Interaction between waves and gravity currents: description of turbulence in a simple numerical model

Antonino Viviano · Rosaria E. Musumeci ·
Enrico Foti

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Abstract Effects of surface waves on gravity current propagation are studied by means of a numerical model. The adopted modeling approach couples a Boussinesq-type of model for surface waves and a gravity current model for stratified flows. In particular two different turbulence closure models are introduced which take into account subgrid turbulence and an additional depth-constant eddy-viscosity. The turbulence parameters are calibrated by means of experimental data on the time evolution of the heavy front, obtained both in the absence and in the presence of regular surface waves. Velocity fields, heavy and light front position, shear stresses, vorticity and entrainment calculated by the model are analyzed.

The turbulence closure which includes both uniform and Smagorinsky type eddy viscosity allows a better description of the actual gravity current propaga-

A. Viviano, R. E. Musumeci, E. Foti

Department Civil Engineering and Architecture, University of Catania, Catania, Italy

Tel.: +039-095-7382729

Fax: +039-095-7382748

E-mail: antonino.viviano@dica.unict.it

tion. In particular, the results highlight the fact that the presence of the oscillatory motion causes, simultaneously, a reduction in turbulence and an increase in the mixing of heavy and light fluids. Such a result is in agreement with the experimental observations.

Keywords density currents · wave motion · turbulence

1 Introduction

Buoyancy driven flows in the presence of surface waves are common in estuarine and coastal regions, in natural, urban and industrial zones [33]. The dynamics of gravity currents in the absence of waves has been often investigated by observing the well-known lock-exchange phenomenon [9,23,24], i.e. two fluids, having different densities initially separated by a vertical gate, which move toward a more stable condition with the lighter fluid flowing at the top and the higher density fluid at the bottom. A fundamental description of gravity currents and intrusions can be found in [35], where a wide variety of applications was investigated. 3D lock-exchange gravity currents were studied in [11] by means of experimental and numerical simulations. Furthermore, an innovative numerical Lattice-Boltzmann method was used in [10,30] to simulate such a phenomenon. The mixing mechanism, related to lock-exchange gravity currents, was studied through laboratory gravity currents [7] and 3D numerical models [27,28].

The effect of surface waves on the propagation of gravity currents has not been extensively investigated yet, as noted in [3]. Among the few available studies, [22] developed an asymptotic theory for the spreading of a layer of heavy immiscible liquid at the bottom due to the effects of surface waves. These authors considered a

multiple scale perturbation analysis, by assuming that surface waves and gravity currents act at different time scales, since sea waves are faster than buoyancy-driven flows and streaming effects. Evolution equations of profile distribution of the heavy liquid are obtained as a function of slow-time variables. They found that the streaming current, induced by surface waves, usually predominates and drives the heavy liquid to propagate together with surface waves. In the case of standing waves, the alternating component of the streaming current can induce the development of an undulating interface between the two fluids. The formation of such an undulation can dramatically change the local structure of mass transport near the bottom and it is related to the presence of circulating cells, whose characteristics depend on the thickness of the dense layer.

[31] studied the evolution of the release of a heavy liquid at the bottom of surface waves. They showed that, after the initial fall, the gravity current propagates with two fronts moving in opposite directions, whose overall length was weakly affected by wave action, since it is mainly determined by buoyancy. The position of the middle point and the shape of the heavy front were significantly affected by the presence of surface waves: i) Stokes drift dominates for long waves and the middle point of the heavy liquid is moved downstream; ii) the Eulerian-averaged velocity is most important near the bed for short waves and the gravity current moves upstream, opposite to the direction of wave propagation. Finally, [31] found that the centre of the current is advected at a fraction of the Lagrangian velocity at the bed.

More recently, [21] proposed a numerical model to study the combined surface waves - gravity current flow. The model tries to fill the gap between two-layer shallow water models [12] and more complex LES and DNS models able to mimic

the complete hydrodynamics of the non-homogeneous density flow [25,8]. The model decouples buoyancy-driven flow and wave motion, by separating the orbital velocity from the gravity current velocity. Such a model was derived under the assumption of inviscid flow and free slip condition at the bottom. A preliminary validation of the numerical model was carried out by means of the experimental data of [21]. In both experimental and numerical data, the heavy front was slower in the presence of waves with respect to the calm water case. The behaviour of the undulations predicted by the model at the interface between the two fluids, in the rear part of the heavy front, was qualitatively in agreement with the experimental observations, notwithstanding the fact that turbulent effects were neglected in the gravity current model. However, the velocity of the heavy front computed by the numerical model was overpredicted as a consequence of the inviscid flow assumption.

In the present work, such an assumption has been eliminated. Turbulence has been modeled by two alternative formulations: the first considers only a Smagorinsky type eddy viscosity [32]; in the second formulation, eddy viscosity is the sum of both a Smagorinsky component and a depth-constant component. The rationale behind the latter formulation is that the presence of an additional uniform viscosity component should better simulate the background turbulence due to the combined flow resulting from gravity currents interacting with surface waves. Both formulations have been considered for each of the tests carried out, in order to verify which turbulence closure is the most reliable one.

Gravity currents generated by a lock-exchange scheme are studied here, since such a scheme presents a constant-speed phase and it has been studied comprehensively in scientific literature.

The paper is organized as follows. Section 2 describes the proposed numerical approach, focusing on the turbulence model. Section 3 presents the benchmark experimental data, the setup and the calibration of the model. Section 4 discusses the results of the numerical model both in the absence and in the presence of surface waves, by focusing on the dynamics of the density, velocity, vorticity and shear stress fields. Results are also shown on the entrainment process. Concluding remarks are given in Section 5.

2 Proposed numerical approach

A new numerical model has been developed to analyze the combined two-dimensional flow of gravity currents and free surface waves. The governing equations are obtained under the assumption of Boussinesq gravity currents. The adopted reference system is shown in Figure 1 where the two-dimensional Cartesian coordinate system (x, z) is located on the still water level, from which the water depth h is measured. By using an approach similar to that in [35], the continuity equation, the two momentum Reynolds-Averaged equations and the density transport equation are:

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0 \quad (1)$$

$$\rho \frac{du}{dt} = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial z} (\tau_{xz} + \tau'_{xz}) \quad (2)$$

$$\rho \frac{dw}{dt} = -\frac{\partial p}{\partial z} + \frac{\partial}{\partial x} (\tau_{xz} + \tau'_{xz}) - \rho g \quad (3)$$

$$\frac{d\rho}{dt} = \kappa \left(\frac{\partial^2 \rho}{\partial x^2} + \frac{\partial^2 \rho}{\partial z^2} \right) \quad (4)$$

where u and w are the horizontal and vertical components of ensemble-averaged velocity; p is the total pressure; τ_{xz} and τ'_{xz} are the viscous and turbulent stresses

Fig. 1 Sketch of gravity current propagation under surface waves with indication of the reference system and of the main variables.

respectively; ρ is the local density, whose value is comprised between the density of the ambient fluid ρ_0 and the density of the heaviest fluid $\rho_1 > \rho_0$; g is the gravity acceleration; κ is the diffusion coefficient of the solute (e.g. sodium chloride in the case analysed in this paper), which is equal to zero in the case of immiscible fluids.

Modelling the surface wave motion in a lock-exchange release is a complex task, which, thanks to the adopted Boussinesq approximation i.e. $\gamma = \rho_0/\rho_1 \sim 1$, is tackled here by considering that surface waves are not influenced by gravity currents. Indeed the phase speed of surface waves is proportional to gravity (g), whereas the bulk velocity of the gravity current is proportional to the reduced gravity g' , since $g' \ll g$ as a consequence of the Boussinesq approximation. As a consequence of that it is possible to decouple the homogeneous density wave motion and the gravity current propagation forced by actual density gradients, by neglecting, at leading order, the influence of buoyancy on the wave field. Such an assumption is also confirmed by the present experimental data.

It follows that the total velocity field $\mathbf{u}$ is obtained by linearly adding up the effects of the external wave motion and of the gravity current:

$$\mathbf{u} = \mathbf{u}_B + \mathbf{u}_d \quad (5)$$

where $\mathbf{u}_B \equiv (u_B, w_B)$ is the wave orbital velocity, which is independent from the gradient of ρ and it is calculated through a Boussinesq-type wave model for homogeneous flow, and the gravity current $\mathbf{u}_d \equiv (u_d, w_d)$ which is evaluated as a function of the pressure field induced by both buoyancy and waves.

A splitting procedure between the orbital wave motion and the gravity current is applied to eqs. (1) and (2), giving respectively:

$$\frac{\partial(u_B + u_d)}{\partial x} + \frac{\partial(w_B + w_d)}{\partial z} = \left(\frac{\partial u_B}{\partial x} + \frac{\partial w_B}{\partial z} \right) + \left(\frac{\partial u_d}{\partial x} + \frac{\partial w_d}{\partial z} \right) = 0 \quad (6)$$

$$\begin{aligned} \rho_0 \frac{d(u_B + u_d)}{dt} + \frac{\partial(p_B + p_d)}{\partial x} - \frac{\partial}{\partial z} [(\tau_{xz} + \tau'_{xz})_B + (\tau_{xz} + \tau'_{xz})_d] = \\ \left[\rho_0 \frac{du_B}{dt} + \frac{\partial p_B}{\partial x} - \frac{\partial}{\partial z} (\tau_{xz} + \tau'_{xz})_B \right] + \left[\rho_0 \frac{du_d}{dt} + \frac{\partial p_d}{\partial x} - \frac{\partial}{\partial z} (\tau_{xz} + \tau'_{xz})_d \right] \\ + u_B \frac{\partial u_d}{\partial x} + u_d \frac{\partial u_B}{\partial x} + w_B \frac{\partial u_d}{\partial z} + w_d \frac{\partial u_B}{\partial z} = 0 \end{aligned} \quad (7)$$

in which pressure and shear stresses are split similarly to the velocity. Further details on such components will be given in Section 2.1 and 2.2, when describing the two decoupled models.

Since the above velocity decomposition is adopted, the horizontal momentum equation presents mixed convective terms, which are reported in the last row of eq. (7). As these mixed terms are neglected in the proposed model, the model may fail when it is applied to reproduce highly nonlinear phenomena. In the lock-exchange process, the greatest nonlinearities occur during the initial acceleration phase after the gate opening. Therefore, it is expected that the proposed model

is more reliable once the constant - speed phase is reached. An analysis on the importance of neglected convective terms has been carried out. The results confirm the above limit of the model, particularly up to the point when the distance between the heavy and lighter front is similar to the value of the water depth. After such an initial stage, the mixed convective terms are negligible and do not influence the physics of the gravity currents under surface waves.

The modelling procedure is schematized by the flow-chart shown in Figure 2, in which the incident surface wave height (H) and period (T) represent the input of the Boussinesq-type wave propagation model. The initial density distribution corresponds to a lock-exchange configuration, i.e. the fluid with density ρ_1 is separated by a fluid having density $\rho_0 < \rho_1$ by a vertical sluice gate. Such a density distribution, along with the orbital velocity field $\mathbf{u}_B$ and the free surface elevation due to the waves ζ_B , are entered into the gravity current model, which calculates the evolution of the total velocity $\mathbf{u}$ and of the density distribution.

2.1 Model for surface waves

A one-dimensional weakly dispersive fully nonlinear Boussinesq-type model [20, 13] has been adopted here to describe the hydrodynamics of surface waves. Such a model was originally derived to describe surf zone hydrodynamics and it has been recently extended to deal with two-dimensional horizontal problems [37].

On the basis of scaling arguments for relatively shallow water wave propagation, two dimensionless parameters are adopted, namely the dispersive parameter $\mu = kh$ and the nonlinear parameter $\delta = a/h$, where $a = H_w/2$ is the surface wave amplitude. Since the surface wave model is weakly dispersive, only terms up

Fig. 2 Flowchart of the proposed approach for modelling gravity currents under surface waves.

Input variables are: characteristics of the regular wave motion (i.e. wave period T and wave height H) and initial density distribution. Such a distribution is characterized by two uniform zones having density ρ_0 and ρ_1 respectively, as in the lock exchange initial setup.

167 to $O(\mu^2)$ are retained, whereas the fact that the model is fully nonlinear means
 168 that no assumptions are made about the order of magnitude of δ . It is assumed
 169 also that the flow is irrotational only outside the surf zone, that is wave breaking
 170 is assumed to be the unique source of vorticity within the flow. Within the surf
 171 zone the velocity field is influenced by the effects of breaking-induced vorticity
 172 and the vorticity transport equation is solved analytically. The amount of vortic-
 173 ity introduced by the breaking process is determined through a similarity with the
 174 hydraulic jump, by using the surface roller concept [34]. The governing equations

of the Boussinesq-type wave model, applied here, have been derived in [20]. In particular, it is possible to separate the wave related horizontal velocity (u_B) into a potential component u_p and a rotational component u_r , which is function of surface wave related vorticity.

It is assumed [36] that potential velocity component can be represented as a sum of the mean potential velocity $\bar{u}_p$ and of a quadratic function of vertical coordinate z . Consequently, the total wave related horizontal velocity can be expressed as follows

$$u_B = \bar{u}_p + \mu^2 (h\bar{u}_p)_{xx} \left(\frac{\Delta_1}{2} - z \right) + \frac{\mu^2}{2} (\bar{u}_p)_{xx} \left(\frac{\Delta_2}{3} - z^2 \right) + u_r + O(\mu^4) \quad (8)$$

where the subscript x represents partial derivative along the horizontal direction.

In the present case, $u_r = 0$ since waves are non-breaking; Δ_1 and Δ_2 are coefficients given by the following expressions:

$$\Delta_1 = \delta\zeta_B - h, \quad \Delta_2 = \delta^2\zeta_B^2 - \delta\zeta_B h + h^2 \quad (9)$$

in which ζ_B is the free surface elevation under homogeneous flow conditions.

The Reynolds-Averaged Navier-Stokes equations, used in the derivation of the adopted Boussinesq-type of model, can be expressed as:

$$\frac{\partial u_B}{\partial x} + \frac{\partial w_B}{\partial z} = 0 \quad (10)$$

$$\rho_0 \frac{du_B}{dt} + \frac{\partial p_B}{\partial x} - \frac{\partial}{\partial z} (\tau_{xz} + \tau'_{xz})_B = 0 \quad (11)$$

$$\rho_0 \frac{dw_B}{dt} + \frac{\partial p_B}{\partial z} - \frac{\partial}{\partial x} (\tau_{xz} + \tau'_{xz})_B + \rho_0 g = 0 \quad (12)$$

where eqs. (10) and (11) correspond, respectively, to the wave component of eq. (6)

and (7); eq. (12) is the vertical momentum equation used to derive the wave component of pressure p_B . All these equations are valid under a uniform density condition.

In order to obtain the Boussinesq-type wave model [20], such equations can be integrated over the depth and it is possible to apply the kinematic and dynamic boundary conditions at the bottom and at the free surface. Consequently the following equations can be obtained, as a function of surface elevation ζ_B and of depth-averaged horizontal orbital velocity, $\bar{u}$:

$$\frac{\partial \zeta_B}{\partial t} + \frac{\partial}{\partial x} [\bar{u}(h + \delta \zeta_B)] = 0 \quad (13)$$

$$\begin{aligned} & \bar{u}_t + \delta \bar{u} \bar{u}_x + \zeta_{Bx} + \mu^2 \left[\left(B - \frac{1}{3} \right) h^2 \bar{u}_{xxt} - \frac{1}{2} h h_{xx} \bar{u}_t - h h_x \bar{u}_{xt} \right] \\ & + B \mu^2 h^2 \zeta_{xxx} + \delta \mu^2 \left[-\frac{1}{3} h^2 \bar{u}_{xxx} - h \zeta_{Bx} \bar{u}_{xt} + \frac{1}{3} h^2 \bar{u}_x \bar{u}_{xx} - \frac{2}{3} h \zeta_B \bar{u}_{xxt} \right. \\ & - \frac{3}{2} h h_{xx} \bar{u} \bar{u}_x - \frac{1}{2} h h_{xx} \bar{u}^2 - h h_x \bar{u} \bar{u}_{xx} - \zeta_B h_x \bar{u}_{xt} - h_x \zeta_B \bar{u}_t \\ & - \frac{1}{2} \zeta_B h_{xx} \bar{u}_t + B h^2 (\bar{u} \bar{u}_x)_{xx} \left. \right] + \delta^2 \mu^2 \left[\frac{1}{6} \zeta_B^2 \bar{u}_{xxt} - \frac{1}{3} h \zeta_B \bar{u}_x \bar{u}_{xx} \right. \\ & - \frac{1}{3} h \bar{u}_{xx} (\zeta_B \bar{u})_x + h (\zeta_B \bar{u}_x^2)_x - \frac{1}{2} (\zeta_B^2 \bar{u}_{xt})_x - \frac{2}{3} h (\zeta_B \bar{u} \bar{u}_{xx})_x \\ & - \zeta_{Bx} h_{xx} \bar{u}^2 - \zeta_B h_x \bar{u} \bar{u}_{xx} - \frac{1}{2} \zeta_B h_{xx} \bar{u}^2 - \frac{3}{2} \zeta_B h_{xx} \bar{u} \bar{u}_x - \zeta_{Bx} h_x \bar{u} \bar{u}_x \left. \right] \\ & + \delta^3 \mu^2 \left[-\frac{1}{3} \zeta_B^2 \bar{u} \bar{u}_{xxx} - \zeta_B \zeta_{Bx} \bar{u} \bar{u}_{xx} + \zeta_B \zeta_{Bx} \bar{u}_x^2 + \frac{1}{3} \zeta_B^2 \bar{u}_x \bar{u}_{xx} \right] \\ & + [\delta (\Delta M)_x + \mu^2 (\Delta P)_{xxt} - \mu^2 D_s + \delta \mu^2 (\Delta M_1)_x + \delta \mu^2 D_w \\ & + \delta \mu^2 D_{uw}] (h + \delta \zeta_B)^{-1} + O(\mu^4) = 0 \end{aligned} \quad (14)$$

where the subscript t represents partial derivative over the time; B is a coefficient which allows for the enhancement of the dispersion characteristics of the model in deeper water, with $B = 1/15$ suggested by [17]; the terms $(\Delta M)_x, (\Delta P)_{xxt}, (\Delta M_1)_x, D_w, D_s$ and D_{uw} are the breaking terms, function of the rotational velocity u_r . In the present work all these terms are equal to zero, as well as vorticity and rotational velocity.

For the numerical solution of eqs. (13) and (14), the Adams-Bashforth-Moulton scheme is used since it gives good stability properties [29]. Such a scheme consists of two stages: a first-attempt predictor step, which is accurate up to 3th order, and an iterative corrector step which is accurate up to 4th order.

At the offshore boundary outgoing waves are allowed to leave the domain by using an absorbing-generating boundary condition [5], while at the onshore boundary a sponge layer is used in order to dissipate the wave motion at the shoreward boundary of the domain [20].

Starting from the solution of the above eqs. (13) and (14) of the model, which gives ζ_B and $\bar{u}$, it is possible to extract information on the horizontal component of orbital flow motion u_B by applying eq. (8) with $u_r = 0$.

Similarly, the vertical component of surface wave related velocity (w_B) can be obtained, by inserting u_B , calculated using eq. (8), inside the continuity eq. (10):

$$\begin{aligned}
 w_B = & -(\bar{u}_p)_x(z+h) - 2\bar{u}_p h_x - \mu^2(h\bar{u}_p)_{xxx} \left[\frac{\Delta_1}{2}(z+h) - \frac{z^2}{2} + \frac{h^2}{2} \right] \\
 & - \mu^2(h\bar{u}_p)_{xx} \left[\frac{(\Delta_1)_x}{2}(z+h) + 2hh_x + \Delta_1 h_x \right] \\
 & - \frac{\mu^2}{2}(\bar{u}_p)_{xxx} \left[\frac{\Delta_2}{3}(z+h) - \frac{z^3}{3} - \frac{h^3}{3} \right] \\
 & - \frac{\mu^2}{2}(\bar{u}_p)_{xx} \left[\frac{(\Delta_2)_x}{3}(z+h) - 2h^2 h_x + \frac{2}{3}\Delta_2 h_x \right] + O(\mu^4) \quad (15)
 \end{aligned}$$

2.2 Model for gravity currents

The orbital velocity field, $\mathbf{u}_B \equiv (u_B, w_B)$, and the free surface elevation, ζ_B , obtained through the above wave model, are the inputs of the proposed gravity current model, which is a modified version of the one proposed in [21]. In particular,

here a viscous flow is considered and a turbulence closure is included to better simulate the full depth lock exchange phenomenon. A further assumption is that the presence of a spatial variability of density may influence the flow only through modification of the pressure field. Hence, the Reynolds-Averaged Navier-Stokes momentum equations, i.e. eqs. (2) and (3), are rewritten in terms of a new pressure term related to stratification, called $p_d = p - p_B$, where p_B is the value of the wave-related dynamic in homogeneous non-stratified conditions.

The adopted splitting procedure between the wave related velocity $\mathbf{u}_B \equiv (u_B, w_B)$ and the gravity current velocity $\mathbf{u}_d \equiv (u_d, w_d)$ has been already introduced in eqs. (6) and (7), i.e. in the continuity and horizontal momentum equations. The density-related continuity and horizontal momentum equations which are extracted from eqs. (6) and (7) can be written as follows

$$\frac{\partial u_d}{\partial x} + \frac{\partial w_d}{\partial z} = 0 \quad (16)$$

$$\rho_0 \frac{du_d}{dt} + \frac{\partial p_d}{\partial x} - \frac{\partial}{\partial z} (\tau_{xz} + \tau'_{xz})_d = 0 \quad (17)$$

where the last two terms on the left-hand-side of eq. (17) represent the viscous and turbulent shear stresses related to gravity currents. Such stresses are modeled by means of the kinematic and eddy viscosity, ν and ν_t respectively. Thus eq. (17) reads:

$$\frac{\partial u_d}{\partial t} + \frac{1}{2} \frac{\partial u_d^2}{\partial x} + w_d \frac{\partial u_d}{\partial z} + \frac{1}{\rho_0} \frac{\partial p_d}{\partial x} - \frac{\partial}{\partial z} \left[(\nu + \nu_t) \left(\frac{\partial u_d}{\partial z} + \frac{\partial w_d}{\partial x} \right) \right] = 0 \quad (18)$$

The pressure variable p_d can be computed on the basis of a properly decomposed vertical momentum equation, obtained by subtracting eq. (12) from the total vertical momentum eq. (3), and by applying the Boussinesq approximation:

$$\rho_0 \frac{dw}{dt} + \rho g - \rho_0 \left(\frac{dw_B}{dt} + g \right) + \frac{\partial p_d}{\partial z} - \frac{\partial}{\partial x} [(\tau_{xz} + \tau'_{xz}) - (\tau_{xz} + \tau'_{xz})_B] = 0 \quad (19)$$

The pressure term p_d , due to the in-homogeneous density field, can be calculated by integrating eq. (19) over the water depth, i.e. between the bottom ($z = -h$) and the free surface ($z = \zeta$):

$$p_d = \int_{-h}^{\zeta} -\Delta\rho g - \rho_0 \left(\frac{dw}{dt} - \frac{dw_B}{dt} \right) + \rho_0 \frac{\partial}{\partial x} \left[(\nu + \nu_t) \left(\frac{\partial u_d}{\partial z} + \frac{\partial w_d}{\partial x} \right) \right] dz \quad (20)$$

in which $\Delta\rho = \rho - \rho_0$ is the local density variation with respect to the initial value ρ_0 of the lighter fluid. Eq. (20) is numerically integrated, by means of the trapezoidal rule [29], between the free surface ζ , where the pressure p_d is assumed equal to zero, and the bottom $z = -h$.

Following the hypothesis at the basis of the derivation of the proposed model, it is assumed that at leading order the wave motion dominates and the contribution due to density variations is negligible, i.e. $\zeta_d \ll \zeta_B$. Nevertheless, the contribution to the free surface elevation ζ_d must also be included when computing eq. (20) to find p_d . Such a contribution is calculated by integrating eq. (16) along the water column and by applying the well-known kinematic boundary conditions at the free surface and at the bottom, thus obtaining:

$$\frac{\partial \zeta_d}{\partial t} + \frac{\partial}{\partial x} \int_{-h}^{\zeta} u_d dz = 0 \quad (21)$$

The unknown variables of the gravity current model are p_d , u_d , w_d , ρ and ζ_d . All the other variables are outputs of the previous Boussinesq-type surface wave model. Therefore a total of 5 equations are solved. In particular, w_d is calculated by spatially integrating eq. (16); p_d by integrating eq. (20) over the depth; u_d , ρ and ζ_d are estimated by iteratively solving eqs. (18), (4) and (21), which all contain time derivatives. Similarly to the wave model, also here the Adams-Bashforth-Moulton scheme is used for time-stepping.

In order to solve eq. (18), a free slip boundary condition is used at the bottom. Such a condition allows the model to be efficient since it does not need a very refined grid, with respect to a more rigorous no-slip condition which would need a very fine discretization close to the bed. Preliminary tests have been carried out by considering also a friction velocity at the bottom, proportional to the drag coefficient. It has been found that the advantages of using such a modified boundary condition at the bottom are not significant compared to the velocity field predicted by the model with the mere simpler slip boundary condition adopted here.

2.3 Turbulence closure

The use of a 2D model for gravity currents may cause a non-realistic simulation of the dynamics of the turbulent structures at the interface between the two fluids (e.g. Kelvin-Helmholtz billows), with respect to 3D models. In particular, in 2D models such turbulent structures tend to become bigger than those present in actual gravity current propagation [26]. In the proposed model, which is designed for engineering applications, we decide to partially overcome such a limit by carrying out an adequate calibration of some simple turbulent closure formulations. This allows to achieve a reasonable comparison between the modeled gravity current and reality. Such a 2D calibrated model simulates K-H structures which do not detach from the front between the two fluids, thus giving a behaviour qualitatively similar to laboratory gravity currents [21].

As shown in eqs. (18) and (20), the proposed approach for simulating buoyancy-driven flows considers the effects of turbulence by means of the eddy viscosity concept. This kind of turbulent closure is quite simple, if compared to other more

physically based approaches, such as $k - \epsilon$ or $k - \omega$ methods [2], and it has been widely tested in both steady and oscillatory flow conditions [32, 6, 14, 1].

In particular, in the present work two kinds of eddy viscosity have been adopted. In the first approach, called formulation ‘A’, the eddy viscosity is estimated on the basis of the Smagorinsky approach for sub-grid scale turbulence (SGS) [32], as a function of the local derivatives of the velocity field and of the local grid size:

$$\nu_{t,A} = C_s \Delta x \Delta z \sqrt{\left(\frac{\partial u}{\partial x}\right)^2 \left(\frac{\partial w}{\partial z}\right)^2 + \frac{1}{2} \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)^2} \quad (22)$$

where C_s is a parameter to be determined through calibration, Δx and Δz are the dimension of the numerical grid along the horizontal and the vertical directions, respectively. Using such a formulation the small (sub-grid) scales are assumed to be in equilibrium, thus energy production and dissipation balanced. This kind of approach is widely adopted in large-eddy simulations (LES) in which the effect of large scales is directly computed [6].

Since the formulation A is a mere SGS formulation, it may not be capable of taking into account larger scale effects due to the interaction between the orbital motion and the gravity current. Thus an alternative formulation has been tested (called ‘B’), where a depth-uniform background eddy viscosity is added to the SGS Smagorinsky eddy viscosity of formulation A:

$$\nu_{t,B} = C_u h \sqrt{g' h} + C_s \Delta x \Delta z \sqrt{\left(\frac{\partial u}{\partial x}\right)^2 \left(\frac{\partial w}{\partial z}\right)^2 + \frac{1}{2} \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)^2} \quad (23)$$

where g' is the reduced gravity, defined as $g(\rho_1 - \rho_0)/\rho_0$, and C_u is an additional calibration coefficient. The parameter C_s is similar to the one used in the purely Smagorinsky formulation (eq. 22) but its values are expected to be lower, due to the contribution of the depth-uniform viscosity component.

After a calibration of the two parameters C_s and C_u , the differences between the two proposed formulations (A and B respectively) are discussed in the following by analyzing the hydrodynamics of the propagation of gravity currents both in the absence and in the presence of surface waves. In particular, results are presented on the kinematics of the front propagation, density distribution, velocity and vorticity field, shear stresses and entrainment.

3 Calibration of the model

3.1 Benchmark experiments

A laboratory dataset of lock-exchange experiments performed within a wave flume has been used as a benchmark in order to calibrate the parameters of the two turbulence models described above. The experiments have been carried out in a wave flume, which is sketched in Figure 3, at the Hydraulic Laboratory of the University of Catania. The wave flume is 9 m long, 0.7 m deep and 0.5 m wide. At the offshore end of the flume, a piston-type wavemaker generates regular waves. At the onshore end, a porous beach is used to control undesired wave reflection. At 5.5 m from the initial section of the flume, a Perspex sluice gate is used to obtain a lock of salted colored water having a density equal to ρ_1 . Onshore of the gate, the flume is filled with fresh water, having a density equal to ρ_0 .

The values of ρ_1 and ρ_0 are estimated for each test by using a set of three calibrated 100-mL pycnometers, equipped with a thermometer having a precision of 0.005°C [21]. By following such a procedure, the error in measuring the density is estimated to be smaller than 1 g/m³.

Fig. 3 Sketch of the wave flume in which lock-exchange experiments have been carried out.

Full-depth two-dimensional lock-exchange experiments have been performed both in the absence and in the presence of surface regular waves. In the first case, starting from an hydrostatic condition, the sluice gate is removed and a positive front of denser fluid intrudes in the lower part of the water column under the lighter fluid in the onshore direction while in the upper part a negative front of lighter fluid moves offshore. In the second case, the wavemaker starts generating monochromatic waves propagating in the onshore direction. As soon as the waves reach the sluice gate, the latter is removed and a gravity current develops. In this case, the propagation of the salted front in the onshore direction occurs in the presence of the wave motion. Acoustic wave gauges have been used to measure the characteristics of the waves, i.e. wave height H and wave period T , while a set of HD video cameras has been used to record the propagation of the density front at a frame rate of 50 fps, with 1920×1080 pixel wide images. A detailed description the experimental setup is reported in [21].

The presence of progressive surface waves causes a mean flow, producing a mass transport [15], which influences the propagation of gravity currents. The wave driven mass transport has two components: the Stokes drift due to irrotational free surface flow and the boundary layer component due to vorticity. The first

one is established immediately while the second one takes much longer, depending on the flume dimensions. In all the tests carried out, the duration of the tests is $O(10 \text{ s})$. Such a duration is brief when compared to the time necessary for the development of a steady streaming component, about 5 min in the present case [18]. As a consequence, only the Stokes drift and consequently the offshore directed undertow influences the gravity current propagation, while the unsteadiness of the flow should play a negligible role. By means of Vectrino profiler measurements, it has been estimated that the undertow is about 0.6 cm/s, in agreement with predictions obtained by classic literature models [16, 4].

The characteristics of the benchmark experimental dataset used in the present work to calibrate the parameters of the numerical model are summarized in Table 1.

Table 1 Experimental benchmark tests used to calibrate the proposed numerical model.

Test n.	h	ρ_0	ρ_1	g'	H	T	H/L	U_m
	[m]	[kg/m ³]	[kg/m ³]	[m/s ²]	[m]	[s]	[-]	[m/s]
S004	0.200	997.856	1000.012	0.023	-	-	-	0.022
S007	0.200	998.857	1003.674	0.047	-	-	-	0.046
S009	0.200	998.705	1009.591	0.106	-	-	-	0.061
S111	0.203	997.781	1001.734	0.039	-	-	-	0.045
W004	0.200	998.141	1000.508	0.023	0.020	0.994	0.016	0.017
W007	0.200	998.857	1003.689	0.047	0.020	0.994	0.016	0.037
W009	0.200	998.705	1009.379	0.104	0.020	0.994	0.016	0.068
W123	0.202	997.704	1001.552	0.038	0.008	1.840	0.003	0.034
W108	0.200	996.242	1001.951	0.056	0.031	0.840	0.032	0.048
W109	0.201	997.804	1001.810	0.039	0.043	0.710	0.060	0.039

The first column indicates the name of the test, where the prefix S refers to classical lock-exchange tests while the prefix W indicates that density currents and surface regular waves have been combined. The second column reports the water depth h , the third and the fourth columns give the densities of the light and of the heavy fluids, namely ρ_0 and ρ_1 . The fifth column presents the reduced gravity g' . In the case of regular wave experiments the sixth and the seventh columns show the measured wave height H and wave period T . The eighth column reports the wave steepness H/L , in which the wave length L is computed by using the linear dispersion relationship, derived for surface wave motion under uniform density conditions [4]. Such a relationship has been used here since changes of the wave length in the presence of gravity currents are assumed to be negligible. Indeed the wave motion is dominated by gravity (g), while the gravity current is forced by reduced gravity g' , which is about two order of magnitude lower than g in the present case. Finally, the last column of Table 1 shows the measured time-averaged velocity of the heavy front U_m . This quantity is calculated as the slope of the straight line which best interpolates the front position over time, by considering just the constant-velocity stage. The position of the heavy front (from which velocity is computed) is measured by means of video image analysis. In particular, the initial frame is considered as a reference. The grayscale images are then converted into binary black and white images by using a threshold on the pixel intensity, to obtain the front as a set of black pixels. The front location is determined by counting the number of black pixels at the bottom, from the initial gate position. The errors on the location of the heavy front of the gravity current is smaller than 0.5 mm. Consequently the uncertainty on the measurement of time-averaged (bulk) velocity is always less than 10^{-4} m/s.

A preliminary analysis on the behaviour of the measured front velocities can be carried out as function of the Grashof number, similarly to [26]. Such a dimensionless number is the ratio between buoyancy forces and viscous forces:

$$Gr = \left(\frac{U_b h}{\nu} \right) \quad (24)$$

where $U_b = \sqrt{g'h}$ is the buoyancy velocity defined in terms of the initial water depth h . For large values of Grashof number ($Gr > 10^9$), viscous forces are negligible and the flow regime tends to become fully turbulent.

Figure 4 shows the behaviour of the heavy front bulk velocity as a function of $\sqrt{Gr}$, by separating still water tests from surface wave tests. It is evident that the presence of waves causes a reduction of front velocity, except for the tests having the greatest Grashof numbers, i.e. S009 and W009 ($Gr \approx 10^9$). It is possible to estimate a reduction of 15% of front velocity in the presence of waves, when $Gr < 6 \times 10^8$.

3.2 Numerical model setup and parameter calibration

The 2D computational domain is 14 m long and 0.3 m high, the still water depth is $h = 0.2$ m. At the offshore end of the numerical grid, an absorbing-generating boundary condition allows both the propagation of regular waves inside the domain and the absorption of reflected waves exiting the flume. At the onshore end, a sponge layer is used in front of a vertical wall to minimize wave reflection.

In the numerical simulations, several waves are run before the opening of the gate, in order to have an established wave field before the propagation of the density current. Although different from the experimental procedure, this is re-

Fig. 4 Averaged measured velocity of the heavy front as function of Grashof number square root, by considering both still surface and wave tests.

quired by the wave generation module, and it is numerically possible thanks to the separation between the wave module and the gravity current module.

The opening of the gate corresponds to the start of the gravity current in the numerical model. Initial conditions of such a model are: (i) density current velocity $\mathbf{u}_d$ equal to zero; (ii) free surface elevation as obtained by the wave model; (iii) uniform density distribution at the two sides of the gate, with a density equal to ρ_1 if $0 \leq x \leq 7$ m, and to ρ_0 if $7 < x \leq 14$ m. The spatial discretization is different along the horizontal and vertical directions, in order to allow a fairly reasonable description of the vertical density profile along the tank. In particular, $\Delta x = 0.05$ m and $\Delta z = 0.015$ m.

The calibration of the numerical model has been performed considering both formulations of the turbulence closure, i.e. model A, where only a Smagorinsky-type of eddy viscosity $\nu_{t,A}$, function of C_s , is used, and model B, where a depth-

constant contribution, function of C_u , is also introduced to calculate $\nu_{t,B}$. In the present work just the calibration of the coefficient C_s is discussed, which is shared by both models. Indeed, the numerical results may be significantly affected by the value of coefficient C_s introduced in the Smagorinsky formulation. Similar analyses, not reported here for the sake of brevity, have shown that the numerical results are less sensitive to changes of the coefficient C_u , which is kept constant and equal to 0.008 in all the calculations discussed in the following.

In order to compare the numerical results with the experimental ones, the interface between heavy fluid and ambient fluid is defined, in the model, as the part of the domain in which density is equal to the mean value between that of ambient and of heavy fluid. Consequently, the heavy front position is the intersection between the interface and the bottom. Figure 5 shows two examples of the comparison between the calculated and measured time evolution of the heavy front position from the initial gate position, i.e. $x - x_0$.

Two tests carried out in the presence of the same reduced gravity $g' = 0.047 \text{ m/s}^2$ are considered, namely test S007 in the absence of surface waves (Figure 5a) and test W007 in the presence of a superimposed regular wave motion (Figure 5b). The results of both model A and model B are reported in Figure 5. It should be noted that due to the relatively short length of the flume, in the present work the gravity currents are analyzed only during the slumping or constant-velocity phase.

Calibration of the proposed numerical model has been carried out on the basis of heavy front evolution. Indeed, this gives a bulk measure of the gravity current dynamics, particularly during the constant-speed phase, and its measurements are highly reliable. Density and velocity field are extremely difficult to be measured

Fig. 5 Examples of heavy front position from initial gate section ($x - x_0$) over time. Results of experiments and numerical simulations, with turbulent closure A and B, are superimposed; (a) test S007 without surface waves, $C_s = 2.47$ for model A, $C_s = 2.00$ and $C_u = 0.008$ for model B; (b) test W007 with surface waves having $H = 0.02$ m, $T = 0.994$ s, $C_s = 0.75$ for model A, $C_s = 0.78$ and $C_u = 0.008$ for model B. In both tests $g' = 0.047$ m/s².

over the entire flow domain. Their behaviour is significantly unstable, with the presence of non negligible turbulent structures (as K-H billows).

The optimal value of C_s has been determined by minimizing the value of the relative error Δ , calculated as

$$\Delta = \frac{U_s - U_m}{U_m} \quad (25)$$

where U_s and U_m are the simulated and measured time-averaged velocities of the front of the gravity current respectively, obtained for different values of the C_s coefficient. In particular, by considering the full dataset reported in Table 1, Figure 6 (a) gives the errors obtained in the absence of waves, while Figure 6 (b) shows those obtained in the presence of waves. Similar plots are reported in Figure 7 for model B.

Although the phenomenon is strongly nonlinear, common trends exist and models A and B behave similarly, indicating also that the contribution of the depth-constant eddy viscosity is slightly beneficial, at least in the range of the analyzed values. First of all, as the value of C_s increases, the relative error decreases. Indeed, the larger the eddy viscosity, the smaller the predicted front velocity. If the simulated dissipation exceeds a certain threshold, the numerical model tends to underestimate the average front velocity and the relative error Δ becomes negative. In the present work, an optimal value of C_s has been determined as the value of C_s at which $\Delta = 0$. It should be noted that such an optimal value is a function of the flow conditions. Indeed, while in both models the optimal values of C_s are in the range 2 to 10 in the absence of waves, when the wave motion is superimposed to the gravity current the optimal values of C_s are smaller as well as their range ($C_s = 0.04 - 4$). Hence, in general, even if g' is similar, smaller values of C_s should

Fig. 6 Error Δ as function of the C_s parameter, obtained for the optimization of model A:

(a) tests without waves; (b) tests carried out in the presence of surface waves.

Fig. 7 Error Δ as function of the C_s parameter, obtained for the optimization of model B:

(a) tests without waves; (b) tests carried out in the presence of surface waves.

be used when waves are present. This could be an indication of the fact that when waves are superimposed to the current turbulence is reduced. In the latter case, an additional role is played by the characteristic of the wave motion. Indeed, the results indicate that for steeper waves the optimal value of C_s should be smaller.

Finally, in the presence of the smallest values of g' (e.g. tests S004 and W004, $Re \approx 2000$), i.e. of very slow gravity currents, there is no difference between model A and B. On the other hand, if the Reynolds number is larger than 2000, the inclusion of the depth-constant contribution to the eddy viscosity in model B leads to slightly smaller errors, if the same values of C_s are used in both models.

4 Analysis of results

The calibration procedure, described in Section 3, has been carried out considering only the heavy front position. Indeed, the eddy viscosity coefficients in the numerical model have been chosen in order to adequately reproduce the interface between the two fluids, particularly near the bottom of the flume. After such a calibration procedure, in the present section the numerical model outputs are used to describe the gravity current hydrodynamics by analyzing the distribution of density, velocity, vorticity, shear stresses and entrainment. Such variables are very difficult to measure during the experiments, but are readily available from the numerical solutions.

As explained in Section 2, model results during the initial acceleration stage after the opening of the gate may not be accurate, due also to the different experimental and numerical procedures used to trigger the combined flow. The focus of the present work is on the subsequent inertial constant-speed phase. Due to that,

numerical results on the evolution of the gravity currents are presented for $t > t_0$, where the time scale t_0 is estimated on the basis of the equations for a falling body, by considering the reduced gravity g' and a falling height $h/4$, which is equal to the vertical distance between the centroids of the heavy fluid, respectively at the instant $t = 0$ and during the slumping or constant-speed phase [9]:

$$t_0 = \sqrt{\frac{2(h/4)}{g'}} = \sqrt{\frac{h}{2g'}} \quad (26)$$

4.1 Density distribution

The time evolution of the spatial distribution of density ρ , which is the main forcing of gravity currents and which gives a significant indication of the characteristics of the gravity current hydrodynamics, may be qualitatively compared to experimental findings obtained by means of flow visualization [21].

Density distributions, calculated by the numerical model using the turbulence formulation A, are shown in Figures 8 and 9 for tests carried out without surface waves (test S007) and in the presence of a regular wave motion (test W007), respectively. Such tests have the same reduced gravity $g' = 0.047 \text{ m/s}^2$, thus the different behaviour is caused only by the superimposition of surface waves. The density distribution is shown at the gate section ($x = 7.0 \text{ m}$) in order to visualize both the heavy front and the light one, which are directed respectively onshore and offshore. The plots show the front during the constant-speed phase, i.e. for $t > t_0 = 1.5 \text{ s}$. The time interval between each plot is 2 s, which corresponds to two surface wave periods in the case of test W007.

The density distributions simulated by the model, both without and with surface waves, show the presence of humps at the nearly horizontal interface between

Fig. 8 Results of model A, test S007 without surface waves. Density distribution at four time steps: (a) $t = 1.8$ s, (b) $t = 1.8$ s + $2T$, (c) $t = 1.8$ s + $4T$, (d) $t = 1.8$ s + $6T$, where $2T \approx 2$ s.

Fig. 9 Results of model A, test W007 with surface waves having $H = 0.02$ m, $T = 0.994$ s.

Density distribution at four time steps: (a) $t = 1.8$ s, (b) $t = 1.8$ s + $2T$, (c) $t = 1.8$ s + $4T$, (d) $t = 1.8$ s + $6T$, where $2T \approx 2$ s.

the two fluids behind the steep front. The humps in the density distribution indicate the presence of turbulent structures which influence mixing between the two fluids, and whose size increases with time. Such a behaviour is evident in the absence of waves (see Figure 8 c and d), since in this case the horizontal front between the two fluids is characterized by the presence of alternating cells of high and low density close to the interface. It may be observed that the humps are larger in the rear part of both the heavy and the light fluid fronts. On the other hand, surface waves cause a reduction of the dimension of such structure between the fluids (see Figure 9). Indeed, in this case the mixing layer is smaller and more regular along the horizontal direction, and the dimension of the cells is influenced by the orbital velocity. In order to quantify such a phenomenon, the time-evolution of the maximum amplitude of the interface oscillations (i.e. humps) has been computed for the tests S007 and W007, which have the same reduced gravity. For the initially quiescent case S007 the highest hump over the time have an height equal to 0.085 m and its mean height is 0.045 m. In the presence of surface waves (test W007) the height of maximum humps is always lower than 0.07 m and its average value is in the range 0.030-0.035 m. From the above result, it can be argued that the presence of surface waves causes a 20-30% reduction of the maximum heights of interface oscillations. The result of the presence of waves reducing the thickness of the mixing layer is in agreement with visual observations of lock-exchange experiments reported in [21]. An additional quantitative investigation on mixing between the fluids is shown in Section 4.4.

4.2 Velocity and vorticity distribution

The analysis of the velocity and vorticity field may clarify what occurs at the interface between the two fluids during gravity current propagation, particularly focusing on the thickness of the mixing layer.

Figures 10 - 13 allow for a comparison of the velocity and the vorticity distribution in the absence (test S007) and in the presence of waves (test W007), by considering both turbulence closure models A and B, corresponding to the eddy viscosity formulations reported in eqs. (22) and (23) respectively. In each of such Figures the velocity field and vorticity distribution are plotted at four time steps during two wave periods. The position of the interface between the two fluids is also reported in the Figures. Such an interface is defined as the line along which the density is equal to the mean value $(\rho_0 + \rho_1)/2$. In each plot the absence of contour lines characterizes positive counterclockwise vorticity, while dotted curves refer to negative clockwise vorticity.

It is possible to identify several features of lock-exchange hydrodynamics which are not dependent on the presence of waves, since they also take place in the absence of waves: the maximum values of counterclockwise vorticity are always located across the interface between the two fluids. Such a primary mechanism is more intense just downstream of the two steep fronts and it is weaker at the center of the domain, where the gate is initially located. The reduction of vorticity is not uniform, due to the development of Kelvin-Helmholtz (K-H) billows, at both the heavy and light fronts. Indeed such K-H billows propagate in opposite directions with respect to each front, gradually losing their intensity. Such an effect causes the oscillation of the fronts, highlighted also in the experiments [21, 24].

Fig. 10 Results of model A, test S007 without surface waves. Vorticity distribution (gray scale with dotted contours for negative values), velocity fields (vectors) and front (separation between heavy and light fluid, represented by the thick line), at four time steps: (a) $t = 1.8$ s; (b) $t = 1.8$ s + $2T$; (c) $t = 1.8$ s + $4T$; (d) $t = 1.8$ s + $6T$ ($2T \approx 2$ s).

Fig. 11 Results of model B, test S007 without surface waves. Vorticity distribution (gray scale with dotted contours for negative values), velocity fields (vectors) and front (separation between heavy and light fluid, represented by the thick line), at four time steps: (a) $t = 1.8$ s; (b) $t = 1.8$ s + $2T$; (c) $t = 1.8$ s + $4T$; (d) $t = 1.8$ s + $6T$ ($2T \approx 2$ s).

Fig. 12 Results of model A, test W007 with surface waves having $H = 0.02$ m, $T = 0.994$ s. Vorticity distribution (gray scale with dotted contours for negative values), velocity fields (vectors) and front (separation between heavy and light fluid, represented by the thick line), at four time steps: (a) $t = 1.8$ s; (b) $t = 1.8$ s + $2T$; (c) $t = 1.8$ s + $4T$; (d) $t = 1.8$ s + $6T$ ($2T \approx 2$ s).

Fig. 13 Results of model B, test W007 with surface waves having $H = 0.02$ m, $T = 0.994$ s. Vorticity distribution (gray scale with dotted contours for negative values), velocity fields (vectors) and front (separation between heavy and light fluid, represented by the thick line), at four time steps: (a) $t = 1.8$ s; (b) $t = 1.8$ s + $2T$; (c) $t = 1.8$ s + $4T$; (d) $t = 1.8$ s + $6T$ ($2T \approx 2$ s).

The numerical results show also a secondary effect of the lock-exchange mechanism, which is perceptible both in the absence and in the presence of waves and which is difficult to notice in the experimental data. A negative clockwise vorticity cell develops ahead of each front. Such a cell is quite stable in time and it counterbalances the primary counterclockwise cell located on the front. The two cells, having positive and negative vorticity respectively, show similar dimensions but the latter secondary cell is characterized by absolute values of vorticity always lower than those of the primary cell.

The above described vorticity dynamics is general since it is not affected by the presence of surface waves and by the adopted turbulence closure formulation. However superposition of the wave motion to the gravity current propagation induces a modification of the shape of both primary-positive and secondary-negative vorticity cells. On the one hand the counterclockwise vorticity cells are more clustered across the interface between the two fluids for test W007 compared to test S007. On the other hand the clockwise cells are stretched under surface waves and they tend to occupy the extreme part of the water column which is not covered by the primary counterclockwise vortices. Both effects cause a reduction of the size of vorticity cells, which is not counterbalanced by an increase of vorticity magnitude within such cells. Thus the presence of surface waves always causes a decrease in the total amount of vorticity. Since turbulence is highly related to vorticity, it could be argued that the presence of surface waves causes a reduction of turbulence intensity at the interface between heavy and light fluids. Such an effect is similar to what happens in other flows characterized by wave current interaction [14,19]. Furthermore, the presence of the waves reduces the front velocity. Such a

reduction, as well as the reduction of the intensity of vorticity, is an indicator of the smaller buoyancy net effect induced by the wave orbital motion.

Concerning the two adopted eddy viscosity formulations, the difference between the results obtained using model A and model B is negligible in the absence of waves, as it can be noticed from the comparison between Figure 10 and 11. Also the comparison between the two models in the presence of waves (see Figures 12 and 13) shows that the hydrodynamics simulated by the two models are qualitatively similar. Nevertheless, in such cases the values of vorticity given by model A are slightly greater than those computed by model B. Furthermore, model B allows a better matching of numerical results with experimental measurements, as it has been already shown when studying the front position, in Figure 5.

The intra-wave effect of the orbital motion on the gravity current hydrodynamics can be investigated considering Figures 14 and 15, in which the vorticity and velocity field are plotted at each $T/4$ during a wave cycle, for turbulence closure models A and B respectively.

In both formulations the dimension and the position of the positive vorticity cells have a weak dependency on the wave phases and are clustered at the interface between the two fluids. On the other hand the secondary counterclockwise vorticity cell located ahead of the heavy front at the bottom tends to be stretched and it is forced to enter the region between the positive cells and the free surface, when the wave crest passes over the positive cells. This phenomenon can be observed in plot (a) of both Figures 14 and 15. Conversely, the negative vorticity cell located at the left of the domain (within the heavy fluid) tends to touch the bottom when the wave trough passes over it and it goes up when the wave crest appears.

Fig. 14 Results of model A, test W007 with surface waves having $H = 0.02$ m, $T = 0.994$ s. Vorticity distribution (gray scale with dotted contours for negative values), velocity fields (vectors) and front (separation between heavy and light fluid, represented by the thick line), at four time steps spaced by $T/4$: (a) $t = 5.8$ s; (b) $t = 5.8$ s + $T/4$; (c) $t = 5.8$ s + $T/2$; (d) $t = 5.8$ s + $3/2T$.

Fig. 15 Results of model B, test W007 with surface waves having $H = 0.02$ m, $T = 0.994$ s. Vorticity distribution (gray scale with dotted contours for negative values), velocity fields (vectors) and front (separation between heavy and light fluid, represented by the thick line), at four time steps spaced by $T/4$: (a) $t = 5.8$ s; (b) $t = 5.8$ s + $T/4$; (c) $t = 5.8$ s + $T/2$; (d) $t = 5.8$ s + $3/2T$.

As for the difference between the two turbulence formulations, it is possible to notice that the results of model A, shown in Figure 14, give greater positive vorticity values with respect to the ones of model B, shown in Figure 15. Conversely, the behaviour of negative vorticity cells is not influenced by turbulence closure formulation. Such an effect is probably related to the fact that turbulence affects more the gravity currents and less the wave motion, at least in the case of non-breaking waves, as the ones in the present work.

4.3 Shear stresses

As noticed when discussing the results in Figures 12 and 13, the presence of the waves induces a general reduction of the vorticity intensity. This turns into a reduction of the intensity of the shear stresses along the water column.

The spatial distribution of the shear stresses τ has been derived by the velocity field. In particular, here, just the contribution of the gravity current has been estimated as

$$\frac{\tau}{\rho} = (\nu + \nu_t) \left(\frac{\partial u_d}{\partial z} + \frac{\partial w_d}{\partial x} \right) \quad (27)$$

where ρ is the local value of density.

The spatial distribution of the shear stresses, not shown here for the sake of brevity, reflects that of the vorticity distribution. Indeed, large negative shear stresses are clustered in cells along the interface between the two fluids. Isolated cells of positive shear stresses are located in regions having clockwise vorticity. In general, the intensity of the shear stresses is reduced in the presence of surface waves. In order to provide an overall estimate of such a reduction, Figure 16 shows

the space-time-averaged value $\overline{\tau/\rho}$ as a function of the root square of the Grashof number of the tests, which represents the turbulence levels of the gravity current.

The results of models A and B are similar. Indeed, in both cases it may be noticed that: (i) $\overline{\tau/\rho}$ increases with a parabolic law as the Grashof number increases; (ii) in the presence of waves, the values of $\overline{\tau/\rho}$ are smaller. Moreover, the shear stresses computed by model B under surface waves are more regular than those of model A, since they are close to the interpolation function.

Once again, the latter results indicate that the combined flow undergoes a reduction of shear stresses induced by the waves, as observed in [14,19]. Such an effect increases as the Grashof number increases.

Finally, as before, the results obtained by the two turbulence models collapse for the undisturbed gravity current propagation. If a wave motion is added to the propagation of the current, model B predicts appreciably larger average shear stresses. Such behaviour is a confirmation that the turbulence enhancement introduced by the depth-uniform contribution to the eddy viscosity plays a role only in the presence of the orbital motion, which contributes to stirring up momentum.

4.4 Entrainment

In order to give a quantitative estimate of mixing between heavy and ambient fluid, the results of the optimized model (turbulence closure B) have been used to calculate the entrainment coefficient E , by following the procedure shown in [21]. In particular, E is defined as the ratio between the bulk velocity of the ambient fluid entering into the heavy fluid (w_e) and the relative horizontal bulk velocity of

Fig. 16 Specific mean shear stress $\overline{\tau}/\rho$ as a function of the square root of the Grashof number Gr : (a) results of model A; (b) results of model B.

the heavy and light front:

$$E = \frac{w_e}{u_s - u_l} \quad (28)$$

where u_s and u_l are the velocities of the simulated heavy and light front, computed, respectively, by dividing the position from the gate of the heavy and of the light front by the time t . All the velocities considered in eq. (28) are bulk-averaged at a given time step. The velocity w_e is evaluated by considering the volume of ambient fluid V mixed with heavy fluid. Such a volume is computed by subtracting the initial volume of heavy fluid from the total volume below the interface between the two fluids at time t . To this aim, in this case the interface is defined as that part of the domain in which the concentration $C = (\rho - \rho_0)/(\rho_1 - \rho_0)$ is equal to 0.02. Therefore, the entrained velocity w_e is obtained as the ratio between the bulk entrained discharge V/t and the width of the flume multiplied by the length of the interface between the two fluids.

The time evolution of the entrainment coefficient E is shown in Figure 17, for quiescent ambient fluid tests and wave tests having similar g' . It is possible to notice that mixing is always very intense at the initial stages and afterwards tends to reach a steady condition. In the tests this occurs 6 s after the opening of the gate.

The most important result shown in Figure 17 is that the presence of surface waves causes generally an increment of entrainment and consequently of mixing between the two fluids. In particular, such an increase of entrainment, due to waves, is reduced in time and it is, on average, 33% for $t > 6$ s.

A physical justification of the increment of entrainment under surface waves, notwithstanding the reduction of turbulence, vorticity and shear stresses, is related

Fig. 17 Time evolution of the entrainment coefficient E for tests carried out in the absence of waves and with various wave slope H/L . $g' \approx 0.05 \text{ m/s}^2$.

to the presence of the orbital motion under surface waves. Indeed, the vertical component w_B of the orbital velocity has non-zero values across the interface between the two fluids, thus increasing the amount of ambient fluid entrained into the heavy fluid.

The increment of wave slope (H/L) causes greater instabilities in the entrainment coefficient, since mixing is more influenced by the smaller wave trajectories and faster orbital velocities. Quantitatively, the wave slope does not have a linear effect on entrainment computed at the end of simulations ($t > 12 \text{ s}$), i.e. in steady conditions. In particular, in the tested conditions such a final entrainment reaches its minimum value, about 0.045, in the absence of waves (test S007). The

maximum value of entrainment at the end of simulation is reached for the test W109, having the steepest surface waves ($H/L = 0.06$). Such an upper limit of E oscillates around the value 0.07. All the other surface wave tests, having lower wave slopes, causes entrainment coefficients which follow between the two limits discussed above.

5 Conclusions

The presence of surface waves causes several effects on gravity currents. In particular, the present work analyzes the wave influence on: shape of the interface between the two fluids, mixing, turbulence, velocity and vorticity fields. Such aspects are studied by means of an efficient modeling approach, which uses two separated models for surface wave and gravity current propagation respectively. In such a modeling system the total velocity is decoupled into a wave-related component and a density-gradient-related component. The latter accounts for the presence of the gravity currents. Turbulence is described by two alternative approaches, based on the eddy viscosity concept: the first approach considers a simple subgrid Smagorinsky formulation (model A); the second one contains both the Smagorinsky formulation and a depth-uniform eddy viscosity (model B).

Calibration of the model was carried out with experimental lock exchange tests, proving that the inclusion of the depth-uniform eddy viscosity into a Smagorinsky formulation (i.e. model B) is beneficial, particularly under combined gravity current-surface wave flow, when Reynolds number of gravity current is larger than 2000.

The choice of the calibration parameter C_s , related to the Smagorinsky formulation, is a critical point for the proposed modeling system. This is mainly related to the simplicity of the model, which has been developed as an engineering-oriented tool. For such a reason, a great attention has been paid to the calibration of the model before its application for investigating the effects of surface waves on lock-exchange hydrodynamics. Further works will be focused on a physically-based optimization of the value of C_s and on its effects on the results.

The results of the model showed a reduction in the average velocity of the front under surface waves, in accordance to the experimental outcomes. Also the front oscillations under waves evaluated by the model are similar to those oscillations that are measured. The evolution of the density distribution over time highlighted that the presence of waves causes a reduction of irregularities at the interface between the two fluids. The vorticity fields showed the existence of a region of positive counterclockwise vorticity along the interface. An overall decrease of the total amount of vorticity was registered if the gravity current propagated in the presence of surface waves. Concurrently, a significant reduction of the turbulent shear stresses was observed under waves, which reaches values of about 37% (for model B), at the greatest tested Grashof number, i.e. for $\sqrt{Gr} = 3 \cdot 10^4$.

The reduction in the amount of turbulence and vorticity of gravity current, due to waves, would lead us to presume that mixing is less intense under waves. Nevertheless the estimation of entrainment highlights that the surface orbital motion causes an increase of mixing. Therefore such a phenomenon is not only related to the turbulence of the gravity current, but also to the wave motion itself. In detail, due to the vertical component of orbital velocity lighter fluid particles from the higher region cross the interface, thus forcing the mixing between the two fluids.

Future work will be aimed at extending the range of wave conditions, supported also by new experimental tests, in order to better simulate the interaction of surface waves and turbulent structure at the front.

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