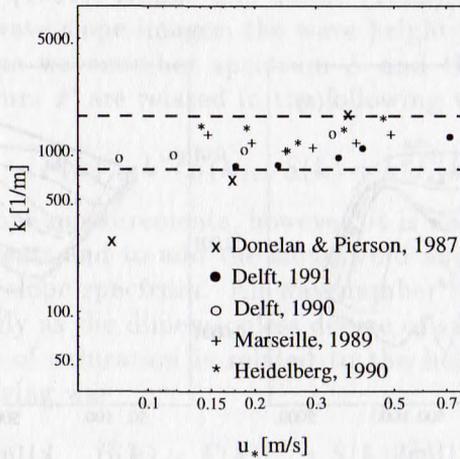


Fig. 2. Cutoff wavenumbers for 100 m fetch at Delft, 29 m fetch at Marseille, and infinite fetch at Heidelberg as a function of the friction velocity in air.

wavenumbers with increasing wind speed. This range covers wavenumbers between  $125 \text{ m}^{-1}$  ( $\lambda = 6 \text{ cm}$ ) and  $1000 \text{ m}^{-1}$  ( $\lambda = 6 \text{ mm}$ ) at the highest wind speeds. In this wavenumber range the degree of saturation increases proportionally to  $k^{0.5}$  for the Delft and  $k^{0.36}$  for the Heidelberg and the Marseille facilities, respectively. These data support a  $k^{-3.5}$  dependence of the height spectra for wave numbers in the small gravity range, which—most surprisingly—extends unchanged well into the capillary range.

*Cutoff.* The unidirectional wavenumber spectra show a remarkable feature. In Fig. 2 the wavenumbers where the spectral densities of each wind-wave facility start to drop, are plotted as a function of friction velocity in air. These so-called cutoff wavenumbers are obtained graphically from the intersection of two straight lines as shown for the 29 m fetch condition in the Marseille facility at 10 m/s wind speed in Fig. 1. While one line describes the asymptotic behavior of the spectral densities towards high wavenumbers, the other line is determined by linearly approximating the wave number dependence of the spectral densities in the equilibrium range. For wind speeds larger than 2.5 m/s, the cutoff wave number in each facility is practically independent of the wind speed. Only for the Delft facility the cutoff wavenumber increases slightly from  $800 \text{ m}^{-1}$  to  $1200 \text{ m}^{-1}$  with increasing wind speed. This tendency was not found for the wind-wave facilities of Marseille and Heidelberg, where the cutoff wavenumbers vary from 1000–1200  $\text{m}^{-1}$  and 1000–1500  $\text{m}^{-1}$ , respectively. In an earlier paper (Jähne and Riemer, 1990), a wind-speed independent cutoff wavenumber of  $800 \text{ m}^{-1}$  was reported. In the light of the new results, it appears that the earlier results were slightly biased towards lower wavenumbers by the limited wave

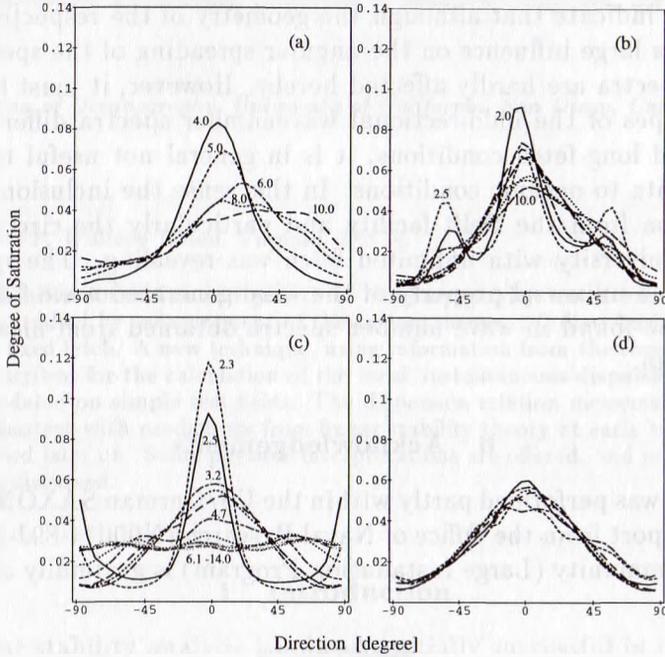


Fig. 3. Angular spreading for 1.6 cm wavelength at wind speeds as indicated. The wind direction is 0°. (a) Heidelberg, 10 m fetch. (b) Heidelberg, infinite fetch. (c) Delft, 7 m fetch. (d) Delft, 100 m fetch + JONSWAP.

number resolution due to the larger image sector and the median filtering of the images. The wind speed independence of the cutoff strongly rules out the possibility that the spectral density of the shortest waves is determined by the balance between wind input and viscous dissipation as assumed in the model of Donelan and Pierson (1987).

*Angular Spreading.* Angular slices of the 2-D wavenumber spectra are shown in Fig. 3 for a wavelength of 1.6 cm. The profiles are scaled in such a way that the area under all curves is the same, independently of the absolute value of the degree of saturation. For pure wind-generated waves one finds a strong dependence of the angular dispersion on the wind speed and the fetch. Generally, increasing wind speed tends to broaden the angular spreading of the waves. At 10 m fetch in the Heidelberg facility, centrifugal effects become visible at higher wind speeds, causing the waves to travel obliquely to the wind direction. A multimodal distribution can be found in the Heidelberg facility at infinite fetch and the lowest wind speeds. When mechanically generated waves

are superimposed on the pure wind waves, the wind speed dependence of the angular spreading disappears almost completely, as can be seen in case of the superimposed JONSWAP-type wave field at 100 m fetch in the Delft facility.

## 5 Conclusions

These results indicate that although the geometry of the respective wind-wave facilities has a large influence on the angular spreading of the spectra, the unidirectional spectra are hardly affected hereby. However, it must be noted that since the shapes of the unidirectional wavenumber spectra differ considerably for short- and long-fetch conditions, it is in general not useful to extrapolate short-fetch data to oceanic conditions. In this sense the inclusion of the 100 m fetch condition from the Delft facility and particularly the circular facility of Heidelberg University with unlimited fetch was revealing. The spectral cutoff appears to be a universal property of the wind-generated wave field and is also expected to be found in wave number spectra obtained from measurements in the open ocean.

## 6 Acknowledgements

This research was performed partly within the US/German SAXON-FPN project. Financial support from the Office of Naval Research (N00014-89J-3222) and the European Community (Large Installation Program) is gratefully acknowledged.

## References

- Donelan, M. A., and Pierson, W. J.: 1987, 'Radar scattering and equilibrium ranges in wind-generated waves with application to scatterometry', *J. Geophys. Res.* **92**, 4971-5029.
- Jähne, B., and Riemer, K.: 1990, 'Two-dimensional wavenumber spectra of small-scale water surface waves', *J. Geophys. Res.* **95**, 11,531-11,546.
- Jähne, B., and Schultz, H.: 1992, 'Calibration and accuracy of optical slope and height measurements for short wind waves', in L. Estep (ed.), *Proceedings Optics of the Air-Sea Interface: Theory and Measurements, SPIE Proceedings 1749*.
- Jähne, B., Waas, S., and Klinke, J.: 1992, 'A critical theoretical review of optical techniques for short ocean wave measurements', in L. Estep (ed.), *Proceedings Optics of the Air-Sea Interface: Theory and Measurements, SPIE Proceedings 1749*.
- Kinsman, B.: 1965, *Wind Waves - Their Generation and Propagation on the Ocean Surface*, Prentice Hall, Englewood Cliffs, N.Y.
- Kitaigorodskii, S. A.: 1983, 'On the theory of the equilibrium range in the spectrum of wind-generated gravity waves', *J. Physical Oceanogr.* **13**, 816-827.
- Klinke, J.: 1991, *2D Wellenzahlspektren von kleinskaligen winderzeugten Wasser-oberflächenwellen, Diploma thesis, Univ. Heidelberg*.
- Klinke, J., and Jähne, B.: 1992, '2D wavenumber spectra of short wind waves - results from wind-wave facilities and extrapolation to the ocean', in L. Estep (ed.), *Proceedings of Optics of the Air-Sea Interface: Theory and Measurements, SPIE Proceedings 1749*.
- Phillips, O. M.: 1985, 'Spectral and statistical properties of the equilibrium range in the spectrum of wind-generated gravity waves', *J. Fluid Mech.* **156**, 505-531.
- van Halsema, D., Snoeij, P., Calkoen, Ch., Oost, W. A., Vogelzang, J., and Jähne, B.: 1991, 'Modulation of the microwave backscatter by long gravity waves as measured in a very large wind/wave flume', *Proceedings IGARSS 91*, pp.885-888.